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COMPUTATION OF ELECTRIC FIELDS AND POTENTIAL ON POLLUTED INSULATORS USING A BOUNDARY ELEMENT METHOD

J.L. Rasolonjanahary, L. Krähenbühl, A. Nicolas
Laboratoire d'Electrotechnique de Lyon, Ecole Centrale de Lyon,
URA CNRS 829, BP 163, 69131 Ecully Cedex, France.

Abstract—This paper presents a calculation method for electric fields and potential usable in the case of polluted insulator. This method based on boundary integral equations is very suitable for three-dimensional geometries and polluted layers. Results given by this method are compared with analytical solutions (potential, currents) and with measured values (potential, leakage current).

INTRODUCTION

The presence of pollution layer on HV insulators is very frequent in industrial and coastal regions. This pollution layer when combined with moisture becomes conductive and a leakage current flows through it. Dry bands can appear and the distributions of electric fields and potential become distorted and flashovers can occur. Therefore, it is important to compute electric fields and potential in studying the behaviour of polluted insulators.

Several numerical computation methods were developed for solving this problem. They are based on domain methods: finite differences[1,2] and finite element methods[3,4,5], charge simulation methods [6,7] and boundary methods[8,9]. Most of these methods are conceived for plane or axisymmetrical arrangements. In this paper we present a boundary integral equation method which can solve three-dimensional arrangements (geometry and pollution). It is a continuation of the work presented by HUANG in [12,13]. This method presents several advantages over the former ones (reduced number of unknowns, these being V and Ψ(=ερ) at the insulator surfaces, taking account of infinite domains).

THEORETICAL FORMULATION

Boundary integral equation method

The governing equation for potential distribution inside a domain Ω with a zero charge density obeys Laplace’s equation

\[ \nabla^2 V = 0 \quad \text{in } \Omega \] (1)

Applying Green’s theorem, we can express the potential V on all points P of Ω in terms of V and the variable Ψ(=ερ) on the boundary Σ of Ω as follows:

\[ cV(P) = \left( \nabla \frac{\delta G}{\delta n} \frac{1}{\varepsilon_f} \nabla \Psi \right)_{\Sigma} \] (2)

In this equation G is the Green’s function, n is the unit normal outward vector to Σ, c = A/4πr where A is the solid angle under which the point P sees the oriented surface Σ and ε_f is the relative permittivity of Ω.

Case of polluted insulator

In the case of polluted insulator, Fig. 1, we assume that the charge density is zero in the air domain Ω_A whose relative permittivity is ε_A = 1 and in insulating material domain Ω_2 which has a relative permittivity ε_A. The polluted area S of the insulator is characterized by its surface conductivity ω.

We write (2) for the air domain whose frontier is S_1 and for the insulating material domain whose frontier is S_2, we obtain for air domain,

\[ cV = \left( \nabla \frac{\delta G}{\delta n} \frac{1}{\varepsilon_A} \nabla \Psi \right)_{S_1} \] (3)

and for insulating material domain,

\[ cV = \left( \nabla \frac{\delta G}{\delta n} \frac{1}{\varepsilon_2} \nabla \Psi \right)_{S_2} \] (4)

Fig. 1. Polluted insulator.
In these equations $\Psi_1$ represents the value of $\Psi$ on the air side and $\Psi_2$ that of insulating material side: Fig. 2.

For clean areas, we have $\Psi_1 + \Psi_2 = 0$ and for polluted ones we use the conservation of charge in taking account of the surface current $J_s = -\sigma_s \text{grad} V$. The conservation of charge is written as

$$\text{div} J_s + \frac{\partial (D.n)}{\partial t} = 0$$

where $D$ is the electric displacement and $t$ is the time. The subscript $s$ indicates a surface operator. If we express $D.n$ in terms of $\Psi_1$ and $\Psi_2$ and if we use a sinusoidal source with an angular frequency $\omega$ ($\frac{\partial}{\partial t} = j \omega$ with $j = \sqrt{-1}$), (5) becomes

$$\text{div}(-\sigma_s \text{grad} V) + j \omega \epsilon_0 (\Psi_1 + \Psi_2) = 0$$

where $\epsilon_0$ denotes the free space permittivity. We introduce an auxiliary unknown $\Theta$ defined by $\Theta = \Psi_1 + \Psi_2$, so (6) becomes

$$\text{div}(-\sigma_s \text{grad} V) + j \omega \epsilon_0 \Theta = 0$$

We solve this equation by using a weighted residual method with the following boundary conditions, Fig. 3:

$$V = V_0$$

on $\Gamma_1$

$$\frac{\partial V}{\partial n} = 0$$

on $\Gamma_2$

Results

In this part, we report computations results which we compare with analytical and measured ones.

Comparison with analytical solutions:

The example we use here for comparison is extracted from [3]. The geometry is reproduced in Fig. 4 with $\epsilon_{rl} = 4$, $\sigma_s = 10^{-9}$ S and $V_0 = 100$ V. The potential $V(x)$ at a point $x$ is given by

$$V(x) = V_0 \frac{\cosh[(1 + j)k(1-x)]}{\cosh[(1 + j)k]}$$

where $k$ is the length of coating, $k = \sqrt{\frac{j \omega \epsilon_{rl} \sigma_s}{2 \epsilon_0}}$ with $a$ the thickness of the insulating material and $\omega = 100 \pi$. The modulus of $\Psi(x)$ and the tangential field are derived from $V(x)$. For solving this problem we add a width of 60 mm, so we obtain a three-dimensional geometry, Fig. 5. We compare the potential values along the line OA, Fig. 6-7. Also comparisons between analytical and computed values of the modulus of $\Psi$ and tangential electric fields show good similarities [11].
TABLE I

COMPARISON OF ANALYTICAL AND COMPUTED CURRENTS

<table>
<thead>
<tr>
<th>Capacitive current values (A)</th>
<th>Conductive current values (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>theoretical</td>
<td>computed</td>
</tr>
<tr>
<td>theoretical</td>
<td>computed</td>
</tr>
<tr>
<td>(1.0 \times 10^{-7})</td>
<td>(-2.10^{-9} + j2.10^{-2})</td>
</tr>
<tr>
<td>(1.0 \times 10^{-7})</td>
<td>(1.0 \times 10^{-7} + j1.1\times 10^{-7})</td>
</tr>
</tbody>
</table>

Comparison with measured values:

For this sort of comparison the geometries used for the measurements are axisymmetric, therefore we used only 1/12 of the real geometries in our calculation.

For the leakage currents, a sample insulator was sprayed with salt fog. Four experiments were done and for each experiment the leakage current was measured. The corresponding surface conductivity was calculated and this value was introduced in the computation. The computed leakage currents are compared with the measured values [12]: Table II.

TABLE II

COMPARISON OF MEASURED AND COMPUTED CURRENTS

<table>
<thead>
<tr>
<th>(V_f(V_{rms}))</th>
<th>(\sigma_s(\mu S))</th>
<th>measured values(mA)</th>
<th>computed values(mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>830</td>
<td>1.077</td>
<td>2.12</td>
<td>2.13</td>
</tr>
<tr>
<td>1070</td>
<td>1.03</td>
<td>2.62</td>
<td>2.64</td>
</tr>
<tr>
<td>600</td>
<td>3.54</td>
<td>5.02</td>
<td>5.07</td>
</tr>
<tr>
<td>1000</td>
<td>3.81</td>
<td>9.05</td>
<td>9.11</td>
</tr>
</tbody>
</table>

For the potential, the measurement was done along a portion \(A'C\) of the pin-insulator (generatrix). This is reported with its mesh in Fig. 8. The part \(ABB'A'\) was coated with a semiconducting layer \((\sigma_s=1.4 \times 10^{-8} \, \text{S})\). The computed \(\text{(PHSD)}\) and measured values \([8]\) are indicated in Fig. 9. Clearly, the values do agree.

These figures show that our results agree with analytical solutions. For the current values (capacitive and conductive), the comparisons are presented in Table I. We note a good agreement with analytical values.
CONCLUSION

We have presented a boundary integral equation method which presents several advantages over the existing ones in terms of the reduced number of unknowns called into play, the taking account of the infinite domain and the obtaining of direct values of potential and electric fields at the insulator surfaces: $V$ and $\Psi(=\epsilon E_n)$. The two kinds of comparison show that the proposed method is appropriate for three dimensional arrangements.

ACKNOWLEDGMENT

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REFERENCES

[10] L. Krahenbühl, A theory of thin layers in electrical engineering; Application to eddy current calculation inside a shell using the BIE software PHID3, 4th International IGTE Symposium and European TEAM Workshop,10-12 October 1990, Graz, Austria.