# A multivariable passivity based control for an electropneumatic actuator

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# A MULTIVARIABLE PASSIVITY BASED CONTROL FOR AN ELECTROPNEUMATIC ACTUATOR

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*This paper develops a systematic methodology for the control of a class of nonlinear systems and applies it to electropneumatic system. It deals with multiple input – multiple ouput (MIMO) systems in the strict feedback form. The approach is conceptually similar to previously developed integrator backstepping methodologies. However, unlike some previous investigations which have relied exclusively on a Lyapunov analysis, this work presents a stability analysis using a passivity formulation. First, the nonlinear model of the electropneumatic system is presented. A class of modeling error is introduced and compensated for with the resulting control able to guarantee specified boundary layer tracking. Then, the control algorithm is implemented on the pneumatic system. Finally, experimental results are presented and discussed.*

**Key word: Pneumatic Systems, Passivity Based Control, Experimental Results.**

## 1 INTRODUCTION

Pneumatic control systems play important role in industrial automation due to their relatively small size, light weight, and high speed. One of the conspicuous trends is the need for the electropneumatic systems that can achieve precise tracking position control. In recent years, research efforts have been directed toward meeting this requirement. Most of them have been in the field of feedback linearization, **Bobrow** (1998). However, reasonably accurate mathematical models for the pneumatic system are required by the feedback linearization. A number of investigations have been conducted on fuzzy control algorithms, **Li Ruihua** (2004), adaptive control, **Errahimi** (2002), **Di Zhou** (2003), backstepping control, **Smaoui** (2006), sliding mode control, **Laghrouche** (2006) and robust linear control, **Mattei** (2001). Passivity based control is a generic name given to a family of controller design techniques that achieve the control of  objective via the route of passivation, that is, rendering the closed-loop system passive with a desired storage function (that usually qualifies as a Lyapunov function for the stability analysis). See the fundamental book, **Ortega** (1998) and **Brogliato** (2007). Passivity based control, have two main advantages of the proposed approach which become significant during implementation. One practical advantage is that the resulting controller leads to synthetic inputs that are decoupled in a certain sense. This leads to a compartmentalization of modeling error effects associated with the controller. A second advantage of this method is that the system model need not be differentiated in the control formulation, **Andrew** (2000). In this paper the model of the electropneumatic system has been presented and equations governing the motion of this plant have been put in a nonlinear affine form. Then, a systematic methodology for the control of a class of nonlinear systems is proposed. Finally, the control algorithm is implemented on the pneumatic system**.**

## 2 ELECTROPNEUMATIC SYSTEM MODEL

The considered system Fig. 1 is a linear inline double acting electropneumatic servo-drive using a single rod controlled by two three-way servodistributors. The actuator rod is connected to one side of the carriage and drives an inertial load on guiding rails. The total moving mass is *17 kg*.
The electropneumatic system model can be obtained using three physical laws: the mass flow rate through a restriction, the pressure behavior in a chamber with variable volume and the equation of motion.

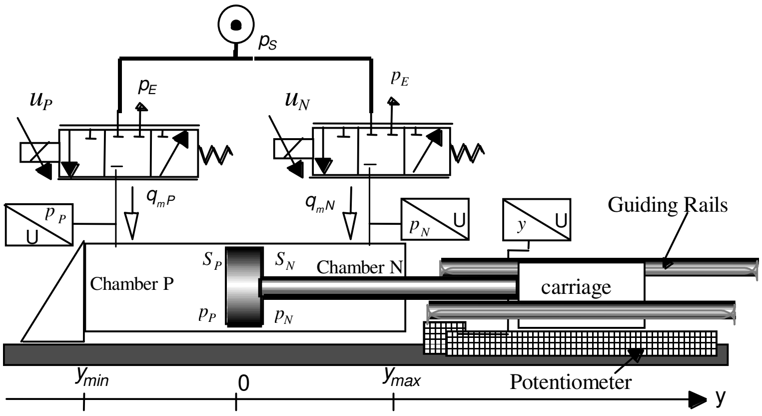


**Fig.1.** The electropneumatic system

The pressure evolution law in a chamber with variable volume is obtained via the following assumptions, **Shearer** (1956): i) air is a perfect gas and its kinetic energy is negligible; ii) the pressure and the temperature are supposed to be homogeneous in each chamber; iii) the process is polytropic and characterized by coefficient *k*. Moreover, the electropneumatic system model is obtained by combining all the previous relations and assuming that the temperature variation is negligible with respect to average and equal to the supply temperature. . Furthermore, we neglect the dynamics of the servodistributors. In such case, the servodistributors model can be reduced to two static relations between the mass flow rates $q_{m^P}(u_P, p_P)$ and $q_{m^N}(u_N, p_N)$,where $u$ is the input voltage, $p_P$ and $p_N$ are the output pressures.
The mechanical equation includes pressure force, friction and an external constant force due to atmospheric pressure. The following equation gives the model of the above system:

$$\begin{cases} \dfrac{dy}{dt} = v \\ \dfrac{dv}{dt} = \dfrac{1}{M}\left[S_P p_P - S_N p_N - bv - F_{ext}\right] \\ \dfrac{dp_P}{dt} = \dfrac{krT}{V_P(y)}\left[q_{m^P}(u_P, p_P) - \dfrac{S_P}{rT} p_P v\right] \\ \dfrac{dp_N}{dt} = \dfrac{krT}{V_N(y)}\left[q_{m^N}(u_N, p_N) + \dfrac{S_N}{rT} p_N v\right] \end{cases} \tag{1}$$

where: $\begin{cases} V_P(y) = V_P(0) + S_P y \\ V_N(y) = V_N(0) - S_N y \end{cases}$ with: $\begin{cases} V_p(0) = V_{DP} + S_P \dfrac{l}{2} \\ V_N(0) = V_{DN} + S_N \dfrac{l}{2} \end{cases}$

are the effective volumes of the chambers for the zero position and $V_{D[P\,or\,N]}$ are dead volumes present at each end of the cylinder.

The main difficulty for model (1) is related to the knowledge of the mass flow rates $q_{mP}$ and $q_{mN}$. In order to establish a mathematical model of the power modulator flow stage, some research works present approximations based on physical laws, **Araki** (1981) by modeling of the geometrical variations of the restriction areas of the servodistributor. Some authors presented an experimental-based characterization model, **Richard** (1996).
In this paper, the results of the global experimental method giving the static characteristics of the flow stage, **Sesmat** (1996) have been used. The global characterization corresponds to the static measurement of the output mass flow rate $q_m$, which depends on the input control *u* and the output pressure *p,* for constant source and exhaust pressure. The global characterization has the advantage of obtaining simply, by projection of the characteristic series $q_m(u, p)$ on different planes:

- The mass flow rate characteristics series (plane $p - q_m$)
- The mass flow gain characteristics series (plane $u - q_m$)
- The pressure gain characteristics series (plane $u - p$)

The authors in **Belgharbi** (1996) have developed analytical models for both simulation and control purposes. The flow stage characteristics were approximated characteristics by polynomial functions affine in control such that:

$$q_m(u, p) = \varphi(p) + \psi(p, sgn(u))\, u \tag{2}$$

where $\psi(.) > 0$ over the physical domain.
From the equation (2) the nonlinear affine model is then given by equation:

$$\begin{cases} \dfrac{dy}{dt} = v \\ \dfrac{dv}{dt} = \dfrac{1}{M}\left[S_P p_P - S_N p_N - bv - F_{ext}\right] \\ \dfrac{dp_P}{dt} = \dfrac{krT}{V_P(y)}\left[\varphi(p_P) - \dfrac{S_P}{rT} p_P v\right] + \dfrac{krT}{V_P(y)}\psi(p_P, \mathrm{sgn}(u_P))u_P \\ \dfrac{dp_N}{dt} = \dfrac{krT}{V_N(y)}\left[\varphi(p_N) + \dfrac{S_N}{rT} p_N v\right] + \dfrac{krT}{V_N(y)}\psi(p_N, \mathrm{sgn}(u_N))u_N \end{cases} \tag{3}$$

With two inputs $u_P$ and $u_N$, the nonlinear model of the system in Fig. 1 has the following form:

$$\dot{x} = f(x) + g(x) \times U \tag{4}$$

where $x$, $f(x)$ and $g(x)$ in $R^4$, $u$ in $R$, and:

$$x = (y, v, p_P, p_N)^T \tag{5}$$

with:

$$f(x) = \begin{pmatrix} v \\ \dfrac{1}{M}\left[S_P p_P - S_N p_N - bv - F_{ext}\right] \\ \dfrac{krT}{V_P(y)}\left[\varphi(p_P) - \dfrac{S_P}{rT} p_P v\right] \\ \dfrac{krT}{V_N(y)}\left[\varphi(p_N) + \dfrac{S_N}{rT} p_N v\right] \end{pmatrix} \tag{6}$$

$$g(x)=\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{krT}{V_P(y)}\psi(p_P,\operatorname{sgn}(u_P)) & 0 \\ 0 & \frac{krT}{V_N(y)}\psi(p_N,\operatorname{sgn}(u_N)) \end{pmatrix} \qquad (7)$$

$$U=(u_P,u_N)^T \qquad (8)$$

The system uses two three-way proportional servodistributors. Generally, it is supposed that these two servodistributors are equivalent to one five-way proportional servodistributor when they are controlled with the inputs of opposite signs, **Brun** (2002). In this case**,** a monovariable position control law can be established. However the validity of the control law depends on the stability of the unobservable subsystem, which is one-dimensional.

With a system of two three-way servodistributors, it is possible to control two different trajectories. For example, it seems useful to control position and pressure without a degradation of the desired specifications (tracking position). Let us define $h(\underline{x})$ the vector consisting of the two chosen outputs: position and pressure in chamber *P:*

$$h(x)=\begin{pmatrix} h_1(x) \\ h_2(x) \end{pmatrix}=\begin{pmatrix} y \\ p_P \end{pmatrix} \qquad (9)$$

In the section 4 we will develop the passivity control law using (3).

## 3 A SYSTEMATIC CONTROLLER DESIGN

Passivity based control, **Ortega** (1998) is a recursive procedure, which enables a control law to be derived for a nonlinear system, associated to the appropriate Lyapunov function, which guaranties passivity. The class of systems to be studied in this work is systems can be transformed into the strict feedback form shown in (10):

$$\begin{array}{lcl}
\dot{x}_1 & = & f_1(x_1)+g_{1,1}(x_1)x_2+g_{1,2}(x_1)x_3+\ldots+g_{1,(s-1)}(x_1)x_s \\
\dot{x}_2 & = & f_2(x_1,x_2)+g_{2,1}(x_1,x_2)x_{s+1}+g_{2,2}(x_1,x_2)x_{s+2}+\ldots+g_{2,l}(x_1,x_2)x_{s+l} \\
\vdots & & \\
\dot{x}_m & = & f_m(x_1,\ldots,x_m)+g_{m,1}(x_1,\ldots,x_m)x_{s+l+z}+\cdots+g_{m,s+l+z+n}(x_1,\ldots,x_m)x_{m+j} \\
\dot{x}_{m+1} & = & f_{m+1}(x_1,\ldots,x_{m+j})+g_{m+1,1}(x_1,\ldots,x_{m+j})u_1+g_{m+1,2}(x_1,\ldots,x_{m+j})u_2+\cdots+g_{m+1,j}(x_1,\ldots,x_m)x_{m+j} \\
\vdots & & \\
\dot{x}_{m+j-1} & = & f_{m+j-1}(x_1,\ldots,x_{m+j})+g_{m+j-1,1}(x_1,\ldots,x_{m+j})u_{j-1}+g_{m+j-1,2}(x_1,\ldots,x_{m+j})u_j \\
\dot{x}_{m+j} & = & f_{m+j}(x_1,\ldots,x_{m+j})+g_{m+j}(x_1,\ldots,x_{m+j})u_j \\
z & = & h(x_1,\ldots,x_{m+j})
\end{array} \qquad (10)$$

Where $x_i,\ \forall i\in[1, \quad m+j]$, $z$ et $u_i,\ \forall i\in[1 \quad \ldots \quad j]$ are states, outputs and inputs of system.
$s$ is the number of state who the first equation of state is depending.
$l$ is the number of state who the second equation of state is depending.
$n$ is the number of state who the $m^{th}$ equation of state is depending.
$z$ is the number of state who the $(m-1)^{th}$ equation of state is depending.
$m$ is the number of equation which depends only on the states (and not of the inputs).
$j$ is the number of equation which depends on the states and the inputs.
$g_i(x)\neq 0\ \forall x,\ \forall i\in[1, \quad m+j]$.

Suppose the output of the system, $z$, is to track some desired value of $z$ and the tracking error is defined as $e = z - z_d$.

Create $m + j$ separate error dynamics as follow:

$$e_i = x_i - x_{id} \qquad \forall i \in [1 \qquad m + j] \tag{11}$$

Differentiating each error equation in (11) once gives:

$$\begin{aligned}
\dot{e}_1 &= \dot{x}_1 - \dot{x}_{1d} &&= f_1 + g_{1,1}x_2 + \ldots + g_{1,s-1}x_s - \dot{x}_{1d} \\
&\vdots \\
\dot{e}_m &= \dot{x}_m - \dot{x}_{md} &&= f_m + g_{m,1}x_{s+l+z} + \ldots + g_{m,s+l+z+n}x_{m+j} - \dot{x}_{md} \\
\dot{e}_{m+1} &= \dot{x}_{m+1} - \dot{x}_{(m+1)d} &&= f_{m+1} + g_{m+1,1}u_1 + \ldots + g_{m+1,j}u_j - \dot{x}_{(m+1)d} \\
&\vdots \\
\dot{e}_{m+j} &= \dot{x}_{m+j} - \dot{x}_{(m+j)d} &&= f_{m+j} + g_{m+j}u_j - \dot{x}_{(m+j)d}
\end{aligned} \tag{12}$$

Equation (12) may be written as:

$$\begin{aligned}
\dot{e}_1 &= f_1 + g_{1,1}e_2 + g_{1,1}x_{2d} + \ldots + g_{1,s-1}e_r + g_{1,s-1}x_{sd} - \dot{x}_{1d} \\
&\vdots \\
\dot{e}_m &= f_m + g_{m,1}e_{s+l+z} + g_{m,1}x_{(s+l+z)d} + \ldots + g_{m,s+l+z+n}e_{m+j} + g_{m,s+l+z+n}x_{(m+j)d} - \dot{x}_{md} \\
\dot{e}_{m+1} &= f_{m+1} + g_{m+1,1}u_1 + \ldots + g_{m+1,j}u_j - \dot{x}_{(m+1)d} \\
&\vdots \\
\dot{e}_{m+j} &= f_{m+j} + g_{m+j}u_j - \dot{x}_{(m+j)d}
\end{aligned} \tag{13}$$

Define the desired values of the states and inputs of system as:

$$\begin{aligned}
x_{2d} &= \frac{1}{g_{1,1}}\left[-\frac{f_1}{s-1} + \frac{\dot{x}_{1d}}{s-1} - k_{1,1}e_1\right] \\
&\vdots \\
x_{sd} &= \frac{1}{g_{1,(s-1)}}\left[-\frac{f_1}{s-1} + \frac{\dot{x}_{1d}}{s-1} - k_{1,(s-1)}e_1\right] \\
&\vdots \\
x_{(s+l+z)d} &= \frac{1}{g_{m,1}}\left[-\frac{f_m}{s+l+z+n} + \frac{\dot{x}_{md}}{s+l+z+n} - k_{m,1}e_m\right] \\
&\vdots \\
x_{(m+j)d} &= \frac{1}{g_{m,(s+l+z+n)}}\left[-\frac{f_m}{s+l+z+n} + \frac{\dot{x}_{md}}{s+l+z+n} - k_{m,(s+l+z+n)}e_m\right] \\
u_j &= \frac{1}{g_{m+j}}\left[-f_{m+j} + \dot{x}_{(m+j)d} - k_{m+j}e_{m+j}\right] \\
u_{j-1} &= \frac{1}{g_{(m+j-1),1}}\left[-f_{m+j-1} - g_{(m+j-1),2}u_j + \dot{x}_{(m+j-1)d} - k_{m+j-1}e_{m+j-1}\right] \\
&\vdots \\
u_1 &= \frac{1}{g_{(m+1),1}}\left[-f_{m+1} - g_{(m+1),2}u_2 - \ldots - g_{(m+1),j}u_j + \dot{x}_{(m+)d} - k_{m+1}e_{m+1}\right]
\end{aligned} \tag{14}$$

Subsisting (14) into (13) leads to a chain of interconnected error dynamics:

$$\begin{aligned}
\dot{e}_1 &= -k_{1,1}e_1 + g_{1,1}e_2 + \ldots - k_{1,(s-1)}e_1 + g_{1,(s-1)}e_r \\
&\vdots \\
\dot{e}_m &= -k_{m,1}e_m + g_{m,1}e_{s+l+z} + \ldots - k_{m,(s+l+z+n)}e_m + g_{m,(s+l+z+n)}e_{m+j} \\
\dot{e}_{m+1} &= -k_{m+1}e_{m+1}^2 \\
&\vdots \\
\dot{e}_{m+j} &= -k_{m+j}e_{m+j}^2
\end{aligned} \tag{15}$$

Now, consider the following positive definite storage function:

$$\phi_i = \frac{1}{2}e_i^2 \qquad \forall i \in [1 \qquad m+j] \tag{16}$$

Differentiating equation (16) gives:

$$\begin{aligned}
\dot{\phi}_1 &= -k_{1,1}e_1^2 + g_{1,1}e_2e_1 + \ldots - k_{1,(s-1)}e_1^2 + g_{1,(s-1)}e_r e_1 \\
&\vdots \\
\dot{\phi}_m &= -k_{m,1}e_m^2 + g_{m,1}e_{s+l+z}e_m + \ldots - k_{m,(s+l+z+n)}e_m^2 + g_{m,(s+l+z+n)}e_{m+j}e_m \\
\dot{\phi}_{m+i} &= -k_i e_i^2 \qquad \forall i \in [1 \quad j]
\end{aligned} \tag{17}$$

Consider $g_i e_{i+1}$ as the input, and $e_i$ as the output. And rearranging (17) result in:

$$\begin{aligned}
\underbrace{\left(g_{1,1}e_2 + \ldots + g_{1,(s-1)}e_s\right)}_{Input}\underbrace{e_1}_{output} &= \dot{\phi}_1 + \underbrace{\left(k_{1,1}e_1^2 + \ldots + k_{1,(s-1)}e_1^2\right)}_{\geq 0} \\
\vdots \qquad & \vdots \quad \vdots \\
\underbrace{\left(g_{m,1}e_{s+l+z} + \ldots + g_{m,(s+l+z+n)}e_{m+j}\right)}_{Input}\underbrace{e_m}_{output} &= \dot{\phi}_m + \underbrace{\left(k_{m,1}e_m^2 + \ldots + k_{m,(s+l+z+n)}e_m^2\right)}_{\geq 0}
\end{aligned} \tag{18}$$

Therefore, it is evident that the relationship between $e_i$ and $e_{i+1}$ is strictly passive and hence BIBO (Bounded Input Bounded Output) stable for any $i \in [1 \qquad m]$. The serial interconnection of strictly passive elements is also strictly passive, Andrew (2001). This implies that the relationship between $e_i$ and $e_{i+1}$ is strictly passive and hence BIBO stable $\forall i \in [1 \qquad m]$.

From the $(m+1)$th till $(m+j)$th error dynamics equation:

$$\dot{e}_{m+i} = -k_{m+i}e_{m+i} \qquad \forall i \in [1 \qquad j] \tag{19}$$

Since $k_{m+i} > 0, \quad \forall i \in [1 \qquad j]$, errors $e_{m+1}$ till $e_{m+j}$ converges exponentially to zero with convergence rate $k_{m+i}$. Therefore, $e_i \to 0$ as $t \to \infty$, $\forall i \in [1 \qquad m+j]$, since the input forcing function $e_{m+1}$ to the chain of strictly passive errors dynamics in (13) decays exponentially to zero.

# 4 APPLICATION TO ELECTROPNEUMATIC SYSTEM

The relative degree associated to the position and the pressure are, respectively, three and one. Thus the sum is equal to the dimension of the system. This is sufficient to affirm that the system is differentially flat. In order to use passivity based control, a coordinate transformation for the pneumatic system (3) is proposed as follow as:

$$
\begin{cases}
\dot{y} &= v \\
\dot{v} &= a \\
\dot{a} &= f_a + g_{a1}U_p + g_{a2}U_n \\
\dot{P}_p &= f_{P_p} + g_{P_p}U_p \\
z &= \begin{bmatrix} y & P_p \end{bmatrix}^T
\end{cases}
\tag{20}
$$

Where

$$
\begin{aligned}
f_a &= \frac{S_p k\, rT}{MV_p(y)}\left[\varphi(p_p) - \frac{S_p}{rT} p_p v\right] - \frac{S_n k\, rT}{MV_n(y)}\left[\varphi(p_n) + \frac{S_n}{rT} p_n v\right] - \frac{b}{M} a \\
g_{a1} &= \frac{S_p k\, rT}{MV_p(y)} \psi(p_p, \quad sign(u_p)) \\
g_{a2} &= -\frac{S_n k\, rT}{MV_n(y)} \psi(p_n, \quad sign(u_n)) \\
f_{P_p} &= \frac{k\, rT}{V_p(y)}\left[\varphi(p_p) - \frac{S_p}{rT} p_p v\right] \\
g_{P_p} &= \frac{k\, rT}{V_p(y)} \psi(p_p, \quad sign(u_p))
\end{aligned}
\tag{21}
$$

Create four separate error dynamics as follow:

$$
\begin{aligned}
e_y &= y - y_d \\
e_v &= v - v_d \\
e_a &= a - a_d \\
e_P &= p_p - p_{pd}
\end{aligned}
\tag{22}
$$

Differentiating each error equation in (22) once gives:

$$
\begin{aligned}
\dot{e}_y &= \dot{y} - \dot{y}_d = f_y + g_y v - \dot{y}_d \\
\dot{e}_v &= \dot{v} - \dot{v}_d = f_v + g_v a - \dot{v}_d \\
\dot{e}_a &= \dot{a} - \dot{a}_d = f_a + g_{a1}U_p + g_{a2}U_n - \dot{a}_d \\
\dot{e}_{P_p} &= \dot{P}_p - \dot{P}_{pd} = f_{P_p} + g_{P_p}U_p - \dot{P}_{pd}
\end{aligned}
\tag{23}
$$

Equation (23) may be written as:

$$
\begin{aligned}
\dot{e}_y &= f_y + g_y e_v + g_y v_d - \dot{y}_d \\
\dot{e}_v &= f_v + g_v e_a + g_v a_d - \dot{v}_d \\
\dot{e}_a &= f_a + g_{a1}U_p + g_{a2}U_n - \dot{a}_d \\
\dot{e}_{P_p} &= f_{P_p} + g_{P_p}U_p - \dot{P}_{pd}
\end{aligned}
\tag{24}
$$

Define the desired values of the states and the input of system as:

$$\begin{aligned} v_d &= \dot{y}_d - k_y e_y \\ a_d &= \dot{v}_d - k_v e_v \\ u_p &= \frac{1}{g_{P_p}}\left(-f_{P_p} + P\dot{p}_{pd} - k_{P_p} e_P\right) \qquad k_y, k_v\ k_a\ et\ k_{P_p} > 0 \\ u_n &= \frac{1}{g_{a2}}\left(-f_a - g_{a1}u_p + \dot{a}_d - k_a e_a\right) \end{aligned} \tag{25}$$

Subsisting (25) into (24) leads to a chain of interconnected error dynamics:

$$\begin{aligned} \dot{e}_y &= -k_y e_y + g_y e_v \\ \dot{e}_v &= -k_v e_v + g_v e_a \\ \dot{e}_a &= -k_a e_a \\ \dot{e}_{P_p} &= -k_{P_p} e_{P_p} \end{aligned} \tag{26}$$

Now, consider the two positive definite storage functions:

$$\begin{aligned} \phi_y &= \frac{1}{2} e_y^2 \\ \phi_v &= \frac{1}{2} e_v^2 \end{aligned} \tag{27}$$

Differentiating equation (27) gives:

$$\begin{aligned} \dot{\phi}_y &= e_y \dot{e}_y = e_y\left(-k_y e_y + g_y e_v\right) \\ \dot{\phi}_v &= e_v \dot{e}_v = e_v\left(-k_v e_v + g_v e_a\right) \end{aligned} \tag{28}$$

Consider $e_y$ and $e_v$ as being the input and $\begin{cases} u_y = g_y e_v \\ u_v = g_v e_a \end{cases}$ as being the output, and rearranging (28) result in:

$$\begin{aligned} \dot{\phi}_y &= u_y^T y_y - \underbrace{k_y e_y^2}_{\geq 0} \\ \dot{\phi}_v &= u_v^T y_v - \underbrace{k_v e_v^2}_{\geq 0} \end{aligned} \tag{29}$$

Therefore, it is evident that the relationship between $e_y$ and $e_v$ is strictly passive and hence BIBO (Bounded Input Bounded Output) stable.
From the acceleration and pressure errors:

$$\begin{cases} \dot{e}_a = -k_a e_a \\ \dot{e}_{P_p} = -k_{P_p} e_{P_p} \end{cases} \tag{30}$$

Since $k_a > 0$ and $k_{P_p} > 0$, errors $e_a$ and $e_{P_p}$ converges exponentially to zero. Therefore, $e_v$ and $e_y$ converges exponentially to zero as $t \to \infty$. Then all errors converge to zero.

# 5 EXPERIMENTAL RESULTS

Before the implementation of the control law (25) in the electropneumatic system, the co-simulation was used. This technique consists in using jointly, the software developed by the researchers in modeling, and the software dedicated for system control. Thus, the physical model of electropneumatic system (1) was treated by AMESim, and the control law (25) was developed on Simulink. A satisfactory simulation results are obtained. Then, the control law is implemented using a Dspace 1104 controller boar with the dedicated digital signal processor. The measured signals, all analog, were run through the signal conditioning unit before being read by the 16 bits analog/digital converter. Two pressure sensors are used, their precision is equal to *700 Pa* (*0.1%* of the extended measurement) and their combined non linearity and hysteresis is equal to $\pm\, 0.1\%$ of the extended measurement. The cylinder velocity is determined by analog differentiation and low-pass filtering of the output of the position given by an analog potentiometer (Its precision and repeatability is equal to $10\mu m$ and its linearity is *0.05%* of the extended measurement.). The acceleration information is obtained by differentiating numerically the velocity. In order o assume the system convergence, the gains must be only positive. The gains $k_y = 30, k_v = 100\ k_a = 500\ et\ k_{P_p} = 500$ have been tuned in order to minimize the position and the pressure tracking error. These values ensure good static and dynamic performance. Some experiment results are provided here to demonstrate the effectiveness of the passivity controller.

Fig. 2 shows the position, the desired position, the position error. The maximum dynamic position tracking error is about $3.3\ mm$. In steady state, the average position error is about $0.06\ mm$. Fig. 3 shows the pressure, the desired pressure, the pressure error. The maximum dynamic pressure tracking error is about $0.06\ bar$. In steady state, the average pressure error is about $0.01\ bar$. Fig. 4 shows the control inputs $u_p$ et $u_N$. The chattering phenomena in the control law are undesirable and seem considerably to decrease the lifetime of some components. This chattering is du to the differentiator who estimates the velocity and the acceleration. In **Brun** (2002), a non linear multivariable control strategy is proposed on the same experimental set-up and in the same conditions. The static position error obtained is about $1.59\ mm$, and the maximum position error is to $5.35\ mm$. So, from this point of view, the obtained results with the passivity based controller are more attractive.

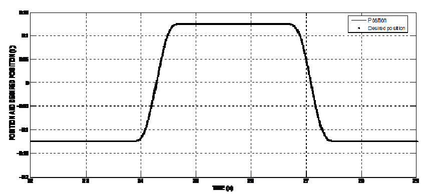

**Fig. 2.a:** Position and Desired Position (m)

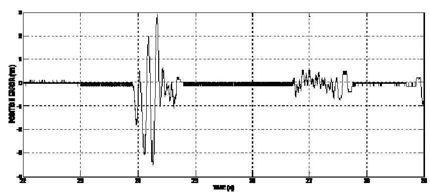

**Fig. 2.b:** Position Error (mm)

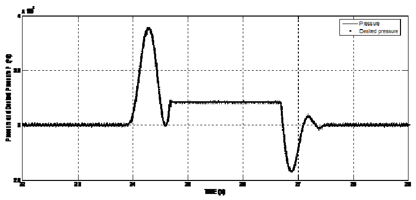

**Fig. 3.a:** Pressure and Desired Pressure (Pa)

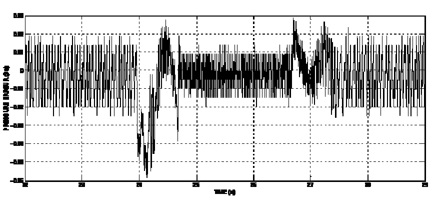

**Fig. 3.b:** Pressure Error (bar)

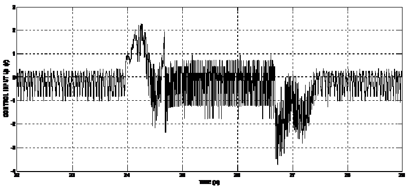

**Fig. 4.a:** Control input $u_P$ (V)

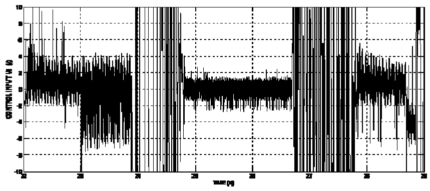

**Fig. 4.b:** Control input $u_N$ (V)

# 6 CONCLUSION

This paper has successfully demonstrated the application of a MIMO passivity to control the position and the pressure of an electropneumatic system. Firstly, the mathematical model of the electropneumatic system was introduced. Then, the theoretical background for the controller synthesis was described in detail. Experiments were carried out to test the effectiveness of the proposed controller. Satisfactory control performance has been obtained by the passivity based controller. Future work will focus on the optimization of energy for the MIMO (multi-input, multi-output) systems applied to electropneumatic system.

# 7 LIST OF NOTATIONS

| | | |
|---|---|---|
| D | dead volume | |
| M | total load mass | |
| N | chamber N | |
| P | chamber P | |
| S | area of the piston cylinder | $m^2$ |
| T | temperature | K |
| V | volume | $m^3$ |
| a | acceleration | $m/s^2$ |
| b | viscous friction coefficient | N/m/s |
| d | desired | |
| ext | external | |
| j | jerk | $m/s^3$ |
| k | polytropic constant | |
| l | length of stroke | m |
| p | pressure in the cylinder chamber | Pa |
| $q_m$ | mass flow rate provided from servodistributor to cylinder chamber | kg/s |
| r | perfect gas constant related to unit mass | J/kg/K |
| u | control input | |
| v | velocity | m/s |
| y | position | m |
| $\varphi(.)$ | leakage polynomial function | kg/s |
| $\psi(.)$ | polynomial function | kg/s/V |

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