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# BUILDING 3D INDOOR SCENES TOPOLOGY FROM 2D ARCHITECTURAL PLANS

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**Keywords:** 3D modelling, architectural scenes, reconstruction, topological model.

**Abstract:** This paper presents a new method for reconstructing geometry and topology of 3D buildings from 2D architectural plans. A complete topological model expresses incidence and adjacency relations between all the elements. It is necessary for both recovering accurately 2D information and constructing a coherent 3D building. Based on an existing topological kernel, several high-level operations have been developed in 2D for creating walls, portals, stairs, etc. Semantic information is associated with all volumes for specifying openings, walls, rooms, stairs, facade, etc. The resulting 2D model is extruded for generating a 3D environment, taking the semantic information into account since doors are not processed as walls for instance. Floors are superimposed using volumes corresponding to upper and lower ceilings linked according to stairways. The resulting models are suitable for various application such as walkthrough, lighting/wave propagation/thermal simulation.

## 1 INTRODUCTION

Accurate three dimensional descriptions of architectural environments is an important need for many building trades such as lighting engineering, thermal simulations, etc. Generally, the models produced by the architects are handled in two dimensions without any topological information. However, in addition to a three dimensional description of the model, many simulation algorithms require adjacency and incidence relationships between volumes.

Unfortunately, manually reconstructing complex architectural scenes using a 3D modeller is a long and tedious process. This is why we propose a new method for automatically reconstructing 3D buildings from 2D architectural plans (figure 1). Our method inherently integrates a complete topological description of the environment. The resulting scenes can thus be edited in a topological modeller for adding furniture, moving walls, etc. In addition, semantic information is used for defining object attributes such as rooms, floor, corridors and so on.

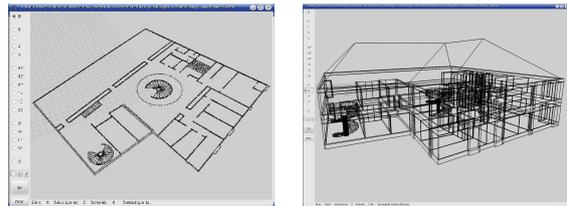


Figure 1: 2D plan and 3D reconstruction.

Our aim is to define a building model corresponding to a 3D partition of space. Each room should correspond to a closed and orientable volume, incident to closed and orientable faces. Amongst the existing topological models, we have chosen generalized maps which allow to represent space subdivisions and incidence and adjacency relations.

The reconstruction method we propose is based on four main phases: (i) 2D edges processing for removing geometrical inconsistencies, (ii) topological reconstruction with semantic information, (iii) 3D building extrusion (iv) superimposing of floors.

During the first phase, most 2D imprecisions (90%) are automatically corrected. However, remaining awkward elements can be semi-automatically processed. Semantic information can be deduced from the plans when they exist, or defined manually by the user. The extrusion operation is guided by the semantics. For instance, walls, doors or stairs are extruded using specific rules. Superimposing of floors is applied using semantics and topology.

The resulting topological indoor models have been designed so that volumes adjacency can be efficiently used for various types of simulation. For instance such models prove efficient in the context of radiosity or photon mapping approaches [11, 6].

This paper is organized as follows. Section 2 describes the existing methods for 3D architectural scenes construction. Section 3 justifies our choice of generalized maps. Section 4 presents our method of geometrical and topological rebuilding in two dimensions. Section 5 details the extrusion and superimposing of several floors. Section 6 discusses the results obtained with several examples of real buildings.

## 2 RELATED WORK

Many methods in the literature propose to rebuild urban environments. For instance, the MATIS team in the research section of the French *Institut Geographique National* (IGN) proposes an elevation method starting from satellite photographs [7]. Ah-Soon et al. processes digitized 2D drawings (i.e. images of plans) for 3D reconstruction [1]. Recognition is based on the detection of vertical and horizontal symbols. The aim is to analyze the interior geometry of a building as well as the openings location (doors, windows, etc). This work primarily concerns methods of image analysis. The geometry reconstruction produces 3D scenes without topological information.

Several methods aim at extracting topological information from a list of polygons, making it possible to reduce calculations of visibility for lighting simulation and visualization. Airey et al. propose a method of binary space subdivision (Binary Space Partitioning or BSP) for axis-aligned environments [2]. Teller et al. present an extension of this method for all types of walls [15]. Meneveaux et al. propose a method containing rules to find the parts of the buildings [12]. All these subdivision schemes produce a set of regions called *cells*, separated by

openings. The topological description corresponds to adjacency relations between 3D cells, there no incidence/adjacency relations between lower-dimensions elements.

Complex urban scenes can also be produced using procedural modeling, such as *cityEngine* [14, 13]. Several parameters can be taken into account: population density or height maps. The road network is generated using a L-System mechanism. A construction grammar is used to create the building facade. Although these methods generate realistic (but not real) geometrical environments, topological information is not managed.

## 3 GENERALIZED MAPS

We wish to represent buildings made up of volumes (floors, walls, rooms, etc), each of them being a *orientable* 3D object. We need a subdivision of space into faces, edges, vertices, defined by their boundaries (boundary representation) in any dimension. Maps and generalized maps offer an implicit representation of cells with efficient operations since a local modification in the map is automatically propagated to the incident edges.

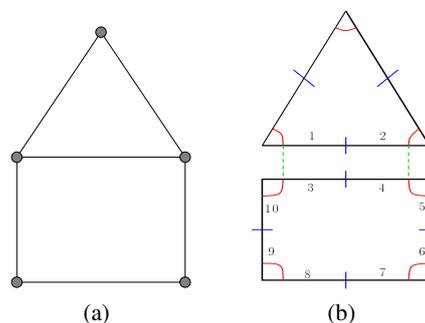


Figure 2: (a) A 2D object containing 2 faces, 6 edges and 5 vertices; (b) corresponding generalized map: the set of darts  $\{1,2,3,4\}$  represents an edge, the set of darts  $\{3,4,5,6,7,8,9,10\}$  represents a face.

Several topological models allowing space subdivisions have been proposed in the literature: structures containing adjacency graphs [4], 2D/3D models based on edges [3, 8, 18] or models capable of handling higher dimensions [4, 10]. As explained in the following section, many reasons have motivated our choice for *generalized maps* [10].

It has been shown in [9] that existing topological models representing 3D subdivisions are comparable with 3D maps (for orientable models without bound-

ary) or with 3D generalized maps (for orientable or not models, with or without boundary). Even though 3D generalized maps are more expensive than 2D maps in the memory, we have chosen this model since it provides a homogeneous representation in all dimensions. This simplifies many operation definitions.

From a single type of basic elements (called *darts*) and one to one mappings  $\alpha$  defined on these darts, generalized maps represent object cells and adjacency/incidence relationships. Each mapping  $\alpha_i$ , with  $0 \leq i \leq n$  ( $n$  being the highest dimension used), represents the adjacency relations between  $i$ -dimensional cells;

**Definition 1 (Generalized map [10])**

A generalized map in dimension  $n \geq 0$  (or  $n$ -G-map) is an algebra  $G = (D, \alpha_0, \dots, \alpha_n)$ , where:

- $D$  is a finite set of darts;
- $\alpha_0, \dots, \alpha_n$  are involutions<sup>1</sup>;
- $\alpha_i \alpha_j$  is<sup>2</sup> an involution for all  $i, j$  such that  $0 \leq i < i+2 \leq j \leq n$ .

Two darts  $d$  and  $d'$  are  $\alpha_i$ -sewed if  $d\alpha_i = d'$  with  $d \neq d'$ , and  $d$  is  $\alpha_i$ -free if  $d\alpha_i = d$ . The  $i$ -cell associated with a given dart  $d$  is composed of all the darts obtained by a coverage starting from  $d$  and using all the involutions except  $\alpha_i$  (see figure 2). The number of distinct edges incident to a vertex defines its *degree*. When the vertex is incident to only one edge, the edge is called a *dangling edge*.

On the basis of this representation, we have used the 3D topological modeller MOKA [17], comprising many operations such as sewing two cells along a face or more complex operations like sweeping or corefining.

## 4 2D RECONSTRUCTION

For extruding a 3D building, a valid topology has to be reconstructed from the 2D plan. Therefore, the dataset has to comply with three fundamental properties: (i) edges should not be merged, (ii) edges should not intersect (iii) edge vertices should all be incident to another edge. (i) and (ii) ensure that the plan is a partition of a 2D space in faces, edges and vertices. In a 2D architectural plan every object is

<sup>1</sup>A bijection  $f$  is an involution iff  $f^2 = Id$

<sup>2</sup>If  $\beta$  and  $\gamma$  are applications of  $E \rightarrow E$ ,  $\beta\gamma$  corresponds to the composition  $\gamma \circ \beta$ , and  $b\beta\gamma$  is the application of this composition to element  $b$  of  $E$ .

usually defined with a given thickness. Consequently edges should not be isolated, which corresponds to (iii). When these 3 properties are verified, the set of edges is said *valid*.

Unfortunately, modeller software used by architects is not devised for 3D *topological extrusion*. Consequently, in most cases, none of the above properties is maintained. The reconstruction robustness of our method highly depends on the detection and correction of all geometrical inconsistencies contained in the 2D plans. Our application corrects geometry and builds up the topological model. It is composed of two parts: the first one consists of geometry error detection and correction while the second one constructs topological information. The final goal is to link edges so as to produce 2D faces.

In practice, for the plans we have used, 90% of imprecisions are automatically corrected. However, some remaining awkward elements have to be processed. Therefore, we propose semi-automatic operations for correcting the plans (see section 4.4).

Semantic information can be deduced from the plans when it exists, or defined manually by the user. Finally, each type of object contained in plans is associated with semantics: walls, rooms, openings, stairs, etc.

The general algorithm of 2D reconstruction is broken up into the following steps: (1) edge extraction from source file (2) geometry correction (3) topological construction (4) semi-automatic finalization (5) semantics association.

### 4.1 Geometrical correction

Once the edges have been identified in the source file, the plan analysis is performed. Therefore, a threshold  $\epsilon$  is defined for testing whether two edges are superimposed and finding all the edges incident to a given vertex. In practice, we have fixed  $\epsilon = 1mm$ .

Two edges are superimposed if they have the same slope, the same origin ordinate and at least one extremity included in the other edge. In this case, both edges are merged into a single one.

All the edge intersections are processed two by two. If an intersection is found, a vertex is added at the intersection point on the concerned edges.

## 4.2 Topological reconstruction

The above processing produces a set of valid segments used to construct topology. All the adjacency and incidence relationships between vertices, edges and faces have to be defined.

### 4.2.1 Edges creation

Each edge is associated with four darts corresponding in 2D to an edge shared by two faces. Links  $\alpha_0$  and  $\alpha_2$  are immediately set on the corresponding darts. Only  $\alpha_1$  remain to be processed for creating faces.

Since buildings are orientable objects, composed of orientable elements, we also need to set an orientation to the whole generalized map. This is why darts are associated with a boolean mark indicating the edge orientation. For a dart  $d$  marked,  $d\alpha_0$ ,  $d\alpha_1$  and  $d\alpha_2$  are not marked.

### 4.2.2 Angle arrangement

For 1 and 2-degrees vertices, the corresponding darts are directly connected by  $\alpha_1$ . For each vertex of higher degree, the incident edges are stored and sorted according to their angle around the vertex. The algorithm is the following:

1. search for dart  $d$   $\alpha_1$ -free;
2. search for all darts  $\{d_i\}$   $\alpha_1$ -free, incident to  $d$ ;
3. sort  $\{d_i\}$  according to the angle with  $d$  (corresponding to the angle formed by the associated edges);
4.  $\alpha_1$ -sew the darts two by two according to this order, with respect to the orientation constraints.

### 4.2.3 Face inclusion

In most plans, some objects are included in others. For instance, stairs are included in rooms. Unfortunately, with boundary representations, these objects are not connected. Consequently, there is no relative position between elements. This is why we have used fictive edges for linking the existing connected components. On the floor, a fictive edge is thus used to link an external face to the included ones. These edges are called fictive edges since they do not represent the boundary of a face.

## 4.3 Accelerating structure

With the process described above, many operations require testing couples of darts according to their location in the plans. The use of an accelerating structure makes it possible to reduce the processing time. Since the plans scale is defined in meters, we choose a uniform grid made up of  $1\text{meter} \times 1\text{meter}$  tiles. Each tile is associated with the list of segments which cross it. Thus, for each segment, tests are performed only in a local neighborhood. Note that segments corresponding to walls only belong to a few tiles.

## 4.4 Additional operations

To eliminate inconsistencies that are not automatically corrected, we propose several semi-automatically operations. Based on the low-level operations *sew* and *removal* defined in [5], we propose higher-level operations for processing several edges at the same time: (i) for sewing two selected edges; (ii) for sewing several selected *dangling edges* to the closest edge; (iii) for sewing all the *dangling edges* to the closest edge; (iv) for topologically removing  $n$  selected edges.

It can be necessary to add doors on the plans. We also propose an operation for creating a door, starting from the selection of two walls. A door is inserted in the plan, and associated with its semantic.

## 4.5 Semantic definition

Semantic information allows the user to know the type of each element in the plan. The objects are classified into various categories: walls, doors, floor, ceiling, stairs, etc. Any type of new information can be conveniently added to the model. In practice, each dart holds a label corresponding to its semantic.

During the reconstruction process, it is possible to use the layers contained in the source file for indicating the semantic of objects. The user can also select part(s) of the building and manually modify semantics. This information is used for guiding the extrusion process described below.

## 5 3D EXTENSION

The starting point of the 3D extrusion is a 2D plan composed of faces, edges and vertices associated

with a consistent topology (i.e. an orientable 2D partition, closed and without *dangling edges*). Each floor is handled using several types of operations. Therefore, we have adapted the extrusion already existing in the MOKA library. Each type of element is specifically processed.

The topological 3D representation has to comply with several important features.

1. The 3D model must be a closed space since each room, wall and portal are defined as a closed volumes. For instance, rooms are defined by volumes with transparent faces corresponding to portals. Consequently, each face should be incident to exactly two volumes. Obviously, faces also have to be closed as well as edges.
2. Each building must be composed of a single 3D connected component. For instance, faces or volumes defining *holes* have to be connected with their respective faces or volumes.
3. The model must be oriented since each part of the building should be clearly identified as the *inside* or the *outside*.

These constraints are guaranteed by the properties of the 2D plan and the extrusion operation.

Finally, the building extrusion is organized as follows: (i) extrusion of the floor (wall, doors), (ii) creation of the ground and ceiling, (iii) superposition of floors, (iv) stairs construction.

### 5.1 Extrusion of walls

For extruding walls, a vertical path is defined with a height equal to 2,5 meters. From each face of the 2D plan, a volume is automatically created and connected using  $\alpha_3$  to the corresponding face (see figure 3). Contrary to existing modeler, our topological extrusion of two faces connected by  $\alpha_2$  produces two volumes connected by  $\alpha_3$ . Note that fictive edges are not extruded so that no useless fictive face be created.

The extrusion operation defined above allows the construction of non vertical walls (the extrusion path has to be properly defined). However, in most cases the slope is not defined on the plans. Moreover, the modeler MOKA can be easily used for modifying the upper wall edges.

### 5.2 Extrusion of openings

On the plans, door volumes are topologically connected to the surrounding walls. The portion of

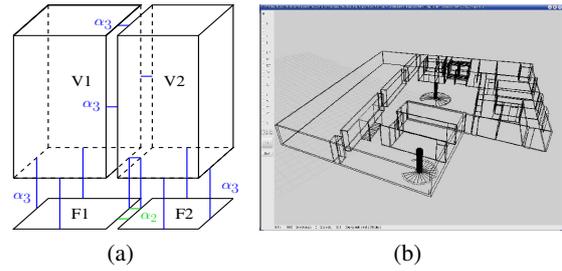


Figure 3: Extrusion of walls: (a) volumes  $V_1$  and  $V_2$  are built and connected to the corresponding faces  $F_1$  and  $F_2$ . Since  $F_1$  and  $F_2$  are connected by  $\alpha_2$ ,  $V_1$  and  $V_2$  are connected by  $\alpha_3$ . (b) sample floor with walls extrusion.

wall above to the door is created. Therefore, the 2D polygon representing the door is extruded according to a vertical path of two components (corresponding to the opening and the portion of wall above the door). The two resulting volumes are superimposed and topologically connected. In a second step, they are connected to the remainder of the building, along the door stiles. Two vertices and an edge must be inserted on the stiles to respect the topological constraints (figure 4).

For windows, the same operation can be applied with an additional component corresponding to the wall part located under the window.

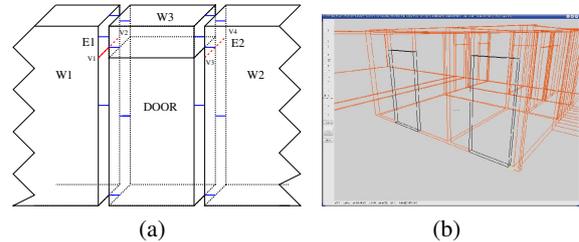


Figure 4: Door extrusion: (a) extrusion and connexion to the walls  $W_1$  and  $W_2$ , the section of wall named  $W_3$  is built above the door. Four vertices ( $V_1, V_2, V_3, V_4$ ) and two edges,  $A_1$  and  $A_2$ , are inserted on  $M_1$  and  $M_2$ . (b) Result of door extrusion.

### 5.3 Creation of grounds and ceilings

With the extrusion system described above, the darts of the 2D plan are  $\alpha_3$ -connected to volumes defining walls or doors (figure 5.a).

The 2D plan is used to create the ground volume (flagstone). Therefore, the contour of the 2D plan is  $\alpha_1$  and  $\alpha_3$ -disconnected (figure 5.b) and the corresponding edges are used to form the desired volume (Figure 6). The external faces of this volume are  $\alpha_2$ -sewed with the faces representing the floor contour (figure 7).

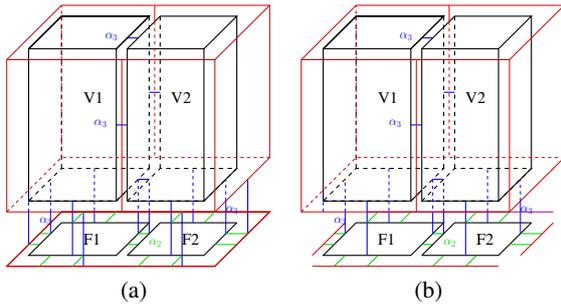


Figure 5: (a) Contour faces are marked for being used during the creation of the floor; (b)  $\alpha_1$  and  $\alpha_3$  are unsewed for the 2D plan contour.

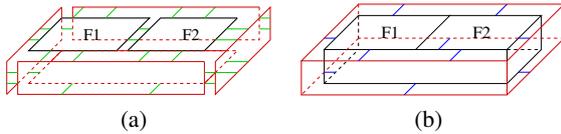


Figure 6: Ground creation: (a) the 2D plan contour is used to construct the ground volume; (b) the ground volume is closed (red volume corresponding to the outer part).

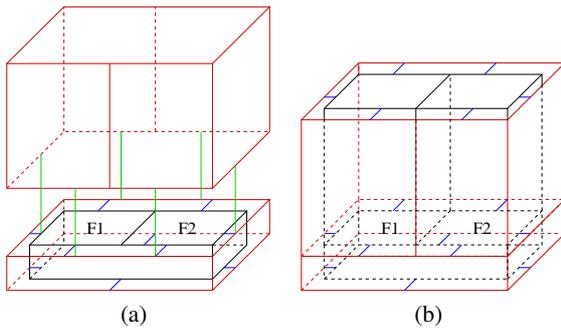


Figure 7: (a) The outer ground volume is sewed by  $\alpha_2$  to the external floor volume (facade); (b) the resulting open volume (in red) represents the floor facade and the ground.

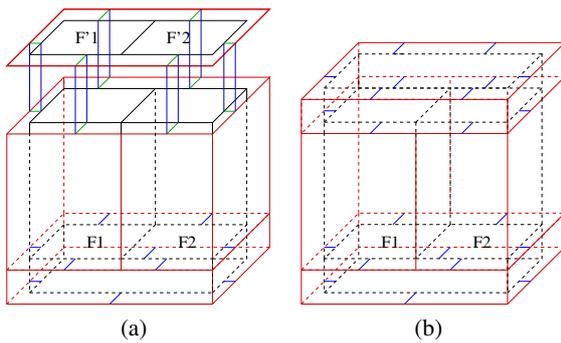


Figure 8: Ceilings creation: (a) the horizontal faces at the top of the floor are duplicated; (b) duplicated faces are used for creating the ceiling volume with the same method as for grounds. The resulting closed volume describes the facade and actually defines the rest-of-the-world volume.

The construction of the ceiling requires the copy of the 2D plan at the top of the floor (Figure 8.a). During duplication, each new dart is sewed by  $\alpha_3$  to its corresponding dart. The external segment is used to construct the ceiling volume (Figure 8.b). The external faces are sewed by  $\alpha_2$  with the darts of the 3D floor contour. This operation produces 4 types of volumes: ground, ceiling, indoor description, facade.

## 5.4 Superimposing of floors

For superimposing two floors with same outer 2D shape, the ground of the upper floor is connected to the ceiling of the lower floor (Figure 9). In practice, the two volumes are sewed by  $\alpha_3$  and the shared face is removed. Thus, only one volume defines the space between the two floors.

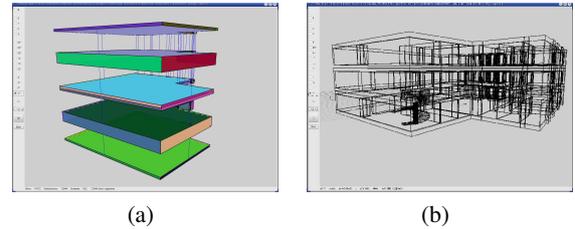


Figure 9: Creation of grounds and ceilings. (a) Visualization of the connections between the grounds and ceilings of two floors. The ceiling of the lower floor and the ground of the upper floor are merged. (b) Building made up of two superimposed floors.

## 5.5 Creation of stairs

Stairs can be defined with various shapes on the 2D plans: straight, snail, elliptic, etc. They are often disconnected from the rest of the plan or joined to the walls. We propose a generic method for creating the stairs topology. The steps geometry is computed according to the data recovered on the plan (length, width, or diameter in the case of spiral stairs). Presently, our method does not provide any automatic system for detecting the geometric type of stairs, the user manually selects the appropriate method.

Each step is composed of two volumes (figure 10.a). A surrounding volume ensures the model closeness (figure 10.b). Once created, the 3D stair is connected to the remainder of the plan by a fictive edge. The ceiling is perforated according to the stair shape using of a boolean operation. Therefore the stair contour is extruded according to the ceiling height and the resulting volume is subtracted from the ceiling so as to create the opening (Figure 11).

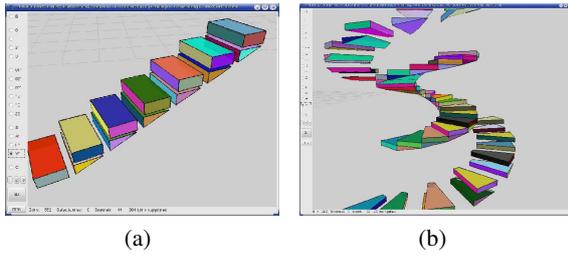


Figure 10: Construction of stairs. (a) Steps of a straight staircase composed of two volumes. (b) Volumes defining snail stairs. (c) Result of snail stairs in a building with ceiling opening.

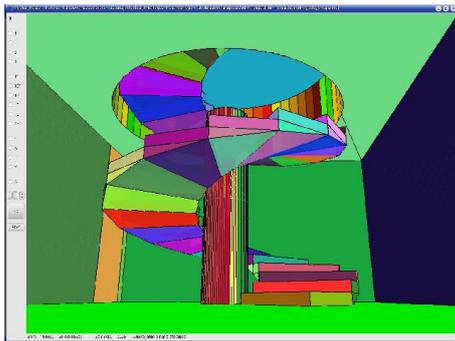


Figure 11: Result of 3D stair reconstruction.

## 6 IMPLEMENTATION AND RESULTS

Our reconstruction system has been implemented in C++, using the MOKA library [17]. The source files have been produced by architects in *dxf* format. Computing times have been obtained with a centrino processor: 2Ghz and 1GB of RAM.

We have applied our reconstruction method to various 2D architectural plans. The processing times for the 2D analysis are presented Table 1. They depend on the distribution of the segments in the plan and on the uniform grid acceleration. The processing of a floor never exceeds one minute, even for complex plans.

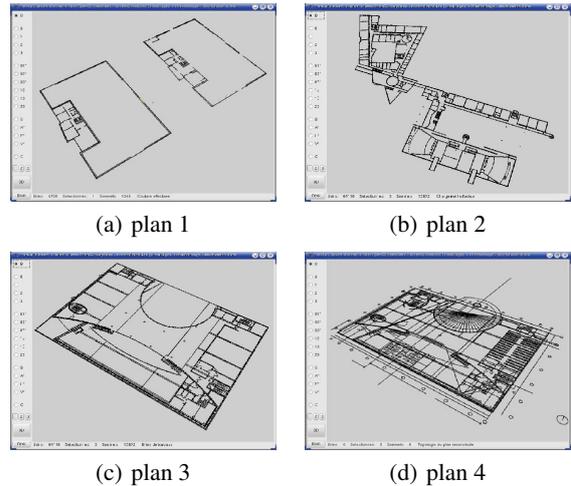


Figure 12: Plans used for geometrical and topological reconstruction.

Buildings	# Segments of the scene	Geometric processing	Topological processing
plan 1	899	2s	1s
plan 2	8050	9s	16s
plan 3	8120	11s	13s
plan 4	11972	56s	59s

Table 1: Processing time of the automatic 2D reconstruction.

With the proposed method, 90% of the plans incoherencies have been detected and corrected automatically. The time required for manually correcting the 2D models incoherencies is about a few hours, depending on the model complexity and the numbers of errors contained in the plan. From this point, the 3D reconstruction becomes completely automatic and takes only a few seconds. Moreover, all the topological and semantical information are automatically propagated.

## 7 CONCLUSION

This paper presents a new method for reconstructing a 3D description of buildings from 2D architectural plans. The resulting description includes geometry and topology so that the whole environment consistency be preserved according to constraints such as closeness, orientability, and connectivity.

The main steps of our method concern: (i) a geometrical correction of source data, (ii) a 2D topology construction, (iii) a 3D extrusion system, (iv) floors

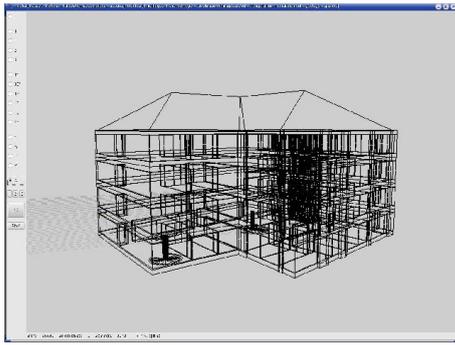


Figure 13: Result of reconstruction 3D.

superimposing. We also propose semi-automatic tools for correcting 2D plans. The results obtained show that 2D and 3D processing require only a few tens of seconds.

The main advantage of our method concerns the use of topology for validating the building structure coherence and editing the model using modelling operations. Furthermore, the resulting structure provides various types of information necessary for visualization or lighting/thermal/low-frequency wave propagation simulations [11, 6, 16].

The next step of this work consists in automatically defining additional semantics (such as rooms or furniture for instance). Thus operations dedicated to volume types can be explored for simplifying the 3D models. We also aim at automatically detecting stairs and their characteristics. Moreover, additional operations have to be defined, for instance related to windows, roofs or superimposed floor with different shapes.

In the future, we wish to apply our system to urban scenes as well, containing furnished buildings, etc. This implies the processing of larger data with missing information. We aim at coupling our system with procedural reconstruction methods.

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