A top-down construction scheme for irregular pyramids

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1 Introduction
2 Recalls
3 A top-down model
4 Operations
5 Results
6 Conclusion and perspectives
### Framework

#### Application
- FoGrImMi project
- Very large medical images (30GB)
- Image processing and segmentation

#### Requirements
- Image representation
- Segmentation and manipulation of regions
- Focus of attention over interesting areas

#### Definition of a data structure
- Topological: process regions
- Hierarchical: multi-resolution images
- Top-down: limit memory requirements


## Timeline

<table>
<thead>
<tr>
<th>Model</th>
<th>Drawbacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadtrees</td>
<td>⇒ Segmentation problems</td>
</tr>
<tr>
<td>Regular pyramids</td>
<td></td>
</tr>
<tr>
<td>Irregular pyramids</td>
<td>⇒ Only bottom-up constructions</td>
</tr>
</tbody>
</table>

⇒ Definition of a top-down and topological framework for irregular pyramids
1. Introduction

2. Recalls

3. A top-down model

4. Operations

5. Results

6. Conclusion and perspectives
Combinatorial maps

Initial image
Disconnected faces
Disconnected edges
Map

Notions
- Dart: $\sim$ half-edge
- $\beta_1$ permutation: turns around a face
- $\beta_2$ involution: gives the other orientation of the edge
Topological maps

Requirements
- Represent any partition
- Describe adjacency and inclusion relationships
- Efficient processing algorithms

Combination of models
- Minimal combinatorial map (topology representation)
- Interpixel matrix (geometry information)
- Tree of regions (inclusion relationships)
Topological maps

Model features
- Complete (topology and geometry)
- Minimal (number of cells)
- Unique (same partition ⇔ same map)
## Pyramids

### Simple graph pyramids
- Stack of successively reduced graphs
- Difficult to update after operations

### Combinatorial pyramids
- Stack of successively contracted combinatorial maps
- Only bottom-up models
- Whole initial partition encoded

### Top-down pyramids
- Only encode upper levels
- Focus of attention: adjust segmentation from first discernable features
1. Introduction
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Goals and definitions

Goals
- Top-down topological model
- No explicit encoding
- Causal structure
- Easy update of the model after splitting

Definitions
- Pyramid ~ stack of linked topological maps
- A level $k$ is deduced from $k - 1$, applying splitting operations
The hierarchical data structure

Up/Down relations:

$G^k$  

Between darts

$G^{k+1}$  

Between regions
Global construction process

Main steps

- Create first map $G^0$
Global construction process

Main steps
- Create first map $G^0$
- $G^1$ is a copy linked to $G^0$
Global construction process

Main steps
- Create first map $G^0$
- $G^1$ is a copy linked to $G^0$
- Split $G^1$
Global construction process

Main steps
- Create first map $G^0$
- $G^1$ is a copy linked to $G^0$
- Split $G^1$
- Merge $G^1$
Duplicating a level

for each dart in $G^m$
- create a copy in $G^{m+1}$ ⇒ geometry
- link it with $G^m$ ⇒ up/down relations (darts)
- $\beta_1$ and $\beta_2$ sewing ⇒ topology

for each region in $G^m$
- create a copy in $G^{m+1}$ ⇒ adjacency relations
- link it with $G^m$ ⇒ up/down relations (regions)
- fill in region relations ⇒ tree of regions
Refining a level

Algorithm 1: Refining

\[
\text{foreach region } R \in G^k \text{ do}
\]

\[
\text{if splitting criterion}(R) \text{ is true then}
\]

\[
\text{Split}(R);
\]

\[
\text{Merge}(G^k, \text{merging criterion});
\]

\[
\text{Simplify the map;}
\]

Splitting criterion: selects one region for burst

Merging criterion: operates on a couple of regions with the same parent
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Splitting operation

Key points
- Initial region
Splitting operation

Key points
- Initial region
- Split edges
Splitting operation

Key points
- Initial region
- Split edges
- Insert dangling edges
Splitting operation

Key points
- Initial region
- Split edges
- Insert dangling edges
- Sew darts
Splitting Operation

**Burst method involvements**

- create one region/pixel
- costly

⇒ But it is necessary to traverse all pixels to compute colorimetric information on new regions
Merging Operation: general case

Key points
- Initial regions
Merging Operation: general case

Key points
- Initial regions
- Turn off geometry
Merging Operation: general case

Key points
- Initial regions
- Turn off geometry
- Relabel darts
Merging Operation: general case

Key points
- Initial regions
- Turn off geometry
- Relabel darts
- Remove darts
Merging Operation: general case

Key points
- Initial regions
- Turn off geometry
- Relabel darts
- Remove darts
- Result (after simplify)
Merging Operation: constraint

Constraint
Only merge regions resulting from the splitting of a same parent

Test
Does the shared edge have a parent?
Merging Operation: particular case

Multiple adjacency

Merging can be independent of criterion in multi-adjacency situations

Steps
Multi-adjacency between $R_1$ and $R_2$
Merging Operation: particular case

Multiple adjacency

Merging can be independent of criterion in multi-adjacency situations

Steps

Shared edges
Merging Operation: particular case

Multiple adjacency
Merging can be independent of criterion in multi-adjacency situations

Steps
Merging criterion
Merging Operation: particular case

Multiple adjacency
Merging can be independent of criterion in multi-adjacency situations

Steps
Remove edge 1
Merging Operation: particular case

Multiple adjacency

Merging can be independent of criterion in multi-adjacency situations

Steps
Remove edge 2
### Properties

#### Preserves causality
- each element of $G^k$ has *at least* one descendant in $G^{k+1}$
- each element of $G^k$ has *at most* one antecedent in $G^{k-1}$

#### Encode any partition
- unrestricted merging within the region burst results
- any connected set of pixels may be group into a region
- splitting does not depend on an initial geometrical pattern
Experiments

Preliminary results with basic segmentation criteria

Figure: (A) standard-deviation based segmentation; (B) gray levels comparison;
### Statistics

<table>
<thead>
<tr>
<th></th>
<th>$G^1$</th>
<th>$G^2$</th>
<th>$G^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of darts</td>
<td>600</td>
<td>7,728</td>
<td>19,090</td>
</tr>
<tr>
<td>Memory occupation (KB)</td>
<td>306</td>
<td>808</td>
<td>1,604</td>
</tr>
<tr>
<td>Total level construction time (s)</td>
<td>3.11</td>
<td>2.05</td>
<td>1.94</td>
</tr>
</tbody>
</table>

**Table:** Top-down construction applied to the Lena image (512*512)
Conclusion

- Topological maps extended to a top-down hierarchical model
- Applicable implementation with basic criteria
  - compare average gray level
  - refine following the homogeneity of the mother region
- Core operations defined for:
  - level segmentation
  - topology modification
Improve segmentation aspect
  - develop optimized criteria

Improve splitting algorithm
  - avoid burst method (one region/pixel)

Change geometry encoding
  - replace the actual explicit encoding

Define a tiled structure for each level
  - manage memory by swapping tiles on disk