



Infrared drying process of an experimental water painting: Model predictive control

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MODEL BASED PREDICTIVE CONTROL OF INFRARED DRYING OF WATER-BASED PAINT

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ABSTRACT

This paper deals with the experimental control of an infrared drying process of a water based epoxy-amine painting. This approach is based on a unidirectional diffusional modelling of infrared drying phenomena where both heat and mass transfers under shrinkage conditions are accounted for. The control problem is concerned with the tracking of any given trajectory for one of the characteristics (i.e. the temperature or the mean water content) during the drying cycle. This is solved using the well-known model predictive control framework where the non-linear diffusional model is directly used in the control formulation. Experimental results show the efficiency of the trajectory tracking. This method can be extended for more general constrained control problem.

INTRODUCTION

Reactive painting drying is an important industrial problem through its impact for the quality of the final aspect of products in many industries (cars, cans, etc.). The main difficulty encountered during this operation is the evacuation of the solvent. This solvent is needed to simultaneously build the painting film and to ensure the film polymerization but one should avoid trapping it in the final dried product. Until now, the use of volatile organic compounds (VOCs) allowed the control of this evacuation during the polymerization reactions. But the enforced laws concerning environment lead now to replace the use of VOCs by the use of water based solvent that evacuates less rapidly and at higher temperatures than VOCs. To overcome this problem, infrared drying has been widely developed in industrial processes. Some studies [1,2] deal with diffusion problems with both infrared and water used as solvent.

The problem treated here is dealing with the on-line control of the drying cycle, which has recently become a need in drying. A control strategy has to be developed and has to be able to be implementable for on-line control, taking into account of natural physical limitations or specifications (constraints) inherent to any process. An advanced model based control strategy is well dedicated to solve this constrained problem: model predictive control (MPC). MPC or receding horizon control refers to a class of control algorithms in which a dynamic process model is used to predict and optimize process performance. The idea is to solve, at each sample time, an open-loop optimization problem over a finite prediction horizon in order to find the value of the manipulated variable that has to be implemented. The procedure is reiterated at the next sample time with the update of process measurements. Today, MPC has

become an advanced control strategy widely used in industry [3]. Indeed, MPC is well suited for high performance control since constraints can be explicitly incorporated into the formulation of the control problem. Therefore, it is not a surprise to see that it is an important tool for control engineers where plants being controlled are sufficiently slow for its implementation: indeed, one has to be able to solve an on-line optimization problem. Some applications of such optimization techniques in drying may be found in [4,5,6].

The paper is organized as follow: first, the first-principle diffusional model obtained in a previous work is briefly described. Then, the MPC strategy is detailed. Experimental results for both temperature and mean humidity trajectory tracking will be shown.

KNOWLEDGE-BASED MODELLING

Painting Formulation and Characteristics

Due to the great complexity of industrial paintings, the experiments are realized using a water based epoxy-amine painting formulated in our laboratory. This permitted us to experimentally determine every physical, thermal and chemical properties [7]. The painting film is composed of two elements:

- A resin constituted of an "oil in water" emulsion of DGEBA (Diglycidylether of Bisphenol A) which condensation index is equal to 0.15,
- A hardener composed of a primary triamine soluble in water. Currently used in the painting industry, it is named Jeffamine T403.

During the experiments, this painting film is coated in low controlled thickness (between 30 and 300 μm) on an iron substrate (also named the support) that has been first classically treated at its surface like in the automobile industry. The painting characteristics and support characteristics may be found in details in [7]. During the drying, two phenomena occur: the solvent vaporization (the water in the present case) and the reticulation. Given the dynamic of these phenomena (respectively about 100s for the vaporization and about 100 min for the reticulation in the present experimental conditions), the reticulation phenomena are not taken into account in this work. Thus, drying characteristics depend only on the temperature and the humidity. Moreover, a non-negligible deformation of the film happens during the drying due to water content (40% of the humidity in dry basis). Given the low sample thickness with respect to its surface, this deformation phenomena is characterized by the sample thickness variation.

Infrared Dryer

The near infrared panel-curing dryer used during the experiments (Fig. 1) was previously described in details [8]. The instrument part is composed of a pyrometer that allows the on-line temperature measurement of the sample at the upper surface and a precision balance that allows the follow-up of the sample and support set mean mass. The accuracy of the chosen balance is 0.001g since the water loss mass is in the order of 0.4g for a total mass about 100g (painting plus support). The infrared panel (see 2 on Fig. 1) is composed of 9 quartz lamps with a tungsten filament used in the wave length about 1.2 μm

that allows to obtain over the sample flow densities that range between 0 and 12 kW/m². A low convective airflow is produced by a fan (see 3 on Fig. 1) to eliminate the water vapour due to the drying.

Drying Modelling

The dynamic model of the painting film sample infrared drying is characterized by the humidity in dry basis X and by the assumed uniform temperature T . The temperature is assumed to be constant due the low thickness of the sample and due to the thermal characteristics of the support. This assumption has been experimentally checked. The partial differential equation (PDE) model S_{NL} is deduced from the following mass and energy balances.

Mass transfer

In the case of shrinking material, assuming an unidirectional transfer along the thickness z , the transfer of the solvent is diffusional and convective. In an eurlian (fixed) framework (z,t) , this transfer is function of the solid deformation rate. By introducing a lagrangian (mobile) framework (ξ,t) , this leads to write the diffusion equation of the solvent as follow [8]:

$$\frac{\partial X(\xi,t)}{\partial t} = \frac{\partial}{\partial \xi} \left(\frac{D}{(1 + \beta^* X)^2} \frac{\partial X}{\partial \xi} \right) \quad (1)$$

One has introduced the new independent space variables ξ to account for the shrinkage phenomena assumed to be characterized by a linear relation:

$$\frac{d\xi}{dz} = \frac{1}{1 + \beta^* X} \quad (2)$$

The initial condition is, at $t=0$ and for $0 \leq \xi \leq \delta_{dry}$, $X(\xi,t) = X_i$ and the two boundary conditions are:

- $\forall t > 0$ and at $\xi = 0$:

$$\frac{\partial X(\xi,t)}{\partial \xi} = 0 \quad (3)$$

- $\forall t > 0$ and at $\xi = \delta_{dry}$:

$$\dot{m}(\bar{X}, T) = - \frac{D \rho_d}{(1 + \beta^* X)^2} \frac{\partial X}{\partial \xi} \quad (4)$$

where $\dot{m}(\bar{X}, T)$, the drying rate, is expressed by the film theory [9] and $\bar{X}$ is the mean humidity in dry basis. As reported in the literature for polymeric solutions [1], it is assumed that the mass diffusion coefficient varies with the temperature and with the humidity content according to the relation:

$$D = D_0 \exp\left(-\frac{Ea}{RT}\right) \exp\left(-\frac{a}{X}\right) \quad (5)$$

where D_0 represents the pre-exponential factor, Ea is the activation energy and a is the humidity parameter.

Heat transfer

Due to the small painting film thickness and due to the great thermal heat diffusivity of the iron support, the temperature of the whole system (painting film + support) is assumed to be uniform. Consequently, and neglecting the heat due to the reaction, an overall heat balance accounting for the radiative and the convective heat losses and the evaporation losses (Fig. 2) leads to:

$$\left(\rho_p c_{pp} \delta_p + \rho_s c_{ps} \delta_s\right) \frac{dT(t)}{dt} = \alpha_{ir}(\bar{X}) \dot{q} - h_{cu}(T - T_u) - \sigma(T^4 - T_u^4) - h_{cl}(T - T_l) - \alpha_s \sigma(T^4 - T_l^4) - l_v(T) \dot{m}(\bar{X}, T) \quad (6)$$

with the initial condition $T(t=0)=T_i$ and where the infrared flow $\dot{q}(t)$ is the manipulated variable.

PROCESS CONTROL STRATEGY

Control Problem

At the end of the drying cycle, the final product obtained as to be usable: bubbles and fissures phenomena that may happen during the polymerization reaction have therefore to be avoided. To ensure the final product quality, paint producers propose to track a reference temperature trajectory during the drying cycle in order to extract water as best as possible before polymerization reactions starts. Therefore, the control problem considered here is to allow the process controlled variable (i.e. the temperature or the mean humidity in dry basis) to track any kind of time-variant reference trajectory. This leads to determine, on-line, the process manipulated variable (i.e. the infrared flow) to apply during the drying cycle. Moreover, process physical limitations concerning the magnitude and the velocity of the manipulated variable have to be taken into account in the problem resolution. Experimental results for temperature and mean humidity trajectory tracking will be shown.

Model Predictive Control Strategy

In previous works [10,11], we have introduced the MPC strategy to solve this trajectory tracking. The control problem is stated as an on-line optimization problem over a receding horizon Np where the performance index J to be minimized reflects the trajectory-tracking task. Since the problem will be solved numerically, a mathematical discrete time formulation is given. The tracking objective can be written as the initial constrained optimization problem at the actual discrete time k :

$$\begin{cases}
\min_{\tilde{u}} J(\tilde{u}) = \sum_{j=k+1}^{j=k+Np} [y_{ref}(j) - y_p(j)]^2 \\
\tilde{u} = [u(k) = \dot{q}(k) \dots u(j) = \dot{q}(j) \dots u(k+Nc-1) = \dot{q}(k+Nc-1)] \\
\text{and } \forall j \in \{k+Nc, \dots, k+Np-1\}: u(j) = u(k+Nc-1) \\
\text{subject to constraints on the manipulated variable:} \\
u_{\min} \leq u(j) \leq u_{\max} \quad \forall j \in \{k+1, k+Np\} \\
\Delta u_{\min} \leq u(j) - u(j-1) \leq \Delta u_{\max} \quad \forall j \in \{k+1, k+Np\}
\end{cases} \quad (7)$$

In this formulation, it is first to notice that in the performance index J expression, the knowledge of the process output y_p is required over the prediction horizon Np (i.e. for future times). These informations are obviously not available at the present time k , but this problem can be overcome using the IMC structure. In this structure, the manipulated variable is applied to both the process and its model. This structure allows to reformulate the tracking problem as:

$$y_{ref}(j) - y_p(j) = y_d(j) - y_m(j) = (y_{ref}(j) - e(j)) - y_m(j) \quad (8)$$

Assumption: [12]

The error e between the process output y_p and the model output y_m remains constant over the prediction horizon Np . The error value is updated at each sampled time k thanks to new measurements from the plant.

According to this assumption and with the IMC structure, the performance index J to be minimized can be expressed as:

$$J(\tilde{u}) = \sum_{j=k+1}^{j=k+Np} [(y_{ref}(j) - e(k)) - y_m(j)]^2 \quad (9)$$

where the model is introduced into the control algorithm. From a practical point of view, the second problem is the computational time aspect. Indeed, as it can be seen in the new performance index formulation, the model aims to predict the future dynamic behaviour of the process output over a finite prediction horizon Np and therefore has to be solved on-line. To reduce the on-line non-linear PDE model resolution time, we use a linearization method [13] of the non-linear model S_{NL} about a similar non-linear model S_0 computed off-line by choosing its input u_0 [11]. Finally, the off-line solved nonlinear model S_0 and the on-line solved linearized model S_{LTV} replace the initial nonlinear model S_{NL} as depicted Fig. 3 [11]. The control objective is now to find the variation Δu of the manipulated variable u about a chosen trajectory u_0 leading to the best optimisation result. A constrained optimization problem including an on-line resolution of the linear model S_{LTV} has therefore to be solved. The last point is concerned with the way to handle input constraints: this is done replacing the constrained parameter sequence $\Delta \tilde{u}$ by an unconstrained parameter sequence $\Delta \tilde{p}$ through a simple hyperbolic transformation [14] that uses the constraints bounds. Output constraints may also be accounted for through an exterior penalty method [14]. The optimiser argument is now an unconstrained argument and any unconstrained optimisation algorithm can be used to solve the final on-line penalized optimisation problem: widely known and used for its robustness and convergence properties, we apply the Levenberg-Marquardt's

algorithm [14] where the parameter sequence $\Delta\tilde{p}$ are determined at each sample time k using the model prediction and the process measurement [11].

EXPERIMENTAL RESULTS

In this first attempts, the control horizon N_c is tuned to 1: It allows to minimize the on-line computational time. Experiments have been realized to point out the influence of the control tuning parameter N_p over the tracking performance.

Operating Conditions

The operating conditions are the following one:

- the linearization about S_0 is performed with $u_0=5000$ W/m² and with the initial conditions $T_i = 36$ °C and $X_i=0.4$ kg/kg,
- the value for the sampling period T_s is 1s,
- constraints bounds are: $u_{\max}=12,000$ W/m², $u_{\min}=0$ W/m², $\Delta u_{\max}=500$ W/m², $\Delta u_{\min}=-500$ W/m²,
- atmospheric conditions are: $X_{\text{air}} = 20$ %, $T_u = 52$ °C, $T_l = 20$ °C,
- the control algorithm, written in Fortran code, has been combined to C code in order to realize the interface with the sensors and the actuator,
- the processor rate is 400 MHz.

In order to compare results for any value of N_p , one introduces the normalized cost function $J' = \frac{J}{N_p}$.

Temperature Reference Trajectory Tracking

From Fig. 4 and Fig. 5, one can see that the tracking objective is correctly achieved in all cases. Moreover, the intermediate value $N_p=6$ for the horizon prediction gives the best result:

- with a small prediction horizon ($N_p=3$), the discontinuities handling (around $t=80, 120$ and 200 s) is less efficient than with $N_p=6$ as confirmed by the values taken by the normalized cost function. In this case, informations quantity available describing the future process behaviour are insufficient. In a way, with $N_p=3$, the problem is badly stated for its resolution, as we can see on the applied control: when the three discontinuities points appear, the infrared flow is always either saturated in magnitude (Fig. 6) or in velocity (Fig. 7). The optimization procedure does not correctly capture the future behaviour of the process. Then, the algorithm tends too often to find a non-admissible solution and to do bang-bang control. This leads consequently to poor tracking performances ;
- increasing the prediction horizon value to 6 and 12, the infrared flow becomes more and more smooth (Fig. 6), but with a big prediction horizon ($N_p=12$), another problem appears for $0s \leq t \leq 80s$: the model, qualitatively true, is quantitatively false. Since more values calculated by the model resolution are taken into account in the optimization problem, the criteria minimization is less efficient than in the case where the prediction horizon takes an average value ($N_p=6$). This is confirm when tuning N_p to values higher than 12.

Therefore, the prediction horizon value $N_p=6$ is the ``optimal" choice for this main parameter: tracking results with N_p tuned to 5, 7, 8, 9, 10 or 11 were less interesting.

Mean Humidity Reference Trajectory Tracking

As mentioned previously, MPC strategy allows to track any reference for any measured variables. Tracking of the mean humidity in dry basis is now presented. For the operating conditions, u_0 has been retuned to 500 W/m². Tracking results are still acceptable (Fig. 8) and infrared flow determined by the control algorithm is still in agreement with infrared panel physical limitations (Fig. 9). Influence of the horizon prediction can be seen as the intermediate tuning of N_p to 8 gives best overall performance. Moreover, robustness of the control algorithm can be seen: an underestimation of 200% between the real initial thickness of the sample and the initial thickness fed into the model does not seem to affect tracking results. However, overall tracking results are now less efficient: it is mostly due to the strongly noise sensitive measure of mass used in the feedback law [11]. To overcome this problem, use of the receding horizon observer using the unnoisy measure of temperature to estimate the mean humidity in dry basis like in [6] is under investigation.

CONCLUSION

In this paper, an efficient approach for the on-line control of an infrared drying process has been shown. The model predictive control approach uses a diffusional model leading to the knowledge of the drying characteristics, i.e. the temperature or the mass content. To allow the on-line application of this method, the nonlinear diffusional model is first solved off-line. Adjustments in the infrared flow to apply are then computed on-line using a linearized model involved in the constrained optimization problem. The final tracking experimental results are very interesting. It shows that this two-phase MPC approach allows to determine a physically applicable infrared flow. The influence of the MPC tuning parameter, i.e. the prediction horizon, has been point out even if the results are acceptable in all cases. Moreover, since the balance measurements may be very noisy, a new approach is under study for a better humidity set points tracking: it is concerned with a soft sensing approach where both the model and the unnoisy temperature measurements are needed instead of the balance measurements. Moreover, since any objective function may be stated, one could introduce a new one that handle directly the final properties of the product through constraints during drying.

NOMENCLATURE

a	exponential factor	kg.kg ⁻¹
α_{ir}	infrared absorption coefficient	-
β^*	linear shrinkage coefficient	-
c_p	specific heat	J.kg ⁻¹ .°C ⁻¹
D	water or organic solvent apparent diffusion coefficient	m ² .s ⁻¹
D_0	water or organic solvent apparent diffusion parameter	m ² .s ⁻¹
δ	coating thickness	m

E_a	activation energy	J.mol^{-1}
h_c	convective heat transfer coefficient	$\text{W.m}^{-2}.\text{K}^{-1}$
J	performance index	-
j, k	discrete time indexes	-
l_v	latent heat of vaporization	J.kg^{-1}
$\dot{m}$	drying rate	$\text{kg.m}^{-2}.\text{s}^{-1}$
N_p	prediction horizon	-
N_c	control horizon	-
p	unconstrained parameter	
$\tilde{p}$	unconstrained parameter sequence	
q	infrared flow	W.m^{-2}
R	universal gas constant	$\text{J.mol}^{-1}.\text{K}^{-1}$
ρ	material density	kg.m^{-3}
S_{NL}	initial nonlinear system	
S_0	nominal system computed off-line	
S_{LTV}	linear time variant system computed on-line	
σ	emissivity coefficient	$\text{W.m}^{-2}.\text{K}^{-4}$
t	time	s
T	temperature	$^{\circ}\text{C}$ or K
u	manipulated variable, control	
$\tilde{u}$	control sequence	
ξ	coordinate (mobile frame)	m
X	local moisture content (dry basis)	kg.kg^{-1}
$\overline{X}$	mean moisture content (dry basis)	kg.kg^{-1}
y	signal	
z	coordinate (fixed frame)	m

Subscripts

d, dry	dry solid or desired
i	initial (at $t=0$)
l	lower surface of the sample
m	model
p	process
ref	reference
s	support
u	upper surface of the sample

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LIST OF FIGURES

Figure 1: Laboratory Scale Tunnel Dryer.

Figure 2: Thermal Flows.

Figure 3: Linearized Time Variant Internal Model Control (LTVIMC) structure.

Figure 4: Temperature Trajectory Tracking: Tracking result for $Np=3, 6$ and 12 .

Figure 5: Temperature Trajectory Tracking: Tracking error for $Np=3, 6$ and 12 .

Figure 6: Temperature Trajectory Tracking: Infrared Flow Magnitude for $Np=3, 6$ and 12 .

Figure 7: Temperature Trajectory Tracking: Infrared Flow Velocity for $Np=3, 6$ and 12 .

Figure 8: Mean Humidity Trajectory Tracking: Tracking result for $Np=5, 8$ and 15 .

Figure 9: Mean Humidity Trajectory Tracking: Infrared Flow Magnitude for $Np=5, 8$ and 15 .

















