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## Abstract

*In this paper we present a method for impulse noise removal that makes use of spectral clustering and graph regularization. The image is modeled as a graph and local spectral analysis is performed to identify noisy and noise free pixels. On the set of noise free pixels, a topology adapted graph regularization is performed. Experimental results show the benefits of the proposed approach regarding the standard VMF when noise proportion is high.*

## 1. Introduction

Images are often corrupted by impulse noise due to noisy sensors or channel transmission errors. There are many works on the restoration of images corrupted by impulse noise. The most popular approach for removing impulse noise is the Vector Median Filter (VMF) [1] because of its good denoising power and computational efficiency. Many improvements of the VMF have been proposed to avoid the modification of pixels not affected by impulse noise [7]. Various decision-based filters have been proposed where noisy pixels are first identified and then replaced by using the VMF [6]. However, such approaches still have the same drawbacks of the VMF: blurring and low performances when noise ratio is high. In the spirit of [3], we propose an iterative approach that, at each iteration, identifies corrupted pixels by spectral clustering on the filtering window and performs graph regularization by topology adaptation to take only into account uncorrupted pixels.

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## 2. Preliminaries on graphs

We provide some basic definitions on graph theory. A graph $G$ is a couple $G = (V, E)$ where $V$ is a finite set of vertices and $E$ is a set of edges included in a subset of $V \times V$. Two vertices $u$ and $v$ in a graph are adjacent if the edge $(u, v)$ exists in $E$. $u \sim v$ denotes the set of vertices $u$ connected to the vertex $v$ via the edges $(u, v) \in E$. In the rest of this paper, we consider only simple graphs for which maximum one edge can link two vertices. These simple graphs are always assumed to be connected and undirected. A graph, as defined above, is said to be weighted if it is associated with a weight function $w : E \to \mathbb{R}^+$ satisfying $w(u, v) > 0$ if $(u, v) \in E$, $w(u, v) = 0$ if $(u, v) \notin E$. The volume of a set of vertices $A$ can be computed by $vol(A) = \sum\limits_{v_i \in A} d(v_i)$ with $d(v_i) = \sum\limits_{u \sim v_i} w(u, v_i)$ the degree of a vertex.

## 3. Noise detection by spectral clustering

The transition probability from a vertex $v_i$ to a vertex $v_j$ is provided by $p(v_i, v_j) = \frac{w(v_i,v_j)}{d(v_i)}$. The associated transition matrix $P$ is then defined by $P = D^{-1}S$ where $D$ is the degree matrix and $S$ a similarity matrix associated to the graph $G$. The eigenvectors $\mathbf{v}$ of $P$ are obtained by solving $P\mathbf{v} = \lambda\mathbf{v}$. The associated eigenvalues of $P$ are $\lambda_1 = 1 \geq \lambda_2 \geq \cdots \lambda_n \geq -1$ and one has $P = \sum\limits_{i=1}^{|V|} \lambda_i \mathbf{v}_i \mathbf{v}_i^T$. The eigenvector $\mathbf{v}_2$ of $P$ associated to $\lambda_2$ is known as the FIEDLER vector of $P$ and provides most of geometrical information. Indeed, according to the sign of the FIEDLER vector, the graph can be partitioned into two sets. This type of analysis is known as spectral clustering [8]. Spectral clustering can be used to partition a filtering window into two sets to identify noisy pixels. Given an arbitrary graph $G = (V, E)$

that models an image where vertices correspond to pixels, the neighborhood set $\mathcal{N}(G,u)$ of a vertex $u$ is defined as $\mathcal{N}(G,u) = \{v \in V : (v,u) \in E\} \cup \{u\}$. $\mathcal{N}(G,u)$ provides the set of vertices in a filter window centered on $u$. Then, we define the complete graph associated to a filtering window centered on $u$ by $F(G,u) = (\mathcal{N}(G,u),\ \mathcal{N}(G,u) \times \mathcal{N}(G,u))$. A spectral clustering of graph $F(G,u)$ is performed and according to the sign of the FIEDLER vector $\mathbf{v}_2$, the set of vertices of $F(G,u)$ is partitioned into two disjoint sets $\mathcal{N}_1(G,u) = \{v \in \mathcal{N}(G,u) : \mathbf{v}_2(v) \geq 0\}$ and $\mathcal{N}_2(G,u) = \{v \in \mathcal{N}(G,u) : \mathbf{v}_2(v) < 0\}$. This can be considered as a natural nonparametric impulse noise detector since we use a complete graph and noise free pixel tend to form a single cluster while noisy pixels are outliers. From these sets, we then want to obtain two sets $\mathcal{N}^c(G,u)$ and $\mathcal{N}^{uc}(G,u)$ that respectively denote noisy and noise free candidates for a filtering window centered on $u$ in a graph $G$ that models an image. To that aim, we apply the following rules:

- If $\mathcal{N}_1(G,u)$ or $\mathcal{N}_2(G,u)$ has a cardinality lower or equal to 1, it is considered as being $\mathcal{N}^c(G,u)$. This corresponds to evident outliers.
- If $\left(\frac{vol(\mathcal{N}_1(G,u))}{vol(\mathcal{N}(G,u))} \leq \frac{vol(\mathcal{N}_2(G,u))}{vol(\mathcal{N}(G,u))}\right)$, $\mathcal{N}^c(G,u) = \mathcal{N}_1(G,u)$ and $\mathcal{N}^c(G,u) = \mathcal{N}_2(G,u)$ otherwise.

$\mathcal{N}^{uc}(G,u)$ is then obtained as the set $\mathcal{N}_1(G,u)$ or $\mathcal{N}_2(G,u)$ that was not retained as $\mathcal{N}^c(G,u)$. These rules first treat the cases of one or less noisy pixels. Otherwise, the noisy set is determined as the less coherent set of vertices.

## 4. Topology adapted graph regularization

We consider a general function $f^0 : V \subset \mathbb{R}^n \rightarrow \mathbb{R}^m$ defined on graphs of the arbitrary topologies and we want to regularize this function. The regularization of such a function corresponds to an optimization problem which can be formalized by the minimization of a weighted sum of two energy terms:

$$\frac{1}{2} \sum_{u \in V} \left( |\nabla_w f(u)|^2 + \lambda \|f(u) - f^0(u)\|_2^2 \right) \qquad (1)$$

With the following definition of the weighted gradient $|\nabla_w f(u)| = \sqrt{\sum_{v \sim u} w(u,v)(f(v) - f(u))^2}$, this problem (1) has a unique solution and can be solved with the following algorithm $\forall u \in V$ [4]:

$$\begin{cases} f^{(0)}(u) = f^0(u) \\ f^{(t+1)}(u) = \dfrac{\lambda f^0(u) + \sum_{v \sim u} w(u,v) f^{(t)}(v)}{\lambda + \sum_{v \sim u} w(u,v)}. \end{cases} \qquad (2)$$

Such a graph regularization enables local and non-local [2] regularization by using appropriated graphs topologies and edge weights [4]. However, it efficient for Gaussian noise removal but not for impulse noise. Therefore, we propose to combine graph regularization with spectral clustering to obtain a general algorithm of impulse noise removal. The key point is that the topology of the graph changes along the iterations and this is performed by retaining only noise free pixels (the $\mathcal{N}^{uc}(G,u)$ set) on a given neighborhood centered on a vertex $u$. The major problem we have to face with is the fact that the vertex $u$ does not necessarily belong to $\mathcal{N}^{uc}(G,u)$ since it can be a corrupted pixel and Algorithm (2) is not directly applicable. Once we get the set $\mathcal{N}^{uc}(G,u)$, an adapted neighborhood graph $\mathcal{G}(u)$ is constructed with $\mathcal{N}^{uc}(G,u)$ as vertices and with $\{(v, v_{med}) : v \in \mathcal{N}^{uc}(G,u), v \neq v_{med}\}$ as edges. $v_{med}$ denotes the vertex that correspond to the vector median of the set $\mathcal{N}^{uc}(G,u)$. First part of Figure 2 illustrates these different steps on a toy filter window. This corresponds to consider the following energy for one vertex $u$:

$$\begin{cases} \frac{1}{2}|\nabla_w^{\mathcal{G}(u)} f(v_{med})|^2 + \quad \frac{\lambda}{2}\|f(u) - f^0(u)\|_2^2 & \text{if } u \in \mathcal{N}^{uc}(G,u) \\ \frac{1}{2}|\nabla_w^{\mathcal{G}(u)} f(v_{med})|^2 & \text{if } u \notin \mathcal{N}^{uc}(G,u) \end{cases} \qquad (3)$$

where $|\nabla_w^{\mathcal{G}(u)} f(v_{med})|$ denotes the weighted graph gradient operator on the adapted neighborhood graph $\mathcal{G}(u)$. One has to note that the data fitting term is taken into account only for noise free vertices. Finally we obtain the following algorithm, $\forall u \in V$:

$$f^{(t+1)}(u) = \frac{\chi(u \in \mathcal{G}(u))\lambda f^0(u) + \sum\limits_{(v,v_{med}) \in \mathcal{G}(u)} w(v, v_{med}) f^{(t)}(v_{med})}{\chi(u \in \mathcal{G}(u))\lambda + \sum\limits_{(v,v_{med}) \in \mathcal{G}(u)} w(v, v_{med})} \qquad (4)$$

where $\chi : V \rightarrow \{0,1\}$ is the indicator function.

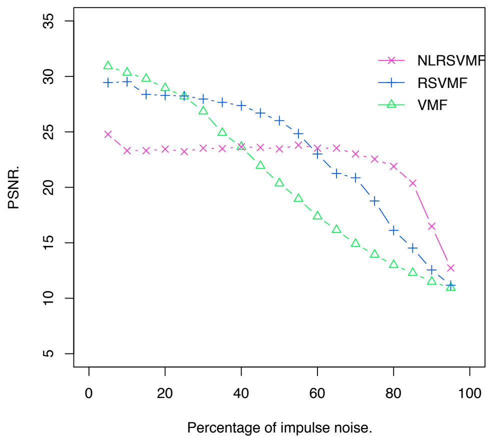
## 5. Experimental results

To test our algorithm, we have considered a part of the Lena image that has been corrupted by impulse noise expressed as [5]:

$$x_{i,j} = \begin{cases} v & \text{with probability } p_v \\ o_{i,j} & \text{with probability } 1 - p_v \end{cases}$$

where $i,j$ characterize the sample position, $o_{i,j}$ is the original sample, $x_{i,j}$ represents the sample from the noisy image, $p_{\mathbf{v}}$ is a corruption probability and $\mathbf{v} = (\mathbf{v}_R, \mathbf{v}_G, \mathbf{v}_B)$ is a noise vector of intensity random values. For the experiments, the considered degree of the impulse noise corruption $p_{\mathbf{v}}$ has ranged from 5% to

90%. To evaluate the achieved results, objective criteria as Peak Signal-to-Noise Ratio (PSNR) and Normalized Color Difference (NCD) have been used [5]. Figure 1 presents these results for the standard VMF, the proposed approach with 8-adjacency grid graphs (called RSVMF for Regularized Spectral VMF) and with a 24-adjacency grid graph (called NLRSVMF for Nonlocal Regularized Spectral VMF). Second part of Figure 2 presents visual results of denoising. Weights are computed according to a measure of similarity $w(u,v) = \exp\left(-\frac{\|F(f^0,u)-F(f^0,v)\|^2}{\sigma^2}\right)$. $F(f^0,u) \in \mathbb{R}^q$ denotes a feature vector associated to each vertex $u \in V$. The feature vector associated to vertices $F(f^0,v)$ can be the initial function value: $F(f^0,v) = f^0(v)$ (classical local processing: RSVMF) or a vector $F(f^0,v) = [f^0(u) : u \in B_{v,s}]^T$ (nonlocal processing: NLRSVMF). For this latter case $F(f^0,v)$ is a patch where $B_{v,s}$ denotes a bounding box of size $(2s+1)\times(2s+1)$ centered at $v$ (3×3 in all our experiments). For all the experiments we set $\lambda = 0.05$. Since the proposed filters are iterative, we provide results at convergence. As depicted in Figure 1 and in the second part of Figure 2, our approach outperforms the VMF once the noise proportion is higher than 20%. Lower performances are obtained for lower noise proportions since our approach tends to sharpen the image while reducing the VMF blurry effect. For very high proportions of noise, the nonlocal version of the proposed approach is very effective. Moreover, nonlocal results show its remarkable robustness as noise proportion increases. Our approach is therefore very effective for removing very high proportions of impulse noise in local or nonlocal configurations.



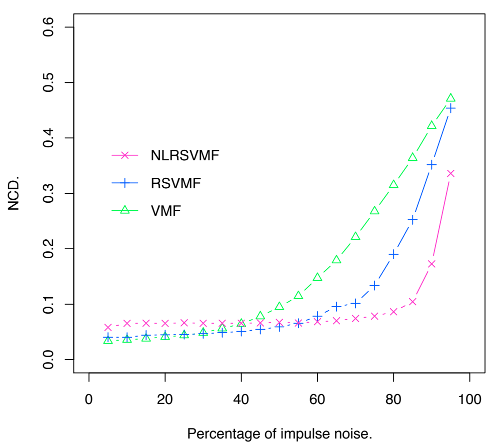




**Figure 1. PSNR and NCD according to the percentage of impulse noise.**

## 6. Conclusion

In this paper, a new algorithm for impulse noise removal has been proposed. The image is modeled as a graph and a regularization process is performed. The latter has the property to locally adapt the topology of the graph under study by a spectral analysis to retain noise free pixels. Experimental results show that the new filter outperforms the VMF when high proportions of impulse noise should be eliminated.

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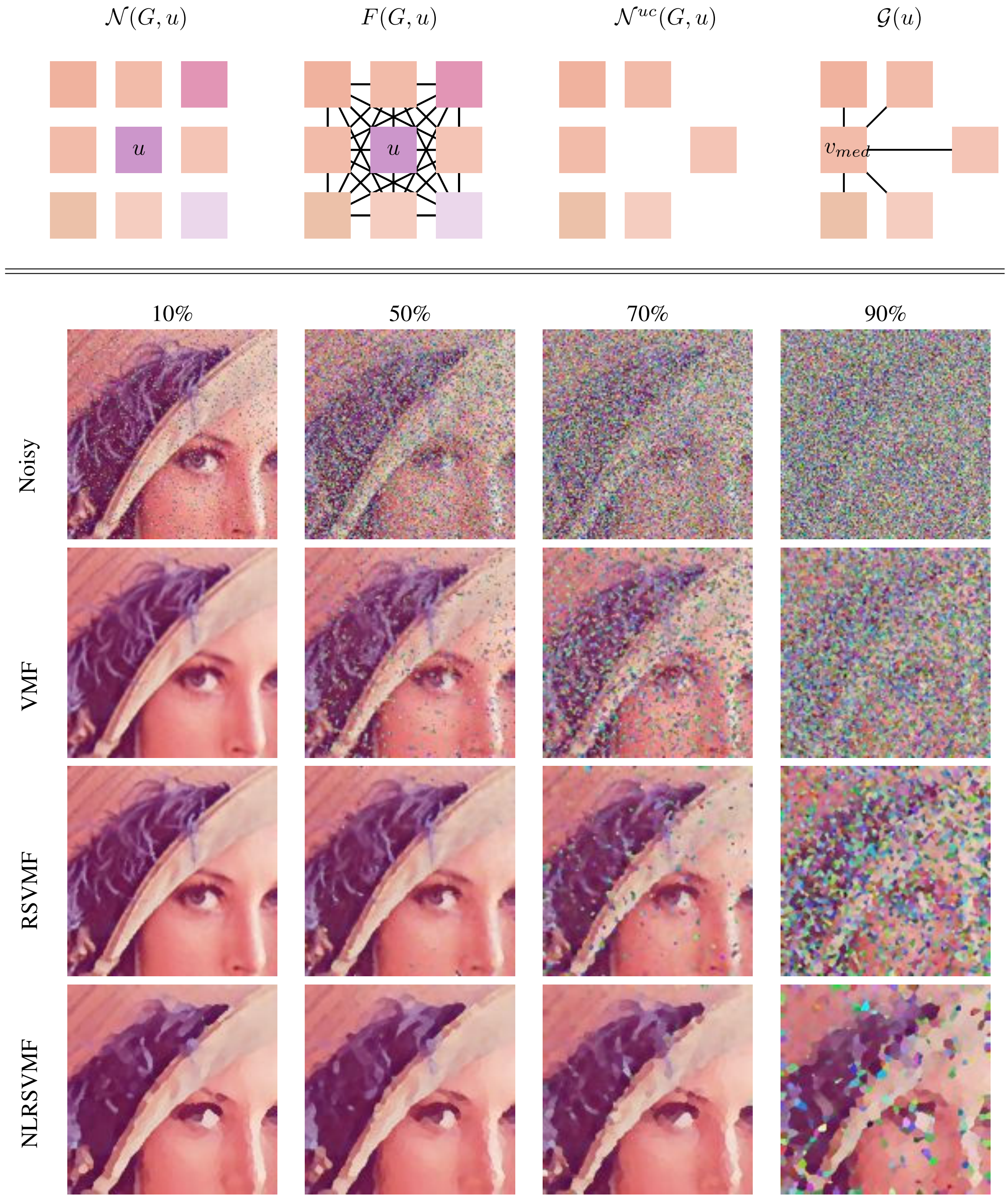


**Figure 2. First part: spectral clustering and adapted graph neighborhood construction. Second part: impulse noise removal results. First line presents corrupted images for different amounts of noise. Next lines present filtering results with the standard VMF, the Regularized Spectral VMF (RSVMF) on a $8$-adjacency grid graph with $F(f^0,v) = f^0(v)$ (local processing) and the Nonlocal Regularized Spectral VMF (NLRSVMF) on a $24$-adjacency grid graph with $F(f^0,v) = [f^0(u) : u \in B_{v,2}]^T$ (nonlocal processing).**