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Rayleigh Time-varying Channel Complex Gains Estimation and ICI Cancellation in OFDM Systems

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SUMMARY
In this paper, we consider an orthogonal-frequency-division-multiplexing (OFDM) mobile communication system operating in downlink mode in a time-varying multipath Rayleigh channel scenario. We present a Mean Square Error theoretical analysis for a multipath channel complex gains estimation algorithm with inter-sub-carrier-interference (ICI) reduction using a comb-type pilot. Assuming the presence of delay-related information, the time average of the multipath complex gains, over the effective duration of each OFDM symbol, are estimated using LS criterion. After that, the time-variation of the multipath complex gains within one OFDM symbol are obtained by interpolating the time-averaged symbol values using low-pass interpolation. Hence, the channel matrix, which contains the channel frequency response and the coefficients of ICI, can be computed and the ICI can be reduced by using successive interference suppression (SIS) in data symbol detection. The algorithm’s performance is further enhanced by an iterative procedure, performing channel estimation and ICI suppression at each iteration. Theoretical analysis and simulation results show a significant performance improvement for high normalized Doppler spread (especially after the first iteration) in comparison to conventional methods.

1. INTRODUCTION
ORTHOGONAL frequency division multiplexing (OFDM) is widely known as the promising communication technique in the current broadband wireless mobile communication system due to the high spectral efficiency and robustness to the multipath interference. Currently, OFDM has been adapted to the digital audio and video broadcasting (DAB/DVB) system, high-speed wireless local area networks (WLAN) such as IEEE802.11x, fixed wireless access WiMax IEEE802.11e, 3GPP/LTE, HIPERLAN II and multimedia mobile access communications (MMAC), ADSL, digital multimedia broadcasting (DMB) system and multi-band OFDM type ultra-wideband (MB-OFDM UWB) system, etc. However, OFDM system is very vulnerable when the channel changes within one OFDM symbol. In such case, the orthogonality between subcarriers are easily broken down resulting the inter-sub-carrier-interference (ICI) so that system performance may be considerably degraded.

A dynamic estimation of channel is necessary since the radio channel is frequency selective and time-varying for wideband mobile communication systems [5] [17] [19]. In practice, the channel may have significant changes even within one OFDM symbol. In this case, it is thus preferable to estimate channel by inserting pilot tones into each OFDM symbol which called comb-type pilot [6]. Assuming insertion of pilot tones into each OFDM symbols, the conventional channel estimation methods consist generally of estimating the channel at pilot frequencies and next interpolating the channel

Part of this work was presented in 50-th IEEE GLOBECOM, Washington, USA, November 2007 [1] and in European Wireless Conference (EW), Paris, FRANCE, April 2007 [2]
frequency interpolation, low-pass interpolation, spline cubic
value-decomposition. The interpolation techniques used
in channel estimation are linear interpolation, second
order interpolation, low-pass interpolation, spline cubic
interpolation, time domain interpolation and Wiener
filtering as 2-D interpolation [19]. In [8], low-pass
interpolation has been shown to perform better than all
the interpolation techniques. In [9] the channel estimator
is based on a parametric channel model, which consists
of estimating directly the time delays and complex
attenuations of the multipath channel. This estimator
yields the best performance among all comb-type pilot
channel estimators, with the assumption that the channel
is invariant within one OFDM symbol. For fast time-
varying channel, many existing works resort to estimate the
equivalent discrete-time channel taps which are modeled
in a linear fashion [17] or more generally by a basis
expansion model (BEM) [15] [16]. The BEM methods [15]
used to model the equivalent discrete-time channel taps
are Karhunen-Loeve BEM (KL-BEM), prolate spheroidal
BEM (PS-BEM), complex-exponential BEM (CE-BEM) and polynomial BEM (P-BEM).

In the present paper, we present an iterative algorithm
for channel estimation with inter-sub-carrier-interference
(ICI) reduction in OFDM downlink mobile communication
systems using comb-type pilots. Our interesting is to
estimate directly the physical channel instead of the
equivalent discrete-time channel. That means estimating the
physical propagation parameters such as multipath delays and multipath complex gains. By exploiting the
nature of Radio-Frequency channels, the delays are
assumed to be invariant (over several OFDM symbols)
and perfectly estimated, and only the complex gains of
the multipath channel have to be estimated as we have
already done in CDMA context [3] [4]. Notice that an
initial very performant multipath time delays estimation
can be obtained by using the ESPRIT (estimation of signal
parameters by rotational invariance techniques) method
[9] [11]. For a Jakes’ time Rayleigh gain, we have
showed that the central value and the time averaged value
over one OFDM symbol are extremely closed even for
high realistic Doppler spread. So, for a block of OFDM
symbols, we propose to estimate the time average of the
complex gains, over the effective duration of each OFDM
symbol of different paths, using LS criterion. After that,
the time variation of the different paths complex gains
within one OFDM symbol are obtained by interpolating the
time averaged symbol values using low-pass interpolation.
Hence, the channel matrix, which contains the channel
frequency response and the coefficients of ICI, can be
computed and the ICI can be reduced by using successive
interference suppression (SIS) in data symbol detection.
The present proposed algorithm, with less number of
pilots and without suppression of interference, gives a
good performance over the conventional methods and
performs better with starting interference suppression. This
proposed algorithm can in fact be considered as a simple
extension of an algorithm for time-invariant channels
but which brings, as we will show, a significant gain in
case of time-varying channels with realistic Doppler
spreads. Moreover, we give a theoretical and simulated
Mean Square Error (MSE) multipath channel complex
gains estimation analysis in terms of the normalized (by
the OFDM symbol-time) Doppler spread. This further
demonstrates the effectiveness of the proposed algorithm.

This paper is organized as follows. Section II introduces
the OFDM baseband model and section III covers the
multipath complex gains estimation and the iterative
algorithm. Next, Section IV presents some simulation
results that demonstrate our technique. Finally, we
conclude the paper in Section V.

Notation: Superscripts $(\cdot)^T$ and $(\cdot)^H$ stand for
transpose and Hermitian operators, respectively. $|\cdot|$, $Tr(\cdot)$
and $E[\cdot]$ are the determinant, trace and expectation
operations, respectively. $||\cdot||$ and $(\cdot)^*$ are the magnitude
and conjugate of a complex number, respectively. $\lbrack$ and $\rbrack$
denote a vector and a matrix, respectively. $A[m]$ denotes
the $m$th entry of the vector $A$ and $A[m,n]$ denotes the
$[m,n]$th entry of the matrix $A$. $I_N$ is a $N \times N$ identity
matrix and $diag\{A\}$ is a diagonal matrix. $J_0(\cdot)$ and $J_1(\cdot)$
denote the zeroth-order and the first-order Bessel functions
of the first kind, respectively. $\circ$ denotes the convolution.
$\delta_{k,m}$ denotes the Kronecker symbol and $(k)_N$ stands
for the residue of $k$ modulo $N$. Letters between brackets $[d]$
or parentheses $(d)$ denote that $d$ and $n$ are indexes or
variables.

2. SYSTEM MODEL
Suppose that the symbol duration after serial-to-parallel
(S/P) conversion is $T_u$. The entire signal bandwidth is
covered by $N$ subcarriers, and the space between two
neighboring subcarriers is $1/T_u$. Denoting the sampling
time by $T_s = T_u/N$, and assuming that the length of the
cyclic prefix is $T_g = N_gT_s$ with $N_g$ being an integer. The
duration of an OFDM symbol is \( T = (N + N_d)T_s \). In an OFDM system, the transmitter usually applies an \( N \)-point FFT to data block normalized QAM-symbols \( \{X_{(n)}[k]\} \) (i.e., \( E[X_{(n)}[k]X_{(n)}[k]^*] = 1 \)), where \( n \) and \( k \) represent respectively the OFDM symbol index and the subcarrier index, and adds the cyclic prefix (CP), which is a copy of the last samples of the FFT output, to avoid inter-symbol-interference (ISI) caused by multipath fading channels. In order to limit the periodic spectrum of the discrete time signal at the output of the FFT, we use an appropriate analog transmission filter \( G_t(f) \). As a result, the output baseband signal of the transmitter can be represented as [9][1]:

\[
x(t) = \sum_{n=-\infty}^{\infty} \sum_{d=-N_d}^{N-1} x_{(n)}[d]g_t(t-dT_s-nT)
\]  

(1)

where \( g_t(t) \) is the impulse response of the transmission analog filter and \( x_{(n)}[d] \), with \( d \in [-N_d, N-1] \), are the \((N+N_d)\) samples of the FFT output and the cyclic prefix of the \(n\)th OFDM symbol given by:

\[
x_{(n)}[d] = \frac{1}{N} \sum_{m=-\frac{N_d}{2}}^{\frac{N_d}{2}-1} X_{(n)}[m]e^{j2\pi \frac{md}{N}}
\]  

(2)

It is assumed that the signal is transmitted over a multipath Rayleigh fading channel characterized by:

\[
h(t, \tau) = \sum_{l=1}^{L} \alpha_l(t)\delta(\tau - \tau_lT_s)
\]  

(3)

where \( L \) is the total number of propagation paths, \( \alpha_l \) is the \( l \)-th complex gain of variance \( \sigma^2 \) and \( \tau_l \) is the \( l \)-th delay normalized by the sampling time \( \tau_l \) is not necessarily an integer). \( \{\alpha_l(t)\} \) are wide-sense stationary (WSS) narrow-band complex Gaussian processes with the so-called Jakes’ power spectrum of maximum Doppler frequency \( f_d \) [10] and uncorrelated with respect to each other. The average energy of the channel is normalized to one (i.e., \( \sum_{l=1}^{L} \sigma^2 \)). At the receiver side, after passing to discrete time through low pass filtering and A/D conversion, the CP is removed assuming that its length is no less than the maximum delay. Afterwards, a \( N \)-point FFT is applied to transform the sequence into frequency domain. The \( k \)th subcarrier output of FFT during the \( n \)th OFDM symbol is given by (see Appendix A):

\[
Y_{(n)}[k] = \sum_{m=-\frac{N}{2}}^{\frac{N}{2}-1} X_{(n)}[m]G_t[m]G_r[m]H_{(n)}[k, m] + W_{(n)}[k]
\]  

(4)

where \( W_{(n)}[k] \) is white complex Gaussian noise with variance \( \sigma^2 \), \( G_t[m] \) and \( G_r[m] \) are the transmitter and receiver filter frequency response values at the \( m \)th transmitted subcarrier frequency, and \( H_{(n)}[k, m] \) are the coefficients of the channel matrix from the \( n \)th transmitted subcarrier frequency to the \( k \)th received subcarrier frequency, and given by (see Appendix A):

\[
H_{(n)}[k, m] = \frac{1}{N} \sum_{l=1}^{L} \left[ e^{-j2\pi \frac{m}{N} \tau_l} \sum_{q=0}^{N-1} \alpha_l^{(n)}(qT_s)e^{j2\pi \frac{m-k}{N} q} \right]
\]  

(5)

where \( k, m \in [-\frac{N}{2}, \frac{N}{2}-1] \) and \( \{\alpha_l^{(n)}(qT_s)\} \) is the \( T_s \) spaced sampling of the \( l \)-th complex gain during the \( n \)th OFDM symbol.

If we assume \( N \) transmission subcarriers within the flat region of the frequency response of each of the transmitter and receiver filters, then, by using the matrix notation and omitting the index time \( n \), (4) can be rewritten as [1][2]:

\[
\mathbf{Y} = \mathbf{H} \mathbf{X} + \mathbf{W}
\]  

(6)

where \( G_t[m] \) and \( G_r[m] \) are assumed to be equal to one at the flat region, where \( \mathbf{X}, \mathbf{Y}, \mathbf{W} \) are \( N \times 1 \) vectors given by:

\[
\mathbf{X} = \left[ X[-\frac{N}{2}], X[-\frac{N}{2} + 1], ..., X[\frac{N}{2} - 1] \right]^T
\]  

\[
\mathbf{Y} = \left[ Y[-\frac{N}{2}], Y[-\frac{N}{2} + 1], ..., Y[\frac{N}{2} - 1] \right]^T
\]  

\[
\mathbf{W} = \left[ W[-\frac{N}{2}], W[-\frac{N}{2} + 1], ..., W[\frac{N}{2} - 1] \right]^T
\]

and \( \mathbf{H} \) is a \( N \times N \) channel matrix, which contains the time average of the channel frequency response \( H[k, k] \) on its diagonal and the coefficients of the inter-carrier interference (ICI) \( H[k, k] \) for \( k \neq m \). Notice that \( \mathbf{H} \) would be obviously a diagonal matrix if the complex gains were time-invariant within one symbol.

### 3. Multipath Complex Gains Estimation and the Iterative Algorithm

In this section, we propose a method based on comb-type pilots and multipath time delays information to estimate the sampled complex gains \( \{\alpha_l[qT_s]\} \) with sampling period \( T_s \).
3.1. Pilot Pattern and Received Pilot Subcarriers

The \( N_p \) pilot subcarriers are fixed during transmission and evenly inserted into the \( N \) subcarriers as shown in Fig. 1 where \( L_f \) denotes the interval in terms of the number of subcarriers between two adjacent pilots in the frequency domain. \( L_f \) can be selected without the need of respecting the sampling theorem (in frequency domain) as opposed to the methods shown in [9][8]. However, as we will see with equation (13), \( N_p \) must fulfill the following requirement:

\[
N_p \geq L_f
\]

Let \( \mathcal{P} \) denote the set that contains the index positions of the \( N_p \) pilot subcarriers defined by:

\[
\mathcal{P} = \{ p_s | p_s = s L_f - \frac{N}{2}, s = 0, ..., N_p - 1 \} \tag{7}
\]

The received pilot subcarriers can be written as the sum of three components [2]:

\[
Y_p = X_p H_p + H_{p\tau} X + W_p \tag{8}
\]

where the \( N_p \times N_p \) diagonal matrix \( X_p \), and the \( N_p \times 1 \) vectors \( Y_p \) and \( W_p \) are given by:

\[
X_p = \text{diag}(X[p_0], X[p_1], ..., X[p_{N_p-1}])
\]

\[
Y_p = [Y[p_0], Y[p_1], ..., Y[p_{N_p-1}]]^T
\]

\[
W_p = [W[p_0], W[p_1], ..., W[p_{N_p-1}]]^T
\]

\( H_p \) is a \( N_p \times 1 \) vector and \( H_{p\tau} \) is a \( N_p \times N \) matrix with elements given by:

\[
H_p[p_s] = H[p_s, p_s] = \sum_{l=1}^{L} \alpha_l e^{-j2\pi \frac{p_s l}{N}} \tag{9}
\]

\[
H_{p\tau}[p_s, m] = \begin{cases} H[p_s, m] & \text{if } m \in \left[ -\frac{N}{2}, \frac{N}{2} - 1 \right] - \mathcal{P} \\ 0 & \text{if } m \in \mathcal{P} \end{cases}
\]

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{Comb-Type Pilot Arrangement with \( L_f = 3 \)}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{MSE between \( \alpha_c \) and \( \overline{\alpha} \) for \( N = 128 \)}
\end{figure}

3.2. Estimation of Multipath Complex Gains

For a block \( \{ \alpha_l(qT_s), q = 0, ..., N - 1 \} \) of \( N \) \( T_s \)-spaced sampling of a gaussian complex gains with a frequency of the Jakes' power spectrum \( f_d \), we have shown in Appendix B that:

a) The \( T_s \)-spaced sampling of the complex gain taken in the middle of the effective duration of the OFDM symbol \( \alpha_l(\frac{N}{2} T_s) \) is closest to the complex gain time average over the effective duration \( \overline{\alpha_l} \) defined in (9)

b) For the whole \( L \) gains, the mean square error (MSE) between exact averaged values \( \overline{\alpha} = [\overline{\alpha_1}, ..., \overline{\alpha_L}]^T \) and exact central values \( \overline{\alpha_c} = [\alpha_1(\frac{N}{2} T_s), ..., \alpha_L(\frac{N}{2} T_s)]^T \) is...
\[ \alpha F c, \text{ which minimizes } F q \text{ is very } \alpha c \alpha LS F LS - \text{ for some OFDM} \]

\[ \alpha LS F LS - Y c = , \text{ in the} \]

\[ \text{We can conclude that, for realistic normalized Doppler}\]

\[ \text{theoretically from (12) and by Monte-Carlo simulation.} \]

\[ \text{duration of each OFDM symbol for the different paths,} \]

\[ \text{these OFDM symbols, for each path.} \]

\[ \text{Notice that, for a normalized channel,} \]

\[ \text{an estimation of} \]

\[ \text{using low-pass interpolation [8], we obtain an estimation} \]

\[ \text{of the sampled complex gains} \]

\[ \text{Interpolation of } K \text{ samples estimated} \]

\[ \text{Interpolation of } K \text{ samples estimated} \]

\[ \text{are estimated using the LS criterion. By neglecting the} \]

\[ \text{ICI contribution, the LS-estimator of} \]

\[ \text{channel matrix estimation block diagram; and (c) the diagram of complex gains estimator} \]

\[ \text{given by:} \]

\[ m se_2 = E [(\alpha - \alpha_L)^H (\alpha - \alpha_L)] \]

\[ = \sum_{l=1}^{L} \sigma_{\alpha_l}^2 \left( \frac{1}{N^2} \sum_{q_1=0}^{N-1} \sum_{q_2=0}^{N-1} J_0 \left( 2\pi f_d T_s (q_1 - q_2) \right) \right) \]

\[ - \frac{2}{N} \sum_{q=0}^{N-1} J_0 \left( 2\pi f_d T_s \left( q - \frac{N}{2} \right) \right) + 1 \]  \hspace{1cm} (12) \]

\[ \text{Notice that, for a normalized channel, } m se_2 \text{ depends only on } f_d T. \]

\[ \text{Fig. 2 shows the evolution of } m se_2 \text{ with } f_d T, \text{ obtained} \]

\[ \text{theoretically from (12) and by Monte-Carlo simulation.} \]

\[ \text{We can conclude that, for realistic normalized Doppler spread} \]

\[ \text{the distance between } \alpha_L \text{ and } \alpha \text{ is very} \]

\[ \text{negligible. Hence, we can assume that an estimation of } \alpha \text{ is} \]

\[ \text{an estimation of } \alpha_L \text{. So, by estimating } \alpha \text{ for some OFDM} \]

\[ \text{symbols and interpolating them by a factor } (N + N_g) \text{ using} \]

\[ \text{low-pass interpolation [8], we obtain an estimation} \]

\[ \text{of the sampled complex gains } \{ \alpha_l(q T_s) \} \text{ at time } T_s \text{ during} \]

\[ \text{these OFDM symbols, for each path.} \]

\[ \text{The time average of the complex gains, over the effective} \]

\[ \text{duration of each OFDM symbol for the different paths,} \]

\[ \text{are estimated using the LS criterion. By neglecting the} \]

\[ \text{ICI contribution, the LS-estimator of } \alpha_L, \text{ which minimizes} \]

\[ \text{is represented by:} \]

\[ \alpha LS = \frac{M Y_p}{M} \]  \hspace{1cm} (13) \]

\[ \text{It should be noted that, the matrix} \]

\[ \text{in the expression of } M \text{ is not invertible if} \]

3.3. **Iterative Algorithm**

\[ \text{The iterative algorithm of channel estimation and ICI} \]

\[ \text{suppression is shown in Fig. 3. The whole algorithm} \]

\[ \text{is divided into two modes: channel matrix estimation mode} \]

\[ \text{and detection mode, as shown in Fig. 3(a). The first mode} \]

\[ \text{includes estimating the sampled complex gains} \}

\[ \text{at time } T_s \text{ via LS-estimator and low-pass interpolation} \]

\[ \text{and computing the channel matrix as shown in Fig. 3(b).} \]

\[ \text{The second mode includes the detection of data symbols} \]

\[ \text{by using successive data interference suppression (SIS)} \]
scheme with one tap frequency equalizer (see Appendix E). A feedback technique is used between these two modes, performing iteratively ICI suppression and channel matrix estimation. In this iterative algorithm, the OFDM symbols are grouped in blocks of $K$ OFDM symbols each one. Each two consecutive blocks are intersected in two OFDM symbols as shown in Fig. 3(c). For a block of $K$ OFDM symbols, the iterative algorithm proceeds as following:

1. $Y_p^{(k,1)} = Y_p^{(k)}$
2. for $i = 1 : N_{\text{iteration}}$ do
3. $\hat{H}^{(k,i)} = M Y_p^{(k,i)}$
4. $\{\alpha_k^{(k,i)}(qT_s), k=2,...,K-1, q=0,...,N_q-1\} = \text{interp}(\alpha_k^{(k,i)}, N + N_q)$
5. compute using (5) the channel matrix $\hat{H}^{(k,i)}$
6. remove the ICI of pilots from the received data subcarriers $Y_p^{(k)}$ in (31)
7. detection the data symbols $\hat{X}_d^{(k,i)}$ using SIS
8. $Y_p^{(k,i+1)} = Y_p^{(k)} - \hat{H}^{(k,i)} \hat{X}_d^{(k,i)}$
9. end for

where $N_{\text{iteration}}$ is the number of iterations, $\text{interp}$ denotes the interpolation Matlab function and, $i$ and $k$ represent the iteration number and the number of OFDM symbol in a block, respectively. Note that, the steps 3 to 6 are executed without considering the first and the last OFDM symbols ($i.e., k = 2$ to $K - 1$) in order to avoid limiting effects of interpolation.

3.4. Mean Square Error (MSE) Analysis

The MSE of the LS-estimator of $\alpha$ is defined by:

$$\text{mse}_1 = E \left[ (\hat{\alpha}_{LS} - \alpha)^H (\hat{\alpha}_{LS} - \alpha) \right]$$

which gives (see Appendix D):

$$\text{mse}_1 = Tr \left( M (R_1 + \sigma^2 I_{N_p}) M^H \right)$$

where the expression of the covariance matrix $R_1$ is detailed in Appendix D. Notice that if ICI are completely eliminated then, $R_1$ is matrix of zeros. Thus, (15) becomes:

$$\text{mse}_1 \text{ (without ICI)} = \sigma^2 Tr \left( M M^H \right)$$

In general, $\text{mse}_1$ depends on the pilot positions and the multipath delays.

It is clear that our LS-estimator is unbiased. So, the CRAMER-RAO BOUND (CRB) [14] is an important criterion to evaluate how good the LS-estimator can be since it provides the MMSE bound among all unbiased estimators. We have shown in Appendix C that the Standard CRB (SCRB) for the estimator of $\alpha$ with ICI known is given by:

$$\text{SCRB}(\alpha) = \frac{1}{SNR} Tr \left( \left( F_p^H X_p^H X_p F_p \right)^{-1} \right)$$

where $SNR = \frac{1}{\sigma^2}$ is the normalized signal to noise ratio. It is easy to show that

$$\left\{ \begin{array}{l}
\text{mse}_1 \text{ (with ICI)} > \text{SCRB}(\alpha) \\
\text{mse}_1 \text{ (without ICI)} = \text{SCRB}(\alpha)
\end{array} \right.$$

So, by iteratively estimating and removing the ICI $\text{mse}_1$ will be closer to $\text{SCRB}(\alpha)$.

The MSE of the assumption that $\hat{\alpha}_{LS}$ is an estimation of $\alpha_c$ is given by (see Appendix D):

$$\text{mse}_c = E \left[ (\hat{\alpha}_{LS} - \alpha_c)^H (\hat{\alpha}_{LS} - \alpha_c) \right] = \text{mse}_1 + \text{mse}_2 + \text{mse}_{12} + \text{mse}_{21}$$

where $\text{mse}_2$ is defined in (12) and, $\text{mse}_{12}$ and $\text{mse}_{21}$ are the cross-covariance terms, which are very negligible, given by (see Appendix D):

$$\text{mse}_{12} = E \left[ (\hat{\alpha}_{LS} - \hat{\alpha}_c)^H (\hat{\alpha}_{LS} - \hat{\alpha}_c) \right] = Tr \left( R_2 M^H \right)$$

$$\text{mse}_{21} = E \left[ (\hat{\alpha} - \alpha_c)^H (\hat{\alpha}_{LS} - \alpha) \right] = \text{mse}_{12}^*$$

where the matrix $R_2$ is computed in Appendix D.

The MSE of the multipath complex gain estimator at time $T_s$ is defined by:

$$\text{mse}_{T_s} = \sum_{k=2}^{K-1} \sum_{q=0}^{N_q-1} E \left[ (\hat{\alpha}_k - \alpha_k)^H (\hat{\alpha}_k - \alpha_k) \right]$$

with $\alpha_k = [\alpha_1^k(qT_s), ..., \alpha_K^k(qT_s)]^T$

For a large value $K$, assuming performant interpolator and respecting sampling theorem in time domain ($f_d T_s \leq 0.5$), we will have (see Appendix D):

$$\text{mse}_{T_s} \approx \text{mse}_c$$

We now study the MSE of the multipath complex gain estimator and the interpolation method versus the
OFDM block length $K$. Fig. 4 gives the $mse_T$, (with exact central values) and the $mse_{int}$ (with exact averaged values) for $f_dT = 0.1$. We notice that the interpolation error $mse_{int}$ decreases with $K$, while the error of the estimator $mse_T$, is constant whatever the interpolation window length $K$. This is due to $mse_2$ (MSE between the central value $e_c$, and the averaged value $\bar{e}$) which is dominant with respect to the interpolation error $mse_{int}$. Moreover, we verify that $mse_T \approx mse_{int} + mse_2$. This means that the cross-covariance terms are very negligible. So, in general, we may say that the interpolation window length $K$ is not necessary to be large and it suffices to choose $K$ such that $K.T = T_{coh} = \frac{1}{f_c}$ (i.e., $T_{coh}$ is the coherence time) in order to have strong correlation between samples $\alpha_l^m(qT_s)$. For example $f_dT = 0.1$, $T_{coh} = 10T$, so we choose $K = 10$.

4. SIMULATION RESULTS

In this section, we verify the theory by simulation and we test the performance of the iterative algorithm.

![Figure 4. MSE of the complex gain estimator in terms of $K$ for $f_dT = 0.1$](image1.png)

![Figure 5. Comparison between MSE for $SNR = 20dB$](image2.png)

The mean square error (MSE) and the bit error rate (BER) performances in terms of the average signal-to-noise ratio (SNR) [9] [8] and maximum Doppler spread $f_dT$ (normalized by $1/T$) for Rayleigh channel are examined. The normalized channel model is Rayleigh as recommended by GSM Recommendations 05.05 [12] [13], with parameters shown in the table below ($\frac{1}{Ts} = 2MHz$).

<table>
<thead>
<tr>
<th>Path Number</th>
<th>Average Power(dB)</th>
<th>Normalized Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-7.219</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-4.219</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>-6.219</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>-10.219</td>
<td>3.2</td>
</tr>
<tr>
<td>5</td>
<td>-12.219</td>
<td>4.6</td>
</tr>
<tr>
<td>6</td>
<td>-14.219</td>
<td>10</td>
</tr>
</tbody>
</table>

![Figure 6. The MSE of the LS-estimator for $SNR = 20dB$](image3.png)
mismatch. Hence, for $f_d T > 0.1$, MSE of complex gains estimator is significant and the ICIs are not estimated and nor removed perfectly.

Fig. 7 shows the evolution of $mse_{c}$ with the iterations in terms of $SN R$ for $f_d T = 0.1$. Note that the x-axis represents the time axis normalized with respect to the sample time $T_s$. After one iteration, a great improvement is realized and $mse_{1}$ is very close to the SCRB especially in low and moderate SNR regions. This is because at low SNR, the noise is dominant with respect to the ICI level, and at high SNR ICI is not completely removed due to the data symbol detection error.

For illustration, Fig. 8 gives the real and the imaginary parts of the exact and estimated (after one iteration) multipath complex gain. This is done for one channel realization over 8 OFDM symbols with $SN R = 20dB$ and $f_d T = 0.1$. Notice how good is the estimation of multipath complex gains for rapidly changing channels.
and becomes better with starting interference suppression. The iterative algorithm performs better than the conventional methods with less pilots and without interference suppression, our performance will be better. Moreover, the results show that, $SNR = 20dB$ symbol detection error.

and ICI because we have an error floor due to the data symbol detection errors. At a high SNR, it is normal to not reach the performance obtained with perfect knowledge of channel and ICI. This result shows that, with all ICI, our algorithm performs better than the conventional methods. Moreover, when we start removing ICI our iterative algorithm offers an improvement in BER after each iteration because the estimation of ICI is improved during each iteration. After two iterations, a significant improvement occurs; the performance of our algorithm and the SIS algorithm with perfect channel knowledge are very close. At a high SNR, it is normal to not reach the performance obtained with perfect knowledge of channel and ICI because we have an error floor due to the data symbol detection error.

Fig. 10 gives the BER in terms of $N_p$ for $f_dT = 0.1$ and $SNR = 20dB$. It is obvious that when using more pilots, performance will be better. Moreover, the results show that, with less pilots and without interference suppression, our algorithm performs better than the conventional methods and becomes better with starting interference suppression.

Fig. 11 shows the BER performance of our proposed iterative algorithm, for $N_c = 2$ and $f_dT = 0.1$ with IEEE802.11a standard channel coding [18]. The convolutional encoder has a rate of $1/2$, and its polynomials are $P_0 = 133_8$ and $P_1 = 171_8$ and the interleaver is a bitwise block interleaver with 16 rows and 14 columns. It can clearly be seen that a significant improvement in BER occurs with channel coding, and that for high SNR there is always an error floor due to data symbol detection errors.

Moreover, when we start removing ICI our algorithm performs better than the conventional methods. Moreover, when we start removing ICI our algorithm performs better than the conventional methods and its BER performance is very close to the performance of SIS algorithm with perfect channel knowledge.

A. RECEIVED OFDM SYMBOL

From equations (1) and (3) the received signal of the $n$th OFDM symbol at the output of the low pass receiver filter $g_r(t)$ is given by:

$$y_{(n)}(t) = \sum_{d=-N_s}^{N_s-1} \sum_{l=1}^{L} x_{(n)}(d) \alpha_l(t) \beta(t - dT_s - nT - T_s) + w(t)$$

where $\beta(t) = (g_r \otimes g_r)(t)$. After A/D conversion and removing the cyclic prefix, the N received samples are given by:

Figure 10. Comparison of BER for $f_dT = 0.1$ and $SNR = 20dB$

Figure 11. Comparison of BER, in the case of the IEEE802.11a convolutional code, for $f_dT = 0.1$

5. CONCLUSION

In this paper, we have analyzed an iterative algorithm to estimate multipath complex gains and mitigate the intersub-carrier-interference (ICI) for OFDM systems. The rapid time-variation complex gains are tracked by exploiting that the delays are assumed invariant (over several symbols) and perfectly estimated. Theoretical analysis and simulation results of our iterative algorithm show that by estimating and removing the ICI at each iteration, multipath complex gains estimation and coherent demodulation can have a great improvement especially after the first iteration for high realistic Doppler spread. Moreover, our algorithm performs better than the conventional methods and its BER performance is very close to the performance of SIS algorithm with perfect channel knowledge.
where \( q \in [0, N - 1] \). Using (2), the N samples of the FFT output are given by:

\[
Y_{(n)}[k] = \sum_{q=0}^{N-1} y_{(n)}[q]e^{-j2\pi \frac{kq}{N}} = \frac{1}{N} \sum_{m=-\frac{N}{2}}^{\frac{N}{2}-1} X_{(n)}[m] \left( \sum_{q=0}^{N-1} e^{-j2\pi \frac{km}{N}} \sum_{d=-N_g}^{N_N} \sum_{l=1}^{L} \alpha_1^{(n)}(qT_s) \beta((q-d-\tau_l)T_s) + w_{(n)}(qT_s) \right) W_{(n)}[k] \tag{22}
\]

where \( k \in \left[ -\frac{N}{2}, \frac{N}{2} - 1 \right] \) and \( W_{(n)}[k] = \sum_{q=0}^{N-1} w_{(n)}(qT_s) e^{-j2\pi \frac{kq}{N}} \).

As a result, for \( \tau_l < N_g \), we have:

\[
\sum_{u=0}^{N_s} \sum_{l=1}^{L} \alpha_1^{(n)}(qT_s) \beta((u-\tau_l)T_s) e^{-j2\pi \frac{ku}{N}} \approx \sum_{l=1}^{L} \alpha_1^{(n)}(qT_s) G_{l}[m] G_{r}[m] e^{-j2\pi \frac{ku}{\tau_l}} \tag{23}
\]

Notice that strict equality would hold if \( u \) varies from \(-\infty\) to \(+\infty\). Inserting (23) into (22) yields the results given in equations (4) and (5).

B. CLOSEST SAMPLE TO TIME AVERAGED COMPLEX GAIN

Let \( \alpha_d = [\alpha_1(dT_s), ..., \alpha_L(dT_s)]^T \) be a vector of samples of the complex gains taken at the time position \( d \in [0, N - 1] \) during the effective duration of the OFDM symbol. The MSE between \( \overline{\alpha} \) and \( \alpha_d \) is defined as:

\[
mse[d] = E \left[ \left( \overline{\alpha} - \alpha_d \right)^H \left( \overline{\alpha} - \alpha_d \right) \right] = \sum_{l=1}^{L} \sum_{u=0}^{N_s} \left[ \sigma_{\alpha_l}^2 \delta_{l,u} + \sigma_{\alpha_l}^2 (dT_s - \alpha_l(dT_s)) \delta_{l,u}^2 + \sigma_{\alpha_l}(dT_s) \sigma_{\alpha_l}(dT_s) \delta_{l,u} \right] \tag{24}
\]

Since \( \alpha_l(t) \) is wide-sense stationary (WSS) narrow-band complex Gaussian processes with the so-called Jakes' power spectrum [10] then:

\[
E \left[ \alpha_1(q_1T_s)\alpha_1^*(q_2T_s) \right] = \sigma_{\alpha_1}^2 J_0 \left( 2\pi f_d T_s (q_1 - q_2) \right) \tag{24}
\]

Using (24), we can calculate \( mse[d] \) as:

\[
mse[d] = \sum_{l=1}^{L} \sigma_{\alpha_l}^2 \left( \frac{1}{N^2} \sum_{q_1=0}^{N-1} \sum_{q_2=0}^{N-1} J_0 \left( 2\pi f_d T_s (q_1 - q_2) \right) \right) - 2 \sum_{q=0}^{N-1} J_0 \left( 2\pi f_d T_s (q - d) \right) + 1 \]

To find the closest \( \alpha_d \) to \( \overline{\alpha} \), we need to find \( d_{\text{min}} \) that minimizes \( mse[d] \). By using the derivative formula of the Bessel function defined as: \( J_0'(t) = -J_1(t) \), we can calculate the derivative of \( mse[d] \) as:

\[
mse'[d] = -\frac{4\pi f_d T_s}{N} \sum_{l=1}^{L} \sigma_{\alpha_l}^2 \sum_{q=0}^{N-1} J_1 \left( 2\pi f_d T_s (q - d) \right) \]

Since \( J_1(t) \) is an odd function then, the solution of the equation \( mse'[d] = 0 \) is obtained when the interval of the index \( u \) in (25) is centered at zero, thus \( d = \frac{N}{2} - \frac{1}{2} \) (not integer). It is easy to show that \( mse[d] \) is symmetric with respect to \( d = \frac{N}{2} \) or \( \frac{N}{2} - 1 \). We denote the minimum of \( mse[d] \) by \( mse_2 = mse[d_{\text{min}}] \). For illustration, Fig. 12 gives the curve of \( mse[d] \) (theoretical and monte-carlo simulation) for \( N = 128 \) and \( f_dT = 0.1 \). It is well observed that \( d = 63.5 \) is an axis of symmetric and \( d_{\text{min}} = 63 \) or 64.
C. CRB FOR THE ESTIMATOR OF $\pi$

Assuming $IC_{lp} = H_{pp} X$ in (8) are known then, the vector $Y_p$ for a given $\pi$ is complex Gaussian with mean vector $\overline{m} = X_p F_p \pi + IC_{lp}$ and covariance matrix $S_1 = \sigma_1^2 I_{N_p}$. Thus, the probability density function $p(Y_p | \pi)$ is defined as:

$$p(Y_p | \pi) = \frac{1}{2\pi S_2} e^{-\frac{1}{2} (Y_p - m)^H S_2^{-1} (Y_p - m)}$$

Since $\pi$ is a complex Gaussian vector with zero mean and covariance matrix $S_2$ then, the probability density function of $\pi$ is defined as:

$$p(\pi) = \frac{1}{2\pi S_2} e^{-\frac{1}{2} \pi^H S_2^{-1} \pi}$$

where $S_2$ is a diagonal matrix of elements $S_2[l, l]$, with $l \in [1, L]$, given by:

$$S_2[l, l] = E[|\pi|^2] = \frac{\sigma_1^2}{N_2} \sum_{q_1 = 0}^{N_1-1} \sum_{q_2 = 0}^{N_1-1} J_0(2\pi f_d T_s (q_1 - q_2))$$

The Standard CRB (SCRB) and the Bayesian CRB (BCRB) for the estimator of $\pi$ are defined as [14]:

$$SCRB(\pi) = Tr \left( -E \left[ \frac{\partial^2}{\partial 2} ln(p(Y_p | \pi)) \right] \right)^{-1}$$

$$BCRB(\pi) = Tr \left( -E \left[ \frac{\partial^2}{\partial 2} ln(p(Y_p, \pi)) \right] \right)^{-1}$$

where $p(Y_p, \pi) = p(Y_p | \pi)p(\pi)$ is the joint probability density function of $Y_p$ and $\pi$ and, the expectation is taken over $Y_p$ and $\pi$. Notice that SCRB and BCRB are for the estimation of deterministic and random variables, respectively.

The results of the second derivatives of $ln(p(Y_p | \pi))$ and $ln(p(Y_p, \pi))$ with respect to $\pi$ are given by:

$$\frac{\partial^2}{\partial \pi^2} ln(p(Y_p | \pi)) = -F_p^H X_p^H S_2^{-1} X_p F_p$$

$$\frac{\partial^2}{\partial \pi^2} ln(p(Y_p, \pi)) = -F_p^H X_p^H S_2^{-1} X_p F_p - S_2^{-2}$$

Hence, substituting (27) in (26) yields:

$$SCRB(\pi) = \sigma^2 Tr \left( F_p^H X_p^H X_p F_p S_2^{-1} + S_2^{-1} \right)^{-1}$$

$$BCRB(\pi) = Tr \left( \frac{1}{\sigma^2} F_p^H X_p^H X_p F_p + S_2^{-1} \right)^{-1}$$

We notice that in our specific problem SCRB is independent of $\pi$. So, SCRB gives the lower bound if the priori distribution of $\pi$ is not used in the estimation method, whereas BCRB takes this information into account. For illustration, Fig. 13 gives the SCRB and BCRB in terms of $SNR$ for the channel given in Table I, $N = 128$, $N_p = 16$ and $f_d T = 0.1$. It is observed that there is a small difference between SCRB and BCRB at low $SNR$. So, we can compare the MSE of our LS-estimator of $\pi$ to SCRB instead of BCRB.

D. MEAN SQUARE ERROR OF COMPLEX GAINS ESTIMATOR

The MSE of the LS-estimator of $\pi$ is given by:

$$mse_1 = E \left[ (\pi_{LS} - \pi)^H (\pi_{LS} - \pi) \right] = Tr \left( M (R_1 + \sigma_1^2 I_{N_p}) M^H \right)$$

where $R_1 = E \left[ H_{pp} X X^H H_{pp}^H \right]$ and the expectation is taken over the data symbols, the noise and the complex gains, since the noise and the ICIs are uncorrelated. The term $IC_{lp} = H_{pp} X$ can be written as the sum of two components:

$$IC_{lp} = H_{pp} X_p + H_{dd} X_d$$

where $X_d$ is the data symbols and, $H_{pp}$ and $H_{dd}$ are a $N_p \times N_p$ and a $N_p \times (N - N_p)$ matrices, respectively, of
\[ R_{pp}[k, m] = E \left[ \sum_{u_1=p_0}^{pN_p-1} \sum_{u_2=p_0}^{pN_p-1} X[u_1]X^*[u_2]H[k, u_1]H^*[m, u_2] \right] \]
\[
= \frac{1}{N^2} \sum_{u_1=p_0}^{pN_p-1} \sum_{u_2=p_0}^{pN_p-1} \sigma^2_{\alpha_1} X[u_1]X^*[u_2] \sum_{q_1=0}^{N-1} \sum_{q_2=0}^{N-1} e^{-j2\pi \frac{(u_1-u_2)(q_1-q_2)}{N}} J_0 \left( 2\pi f_d T_s (q_1 - q_2) \right) \]
\[
R_{dd}[k, m] = E \left[ \sum_{u=-N}^{N} H[k, u]H^*[m, u] \right] \]
\[
= \frac{1}{N^2} \sum_{l=1}^{N} \sigma^2_{\alpha_1} \sum_{u=p_0}^{pN_p-1} \left( \delta_{k,m} - \frac{N}{N} \right) e^{-j2\pi \frac{(u-k)(q-m)}{N}} J_0 \left( 2\pi f_d T_s (q_1 - q_2) \right) \delta_{0,(q_1 - q_2)N_p} \]  (28)

\[ R_2[l, k] = E \left[ \sum_{u=p_0}^{pN_p-1} X^*[u]H^*[k, u] \left( \overline{\pi}_l - \alpha_l \left( \frac{N}{2} T_s \right) \right) \right] \]
\[
= \sigma^2_{\alpha_1} \sum_{u=p_0}^{pN_p-1} \left( \overline{\pi}_l \sum_{q=0}^{N-1} \sum_{r=0}^{N-1} e^{-j2\pi \frac{(u-k)(q-r)}{N}} J_0 \left( 2\pi f_d T_s (q_1 - q_2) \right) \right) \]  (29)

The MSE of the assumption that \( \overline{\pi}_{LS} \) is an estimation of \( \alpha_c \) is given by:
\[
mse_c = E \left[ (\overline{\pi}_{LS} - \alpha_c)^H (\overline{\pi}_{LS} - \alpha_c) \right] = mse_{c1} + mse_{c2} + mse_{c12} + mse_{c21} \]
where \( mse_{c12} \) and \( mse_{c21} \) are the cross-covariance terms given by:
\[
mse_{c12} = E \left[ (\overline{\pi}_{LS} - \overline{\pi})^H (\overline{\pi}_{LS} - \alpha_c) \right]
\[
mse_{c21} = E \left[ (\pi - \alpha_c)^H (\pi_{LS} - \pi) \right] = mse_{c12}^* \]

Since the data symbols are normalized (i.e., \( E \left[ X_d X_d^H \right] = I_{N-N_p} \)), then, \( R_{dd} = E \left[ H_{dd} H_{dd}^H \right] \).

Thus, the elements \( R_{pp}[k, m] \) and \( R_{dd}[k, m] \), with \( k, m \in \mathcal{P} \), are given by (28), shown at the top of the present page.
The MSE of the multipath complex gain estimator at time $T_s$ is defined by:

$$mse_{T_s} = \sum_{k=2}^{K-1} \sum_{q=-N_q}^{N_q-1} E[k^H k]$$

where $\xi_q = \xi^k - \xi^k_0$, with $\xi^k_0 = [\alpha_1^k(qT_s), ..., \alpha_K^k(qT_s)]^T$, is the error of the multipath complex gain estimator at time $T_s$. This error can be written as the sum of two errors:

$$\xi^k_q = \xi^k + e_{int}^k$$

where $\xi^k$ is the error due to the assumption that $\xi_{LS}$ is an estimation of $\xi^k$, and $e_{int}^k$ is the error due to the interpolation method which depends on the number $K$ of OFDM symbols in each bloc and the term $f_d T$.

For a large value $K$, assuming performant interpolator and respecting sampling theorem in time domain (i.e. $f_d T \leq 0.5$), we will have $e_{int}^k \approx [0, ..., 0]^T$, and then:

$$mse_{T_s} \approx mse_c$$

**E. SUCCESSIVE INTERFERENCE SUPPRESSION METHOD**

The received data subcarriers are given by:

$$Y_d = H_d X_d + H'_d X_p + W_d$$

(30)

where $X_d$ the data transmitted, $Y_d$ the data received and $W_d$ the noise at data subcarrier positions are $(N - N_p) \times 1$ vectors, $H_d$ and $H'_d$ are a $(N - N_p) \times (N - N_p)$ and $(N - N_p) \times (N_p)$ matrices, respectively, of elements given by:

$$H_d[k, m] = H[k, m] \text{ if } k, m \in \left\{ \frac{N}{2} \frac{N}{2} - 1 \right\} - \mathcal{P}$$

$$H'_d[k, m] = H[k, m] \text{ if } k, m \in \left\{ \frac{N}{2} \frac{N}{2} - 1 \right\} - \mathcal{P}$$

Note that in (30), the first component is the desired data term with ICI due to data symbols and the second component is the ICI term due to pilot symbols.

By successive interference suppression (SIS) scheme with optimal ordering and one tap frequency equalizer the data will be estimated. The optimal ordering is calculated from the large to the small magnitude of the diagonal elements of the data channel matrix $H_d$ and given by:

$$O = \{ O_1, O_2, ..., O_{N-N_p} \}$$

$$i < j \text{ if } ||H_d[O_i, O_i]|| > ||H_d[O_j, O_j]||$$

The detection algorithm can now be described as follows:

1. $O = \{ O_1, O_2, ..., O_{N-N_p} \}$
3. for $i = 1 : N - N_p$ do
4. $X e_d[O_i] = Y'_d[i]/H_d[O_i, O_i]$  
5. $Y'_d[i] = Y_d[i] - X_d[O_i](H_d[O_i])$
7. end for

where $Y'_d$ is the received data subcarriers without contribution from pilot subcarriers, $Q(.)$ denotes the quantization operation appropriate to the constellation and $(H_d)_{O_i}$ denotes the $O_i$th column of the data channel matrix $H_d$. Note that in our algorithm, we have used the minimum distance criterion as quantization method.

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