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Model-based Bit Allocation for Normal Mesh Compression
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Abstract—In this paper, we propose a powerful bit allocation that optimizes the quantization of the normal mesh geometry. This bit allocation aims to minimize the surface-to-surface distance [1] between the original irregular mesh and the quantized normal one, according to a target bitrate. Moreover, to provide a fast bit allocation, we approximate this surface-to-surface distance with a simple criterion depending on the wavelet coefficient distributions, and we use theoretical models. This provides a fast and low-complex model-based bit allocation yielding results better than the recent state-of-the-art methods like [2].

I. INTRODUCTION

BIT allocation is an essential tool to provide a powerful coding of signals when a multiresolution analysis is performed. This process generally aims to optimize the trade-off between bitrate and quality, by minimizing a distortion due to the signal quantization for a specific bitrate. Among the prior works in 3D mesh compression, King and Rossignac proposed for instance a bit allocation based on relationships between the number of vertices, the bitrate per coordinate, a desired approximation error, and the bitstream size [3]. More recently, Karni and Gotsman [4] proposed to truncate their spectral coefficients according to a given RMS value. We also proposed in [5] a model-based bit allocation controlling the quantization error energy to dispatch the bits across wavelet subbands of meshes obtained with MAPS [6]. Recently, an estimation-quantization algorithm has been proposed to encode the normal mesh geometry [7]. In this paper, we propose a model-based bit allocation for a wavelet coder of normal meshes [8]. We focus on these meshes because of their compact multiresolution representation based on subdivision connectivity. The particularity of these meshes is that most of details are in the normal direction to the surface and are expressed through a single scalar (see Fig. 1). The allocation proposed for these normal meshes optimizes the rate-distortion trade-off during the encoding of the normal mesh geometry. Precisely, we aim to find the best quantization for each wavelet subband such that the global reconstruction error is minimized under a constraint on the global bitrate. A distortion measure is consequently needed to evaluate the reconstructed error of the decoded mesh. Several distortion measures have been exploited for 3D mesh compression of irregular meshes [4], [3]. In order to measure the loss related to quantization, the authors of [4] introduce for instance a metric which captures the visual difference between the original mesh and its approximation: to this purpose, they use a criterion depending on the geometric distance and the laplacian difference between models. Unfortunately, this kind of vertex-to-vertex measures cannot be applied in our case since the proposed coder uses a remeshing technique modifying the topology of the input mesh. In that case, the most frequently quality criterion used is the so-called surface-to-surface (S2S) distance [1]. Based on the Hausdorff distance, this distance does not depend on the mesh sampling. Unfortunately, the S2S distance is a computationally intensive process which does not permit real-time computation during process, particularly from wavelet coefficients. The main contribution of this paper is to show how the S2S distance can be approximated in function of the quantization error of wavelet coefficients, and then theoretically modeled according to the wavelet coefficient distributions. This permits to design a fast and low-complex model-based bit allocation.

This paper is organized as follows. Section II introduces the normal meshes and the proposed coder. Section III deals with the approximation of the S2S distance across a wavelet coder. Section IV introduces the model-based bit allocation. Finally, we show results and conclude in section V.

II. BACKGROUND

A. Normal meshes

A normal mesh $M_{nr} = (V_{nr}, T_{nr})$, where $V_{nr}$ and $T_{nr}$ are respectively the set of vertices and the set of triangular faces, can also be defined by a coarse mesh $M_0$ and several sets of details $\{d_{i,j}\}$, $i$ being the resolution level (see Fig. 1). Computed in function of the normal at the surface, most of the geometry information is concentrated in the coordinates $z$ of the details (normal components), the coordinates $x$ and $y$ (tangential components) being infinitesimal [8].

B. Overall coding scheme

Fig. 2 presents the proposed coder. The normal remesher provides a normal mesh $M_{nr}$, from the irregular input one $M_{ir}$. A $N$-level unlifted butterfly wavelet transform is then applied to obtain the subbands of wavelet coefficients. This scheme corresponds to the optimal wavelet transform for the normal meshes [2]. The sets of tangential and normal components are used as guided remeshing inputs. For each vertex, the coordinates are extracted from the normal mesh $M_{nr}$. The normal mesh $M_{nr}$ is then remeshed to obtain a new normal mesh $M_{nr}'$. The normal components are concatenated together with the tangential components to obtain the refined mesh for each level. The coder then is based on the quantization of these wavelet coefficients. The B. Overall coding scheme
components are now encoded with an uniform scalar quantizer depending on the allocation process, and an entropy coder adapted to the multiresolution mesh [5]. In parallel, the connectivity of the coarse mesh is encoded with the lossless coder of Touma and Gotsman [9]. Hence, we obtain the quantized normal mesh \( \mathcal{M}_{sr} = (\mathcal{V}_{sr}, T_{sr}) \), with \( \mathcal{V}_{sr} \) the set of quantized vertices.

III. CHOICE OF THE DISTORTION MEASURE

Since a remeshing technique is included in the proposed coder, we choose as distortion measure \( D_T \) the energy of the difference of "symmetry" between the input mesh \( \mathcal{M}_{ir} \) and the quantized normal mesh \( \mathcal{M}_{sr} \):

\[
D_T = d_S(\mathcal{M}_{ir}, \mathcal{M}_{sr})^2
\]

where \( d_S(\cdot, \cdot) \) represents the S2S distance.

A. Definition of the surface-to-surface distance

The S2S distance between \( \mathcal{M}_{ir} \) and \( \mathcal{M}_{sr} \) is defined by

\[
d_S(\mathcal{M}_{ir}, \mathcal{M}_{sr}) = \max[d(\mathcal{M}_{ir}, \mathcal{M}_{sr}); \tilde{d}(\mathcal{M}_{sr}, \mathcal{M}_{ir})],
\]

where \( d(\mathcal{M}, \mathcal{M}') \) is the unilateral distance between two meshes, given by

\[
d(\mathcal{M}, \mathcal{M}') = \left( \frac{1}{|\mathcal{M}|} \int_{p \in \mathcal{M}} d(p, \mathcal{M}')^2 d\mathcal{M} \right)^{\frac{1}{2}},
\]

\(|\mathcal{M}|\) is the area of \( \mathcal{M} \), and \( d(p, \mathcal{M}') \) is the distance between a point \( p \) belonging to a surface represented by a mesh \( \mathcal{M} \) and the surface represented by a mesh \( \mathcal{M}' \). This distance is defined by

\[
d(p, \mathcal{M}') = \min_{p' \in \mathcal{M}'} ||p - p'||_2 = ||p - \text{Proj}_{\mathcal{M}'}(p)||_2
\]

with \( ||\cdot||_2 \) the \( L_2 \)-norm, and \( \text{Proj}_{\mathcal{M}'}(p) \) the orthogonal projection of \( p \) over \( \mathcal{M}' \). To avoid a real computation of the S2S distance during the bit allocation, which is a computationally intensive process, we propose to approximate this distortion measure.

B. Proposed approximation of the surface-to-surface distance

First, the normal remeshing provides that the irregular mesh \( \mathcal{M}_{ir} \) and the normal mesh \( \mathcal{M}_{sr} \) are visually very similar. The S2S distance between them is thus negligible, and (1) can be approximated by

\[
D_T \approx d_S(\mathcal{M}_{ir}, \tilde{\mathcal{M}}_{sr})^2
\]

\[
= \max[d(\mathcal{M}_{ir}, \tilde{\mathcal{M}}_{sr}); \tilde{d}(\mathcal{M}_{sr}, \mathcal{M}_{ir})] \]

\[
\approx \max[d(\mathcal{M}_{sr}, \tilde{\mathcal{M}}_{sr}); \tilde{d}(\mathcal{M}_{sr}, \mathcal{M}_{sr})]
\]

\[
\approx \tilde{d}(\mathcal{M}_{sr}, \mathcal{M}_{sr})^2
\]

(5)

Let us study the difference of "symmetry" between the distances \( d(\mathcal{M}_{sr}, \mathcal{M}_{sr}) \) and \( \tilde{d}(\mathcal{M}_{sr}, \mathcal{M}_{sr}) \). Table I presents a mean of the relative errors between these two distances, computed on 5 typical models, and according to different bitrate ranges. The difference being very low (\(< 4\%\)) for each bitrate range, we can assume that \( \tilde{d}(\mathcal{M}_{sr}, \mathcal{M}_{sr}) \approx \tilde{d}(\mathcal{M}_{sr}, \mathcal{M}_{sr}) \), and we can approximate (5) by

\[
D_T \approx \tilde{d}(\mathcal{M}_{sr}, \mathcal{M}_{sr})^2
\]

\[
\approx \frac{1}{|\mathcal{V}_{sr}|} \int_{v \in \mathcal{V}_{sr}} d(\tilde{\mathcal{v}}, \mathcal{M}_{sr})^2 d\mathcal{M}_{sr}
\]

(6)

A normal mesh being densely sampled, the integral in (6) can be numerically approximated by a discrete sum [10], and (6) becomes

\[
D_T \approx \frac{1}{|\mathcal{V}_{sr}|} \sum_{v \in \mathcal{V}_{sr}} d(\tilde{\mathcal{v}}, \mathcal{M}_{sr})^2.
\]

(7)

with \( |\mathcal{V}_{sr}| \) the number of vertices of \( \mathcal{M}_{sr} \). Now, we have to approximate \( d(\tilde{\mathcal{v}}, \mathcal{M}_{sr}) = ||\tilde{\mathcal{v}} - \text{Proj}_{\mathcal{M}_{sr}}(\tilde{\mathcal{v}})||_2 \). Let us introduce the quantization error vector \( \mathbf{q}(v) \) between a vertex \( v \) and its quantized version \( \tilde{v} \). Asymptotically, i.e. for optimal high bitrate coding, this error vector is mostly colinear to the normal at the surface \( \mathcal{M}_{sr} \) in \( v \), since most of tangential components of a normal mesh are infinitesimal [8]. Also,
C. MSE across a wavelet coder

Denoted by $\mathcal{M}_{sr}$, we can make the approximation: $\text{Proj}_{\mathcal{M}_{sr}}(\hat{v}) \approx v$ [11]. Finally, we can state that $\|\hat{v} - \text{Proj}_{\mathcal{M}_{sr}}(\hat{v})\|_2 \approx \|\hat{v} - v\|_2 = \|\mathcal{E}(v)\|_2$, and (7) becomes

$$DT \simeq \frac{1}{|V_{sr}|} \sum_{v \in V_{sr}} \|\mathcal{E}(v)\|^2_2. \tag{8}$$

The right part of (8) is the MSE $\sigma^2_{\text{Qbr}}$ related to the normal mesh vertices. Finally, in case of densely sampled meshes, the energy of the S2S distance between the input mesh and the quantized normal one can be approximated by the MSE related to the quantization of the normal mesh geometry:

$$DT = d_S(M_{tr}, \mathcal{M}_{sr})^2 \simeq \sigma^2_{\text{Qbr}}. \tag{9}$$

To design a fast and low-complexity bit allocation for a wavelet coder, this approximation has to be expressed in function of the wavelet coefficient subbands.

IV. MODEL-BASED BIT ALLOCATION

A. Problem statement and solutions

The idea of the bit allocation across the wavelet coefficient subbands is to perform the best quantization of the coefficients optimizing the rate-distortion trade-off. The general purpose of the bit allocation process is precisely to determine the best set of quantization steps $\{q_{i,j}\}$ that minimizes the reconstruction error $DT$, at a given rate $R_{\text{target}}$. This can be formulated by the following problem:

$$(P) \left\{ \begin{array}{l}
\text{minimize} \quad DT(\{q_{i,j}\}) \\
\text{under constraint} \quad R_T(\{q_{i,j}\}) = R_{\text{target}}
\end{array} \right. \tag{12}$$

By using a lagrangian criterion and the distortion measure (11), this constrained allocation problem can be written as

$$J_\lambda(\{q_{i,j}\}) = \sum_{i=0}^N w_i \sum_{j \in J_i} \sigma^2_{Q_i,j}(q_{i,j}) + \lambda \left( \sum_{i=0}^N \sum_{j \in J_i} \alpha_{i,j} R_{i,j}(q_{i,j}) - R_{\text{target}} \right), \tag{11}$$

with $\lambda$ the lagrangian operator, $R_{i,j}(q_{i,j})$ the bitrate related to the $i, j^{th}$ component set. The coefficients $\alpha_{i,j}$ depend on the subsampling and correspond to $\alpha_{i,j} = \text{size}(\{x_{i,j}\})/(3 \times |V_{sr}|)$. The only way to allocate the bits in different subbands without pre-quantizating each subband is to perform a model-based bit allocation. This model-based bit allocation takes into account theoretical models for distortion and bitrate, depending on quantization steps and probability density functions of each data set. We have shown in [11] that distributions of tangential and normal component sets can be modeled by a Generalized Gaussian Distribution (GGD) [11]. Hence, according to [14], distortion and bitrate can be written as $\sigma^2_{Q_{i,j}} = \sigma^2_{T_{i,j}} D(q_{i,j}, \alpha_{i,j})$ and $R_{i,j}(q_{i,j}) = R(q_{i,j}, \alpha_{i,j})$, with $\sigma^2_{T_{i,j}}$ and $\alpha_{i,j}$ respectively the variance and the GGD parameter of the $i, j^{th}$ set, $\hat{q}_{i,j} = \frac{q_{i,j}}{\sigma_{T_{i,j}}}, D(\hat{q}_{i,j}, \alpha_{i,j})$ and $R(\hat{q}_{i,j}, \alpha_{i,j})$ are simple functions also given by [14].

The solution of this constrained allocation problem is obtained by computing the derivatives of (12) with respect to the normalized quantization steps $\{\hat{q}_{i,j}\}$ and $\lambda$:

$$w_i \sigma^2_{T_{i,j}} \frac{\partial D(\hat{q}_{i,j}, \alpha_{i,j})}{\partial \hat{q}_{i,j}} + \lambda \frac{\partial R(\hat{q}_{i,j}, \alpha_{i,j})}{\partial \hat{q}_{i,j}} = 0 \tag{13a}$$

$$\sum_{i=0}^N \sum_{j \in J_i} \alpha_{i,j} R_{i,j}(q_{i,j}) = R_{\text{target}}. \tag{13b}$$

B. Model-based Algorithm

In order to solve the system (13) and to speed the bit allocation process up, Parisot et al. propose to use, in case
of GGD, two precomputed tabulations of parametric curves \([R; \ln(-h)]\) and \([q'; \ln(-h)]\), with \(h\) given by [14]. The first one permits to verify the constraint on the bitrate (13b), and the second one permits to compute the quantization step verifying (13a). In that case, the algorithm becomes:

1) \(\lambda\) is given. For each set \(i, j\), compute the corresponding bitrate \(R_{i,j}\) with precomputed tabulations of \([R; \ln(-h)]\);
2) While (13b) is not verified, calculate a new \(\lambda\) by dichotomy and return to step 1;
3) For each set \(i, j\), compute the optimal quantization step \(q_{i,j}\) with precomputed tabulations of \([q'; \ln(-h)]\), and \(\lambda\) found in step 1;

The convergence of this algorithm is reached after few iterations, involving a fast and low-complexity process.

V. RESULTS AND CONCLUSION

We compare the performances of the proposed coder with state-of-the-art coders: the multiresolution coders NMC of Normal Mesh Compression [2], PGC of Progressive Geometry Compression [15], the coder of [5] for “meshes from MAPS”, and the single rate coder TG of Touma and Gotsman [9]. Fig. 3 and 4 show the resulting bitrate-PSNR curves for the models Rabbit and Horse. The PSNR is related to the S2S distance between the irregular input mesh and the quantized normal one, normalized by the bounding box diagonal of the input mesh (computed with MESH [16]). The given bitrate is the number of bits per irregular vertex.

We observe that the proposed coder provides better results than all the state-of-the-art coders: our method provides slightly better results than NMC (up to +1.2 dB). As expected, the MSE is a good way to approximate the S2S distance between the input mesh and the reconstructed one when a normal remesher is used to obtain the semiregular mesh. Finally, we design an efficient wavelet coder for 3D meshes including a bit allocation that optimize the quantization of the wavelet coefficients according to a target bitrate.

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