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HAL Id: hal-00331104
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Submitted on 20 Aug 2007

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Spectral scaling in the turbulent Earth’s plasma sheet revisited

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Received: 11 May 2007 – Revised: 30 July 2007 – Accepted: 6 August 2007 – Published: 20 August 2007

Abstract. Bursty bulk flow associated magnetic fluctuations exhibit at least three spectral scaling ranges in the Earth’s plasma sheet. Two of the three scaling ranges can be associated with multi-scale magnetohydrodynamic turbulence between the spatial scales from ∼100 km to several $R_E$ ($R_E$ is the Earth’s radius). These scales include the inertial range and below ∼$0.5R_E$ a steepened scaling range, theoretically not fully understood yet. It is shown that, in the near-Earth plasma sheet, the inertial range can be robustly identified only if multi-scale quasi stationary (MSQS) data intervals are selected. Multiple bursty flow associated magnetic fluctuations, however, exhibit $1/f$ type scaling indicating that large-scale fluctuations are controlled by multiple uncorrelated driving sources of the bulk flows (e.g. magnetic reconnection, instabilities).

1 Introduction

Turbulence is a prominent feature of plasma flows and of the magnetic field in the Earth’s plasma sheet (e.g. Borovsky and Funsten, 2003, Vörös et al., 2003). The variability of near-Earth plasma sheet parameters and “magnetic turbulence” has already been noticed in early papers (e.g. Hruska and Hruškova, 1970, Coroniti et al., 1977). Also, the large variances in the distant tail during the ISEE-3 mission were found to be consistent with the presence of significant amount of turbulence (Tsurutani et al., 1984). It was noticed that, turbulent wave-particle interactions associated with slow shocks can contribute to the dissipation processes through (anomalous) resistivity (Scarf et al., 1984). Magnetic field measurements on board the Geotail and AMPTE/IRM spacecraft revealed that the power-law behaviour of the energy density spectra in frequency space, $P(f)\sim f^{-\alpha}$ depends on the considered frequency range, both in the distant tail (Hoshino et al., 1994), and in the near-Earth plasma sheet (Bauer et al., 1995). This finding was confirmed by Ohtani et al. (1995) using fractal analysis of the fluctuations in the AMPTE/CCE and SCATHA magnetic field measurements. Further analyses based also on Geotail and Cluster data have shown that the spectral index changes around $f \sim 0.01 – 0.08$ Hz ($100 – 13$ s). The observed scaling indices are $\alpha_1 \sim 0.5 – 1.5$ for $f < f_1$ and $\alpha_2 \sim 1.7 – 3$ when $f > f_1$ (Milovanov et al., 2001; Volwerk et al., 2003, 2004; Vörös, 2004b, Weygand et al., 2005). Interestingly, the first results on magnetic fluctuation spectra in the magnetotail were reported by Russell (1972). Using the data from OGO-5 spacecraft he found $\alpha_2 \sim 2 – 2.5$ for $f > f_1$, nevertheless, the fluctuations were ascribed to noise or/and turbulence. In the plasma sheet, no estimate of $\alpha$ gave robustly the value of 5/3 or 3/2, characteristic for inertial range scaling in hydrodynamic or magnetohydrodynamic turbulence (e.g. Biskamp, 2003), respectively. Although, longer solar wind time series can exhibit scalings with a scaling index $\sim 2$ (appearing during intervals with structures and jumps (Burlaga et al. 1998)), the inertial range scaling index is readily discernible in the solar wind (Bruno and Carbone, 2005) or even in astrophysical turbulent plasmas (Cho et al., 2003). Does it prove that fully developed turbulence is not present in the plasma sheet? In fact, there exist several alternative mechanisms which can explain the uncertain estimates of the scaling indices in the plasma sheet. These include boundary effects, the presence of different time scales, jumps or discontinuities in data, 2D turbulence or transitory mechanisms driven by different physical processes involved in the generation of the observed scalings (Vörös et al., 2004a, 2006; Volwerk et al., 2004; Weygand et al., 2005). In this paper we investigate bursty bulk flow (BBF) driven magnetic turbulence placing emphasis on the transiency of fluctuations. BBFs are preferentially organized into sporadically occurring groups of fast flows, typically lasting for a few minutes.
Despite their short duration, BBFs are the carriers of decisive amounts of mass, momentum and magnetic flux (Angelopoulos et al., 1992, 1993; Schödel et al., 2001). Naturally, BBF driven magnetic turbulence is transient in a spacecraft frame. Neglecting this transiency and the associated non-stationarity of multi-scale fluctuations can easily lead to spurious estimates of scaling indices. The distribution of the power of magnetic fluctuations over frequency/time scales depends on the average large-scale plasma velocity. This is because Doppler-shifted power appears at the small-scales and also because of the possible velocity dependent widening of the scaling ranges (Vörös et al., 2004b, 2005). Obviously, the stationarity of any multi-scale process has to be checked over multiple scales, otherwise the scaling indices would be badly estimated over scales which do not belong entirely to a turbulent cascade.

2 The wavelet method

Abry et al., (2000) proposed a semi-parametric wavelet technique based on a fast pyramidal filter bank algorithm for the estimation of scaling parameters $c_f$ and $\alpha$ in the relation $P(f) \sim c_f f^{-\alpha}$, where $c_f$ is a nonzero constant. The algorithm consists of several steps. First, a discrete wavelet transform of the data is performed over a dyadic grid (scale, time) = $(2^j, 2t)$ and $j, t \in \mathbb{N}$. Then, at each octave $j = \log_2 2^j$, the variance $\mu_j$ of the discrete wavelet coefficients $d_x(j, t)$ is computed through

$$\mu_j = \frac{1}{n_j} \sum_{i=1}^{n_j} d_x^2(j, t) \sim 2^{j\alpha} c_f$$

(1)

where $n_j$ is the number of coefficients at octave $j$. Finally, from Eq. (1) $\alpha$ and $c_f$ can be estimated by constructing a plot of $y_j = \log_2 \mu_j$ versus $j$ (logscale diagram) and by using a weighted linear regression over the region ($j_{\text{min}}, j_{\text{max}}$) where $y_j$ is a straight line. In this paper we use the Daubechies wavelets for which finite data size effects are minimized and the number of vanishing moments can be changed. The latter allows to cancel or decrease the effects of linear or polynomial trends and ensures that the wavelet details are well defined. See further details in Vörös et al. (2004b). Here, for simplicity and convenience, instead of octaves the corresponding time scales will be used in logscale diagrams.

3 Transiency of BBF driven magnetic fluctuations

In what follows, we analyze magnetic data (22 and 67 Hz) from the Cluster fluxgate magnetometer (FGM) (Balogh et al., 2001) and spin-resolution (~4 s) velocity data from the Cluster ion spectrometry (CIS/CODIF) experiment (Rème et al., 2001). We will also use magnetic data (16 Hz) from Geotail magnetic field (MGF) experiment (Kokubun et al., 1994).
and 12 s resolution velocity data from Geotail low energy particle (LEP) experiment (Mukai et al., 1994).

We will consider data intervals which, aside from the large-scale plasma flow in the plasma sheet, contain also small-scale magnetic fluctuations driven by the large-scale flow itself (see the precise definition of scales later). We will refer to the persistent occurrence of such cross-scale activity as “multi-scale quasi-stationarity” (MSQS) criteria throughout the text.

The idea of MSQS and the relevance of simultaneous occurrence of plasma flows and small-scale magnetic fluctuations is demonstrated in Figs. 1a–d. Figure 1a shows the $B_X$ component of the magnetic field and Fig. 1b the magnitude of bulk velocity $V$, during the interval 01:24–02:12 UT from Cluster 3, when the spacecraft were near the $Z_{GSM}$=0 plane in the postmidnight magnetotail ($X_{GSM}$~$-19 R_E$, where $R_E$ is the Earth’s radius). Figures 1c,d show the scaling index $\alpha$ and the power $c_f$ Eq. (1) computed from the magnetic field magnitude $B$ within sliding overlapping windows of width $W$=2 min with a time shift 1 s. The scaling index was estimated over the scales 3–6 s and the wavelet power was estimated at 3 and 6 s, respectively. $\alpha$ is strongly fluctuating and there are only few intervals when it shows quasi-stationary behaviour over a longer time than $W$. Whenever $\alpha$ is estimated well only during shorter intervals than $W$, the sliding window estimate cannot be trusted. Data intervals with $c_f$~0 and $\alpha$∈(0, 1) correspond to the noise produced by the magnetometer. It indicates that the magnetic field is quiet over the scales of 3–6 s and the only disturbance which affects the estimate of $\alpha$ in the logscale diagram (not shown) is the 4 s spin of the spacecraft. Such intervals are between 01:24–01:32 UT and 01:50–01:55 UT when $V$ is below a threshold of approximately 150 km/s. Also, there is practically no energy transfer between the plasma flow and the magnetic field when the spacecraft is outside of the plasma sheet or outside of the region where the plasma flow is mostly perpendicular to the magnetic field (e.g. before 01:30 UT C3 is in the lobe, $B_X$>20 nT). See further details in Vörös et al., (2004). There are only two intervals when $\alpha$>2 and it is showing quasi-stationary behaviour during a period longer than $W$, moreover, C3 is in the plasma sheet ($\langle B_X \rangle$<10 nT) and $\langle V \rangle$>150 km/s. These intervals are roughly between 01:35–01:39 UT and 01:47–01:49 UT. In both cases, $c_f$ is increased over the noise level and the larger time scale (6 s) shows more power than the smaller time scale (3 s). All this indicates that the large-scale average flow and the small time scale magnetic fluctuations are energetically connected and magnetic fluctuations exhibit spectral scaling, however, with a scaling index different from that one could expect for the inertial range. It is important to realize that these intervals are rather short. This stimulates the question of whether the estimation of spectral characteristics could not be improved by choosing longer intervals with bursty flows in the plasma sheet.

The Geotail spacecraft has a more suitable trajectory for multiple flow detection, being usually longer time in the

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**Fig. 2.** Spectral estimations within a sliding window from Geotail data: a.) $B_X$ component of the magnetic field; b.) bulk velocity; c.) scaling index $\alpha$; d.) wavelet power.
plasma sheet than Cluster. Figures 2a–d show a seven hour long interval containing multiple flows within the plasma sheet, when the spacecraft were near $X_{\text{GSM}} \sim -23R_E$, $Y_{\text{GSM}} = 4R_E$ and $Z_{\text{GSM}} \sim -2R_E$. The notation is the same as in Fig. 1. Again, there exists good correlation between the occurrence of rapid plasma flows, the small-scale magnetic power and the intervals with steady estimates of $\alpha > 2$, computed from the magnetic field magnitude. The BBFs are organized into groups. During the whole seven hour long period (interval A in Fig. 1), there are four such groups. Between the groups, $(V) \leq 150\text{ km/s}$, $\alpha$ fluctuates around 1 or it is less than 1 and $f \rightarrow 0$. The longest group in time (interval B) occurs between 12:30 and 14:10 UT. Within this group the bulk velocity varies roughly between 50 and 950 km/s. As a consequence the scaling index is fluctuating between 1 and 3. It has to be clear immediately that, in the absence of well defined large-scale mean flow the Taylor hypothesis cannot be used. The Taylor hypothesis ensures the statistical equality of spatial and temporal using one-point measurements when the spatial fluctuations on a scale pass over the spacecraft faster than they typically fluctuate in time. In the solar wind plasma flows are super-Alfvénic, thus the turbulence cascade and the underlying spatial structures associated with Alfvénic fluctuations are discernible from one-point time-shifted measurements. Bulk plasma flows are usually sub-Alfvénic in the plasma sheet. In a few cases, Alfvénic fluctuations with a quasi-period of the order of minutes can be associated with turbulence in the plasma sheet, too. Using Cluster multi-point measurements it was demonstrated, however, that there is no difference between spatial and temporal fluctuation statistics over the time scale of the order of seconds (Vörös et al., 2006), which means that the validity of Taylor’s frozen-in hypothesis is scale-limited in the plasma sheet.

When the plasma flow velocity is strongly fluctuating in the plasma sheet, the spectral scaling indices obtained over frequency ranges or temporal scales cannot be compared with the expected inertial range scaling indices in the wavenumber space. On the other side, such comparison would be possible when MSQS intervals with less fluctuating speed and increased small-scale magnetic power are considered. Indicatively, the fluctuations of $\alpha$ estimated from the magnetic field are immediately smaller, when the large-scale plasma flow is less fluctuating. The interval between 13:52 and 14:06 UT (7xW!), marked by the letter C, is a good example with $(V) \sim 700\pm 150\text{ km/s}$ and $(\alpha) \sim 2.5\pm 0.3$ (the standard deviations are used as error estimates). It demonstrates that, in terms of spectral properties, quasi-steady intervals are rather short in Geotail data, too. More importantly, however, when the spectral properties are estimated from the data one has to answer the question if the processes over intervals A or B belong physically together forming a widespread intermittent turbulence in the plasma sheet. In fully developed intermittent turbulence bursts of activity and quiescent periods occur alternately and continually in the fluctuating velocity field. Therefore, multiple flow intervals in the plasma sheet might also represent a realization of intermittent dynamical process, though due to the breaking of Taylor hypothesis the underlying scalings could not be compared with turbulent models. Alternatively, each MSQS interval of type C would represent a moving localized blob of turbulent plasma, appearing transient in the reference frame of the spacecraft. In the latter case, the statistical properties of fluctuations over longer intervals containing multiple flows have to be related to the dynamics of driving sources of multiple flows (e.g. reconnection, instabilities) and not to the internal intermittency in turbulent flows. Notwithstanding, the localized and moving blobs of plasma can contain intermittent turbulence.

4 Multiple flow versus individual flow related scalings

In order to investigate the difference between fluctuation statistics associated with multiple (types A, B) and individual flows (type C), as explained above in Fig. 2, we analyse several MSQS intervals from both Cluster and Geotail spacecraft first. Figure 3 shows the logscale diagrams (see the explanation of Eq. 1) obtained from wavelet analysis of five MSQS intervals from Cluster and four MSQS intervals from Geotail spacecraft. For the sake of lucidity the curves corresponding to Geotail were shifted up. The analysed intervals are 6 min long, close to the maximum available length of MSQS intervals. The logscale diagrams show two scaling regions with $\alpha = 2.6 \pm 0.2$ below and $\alpha = 1.8 \pm 0.4$ above time scales of $\sim 6$–10 s, respectively. The mean values of scaling
indices and the standard deviations were estimated using all the data intervals from both spacecraft.

The spectral break over the 6–10 s time scales roughly coincides with the proton gyroperiod of ~ 5–15 s in the plasma sheet. The inertial range in turbulence is expected to occupy time scales larger than the proton gyroperiod (e.g. Borovsky and Funsten, 2003). In fact, over this time scale α = 1.8 ± 0.4 is observed for MSQS intervals which, within the uncertainties, is comparable to both hydrodynamic or magnetohydrodynamic inertial range scaling exponents (5/3 and 3/2, respectively). Because of the Doppler shift the inertial-range time scales can be a bit larger. Below the proton gyroperiod the scaling becomes steeper. This steepening cannot be caused by dissipation via kinetic wave damping, because it would result in a strong cutoff in the power spectra rather than a power law (Li et al., 2001). Spectral scaling could be produced by the Doppler shift of different wave modes, but steepened broadband spectra below the proton gyroperiod were also observed e.g. in the high-altitude cusp during intervals with no plasma flow and no Doppler effect (Nykkyri et al., 2006). When the energy is not dissipated it has to be transferred further towards smaller time scales. The spectral energy transfer in the small-scale cascade might be controlled by other physical processes exhibiting different characteristic time scales than the inertial range magnetohydrodynamic turbulence. For example, the Hall effect could modify the nonlinear cascade when ion and electron motions are decoupled. The Hall term may be responsible for dispersive and polarized Alfvén waves and the frequency steepening may be attributed to dispersive nonlinear processes (Ghosh et al., 1996). Matthaeus et al. (2003) have shown in a one-fluid compressible 3D magnetohydrodynamic model that the Hall effect is becoming important over the scales larger or equal than the dissipation scale so that the cascade or/and dissipation processes should be affected by the Hall term, which can lead to two different scaling ranges in magnetic fluctuation spectra. As a matter of fact, however, the Hall term has no effect on decay rates in homogeneous turbulence in these simulations. The results of Matthaeus et al. (2003) cannot be fully generalized, however, because e.g. the effects of high Reynolds numbers and nonzero cross and magnetic helicities were not fully explored. Actually, Hall magnetohydrodynamics can support three quadratic invariants: total energy, magnetic helicity, generalized helicity, and the velocity and magnetic fluctuations become wave number dependent (Krishan and Mahajan, 2005). A possibility is that viscosity (ν) is large in comparison with resistivity (η), in which case there will be a range of scales below the viscous cut-off where resistivity is negligible and magnetic structures can evolve towards smaller scales. In this case magnetic and kinetic spectra decouple and the magnetic spectrum follows a new inertial range (Cho et al., 2003). It is not fully clear how the scaling index might evolve with changing ratio of viscosity and resistivity ν/η in the plasma sheet. In the simulations of Matthaeus (2003) the dependence of scalings on ν/η was not investigated, nevertheless, double scaling appears in their Fig. 3 with a break at the wavenumber k ~ 30, with and without the Hall term. Small wavenumbers (large scales) exhibit roughly the 5/3 scaling while over large wavenumbers (small scales) the scaling steepens. In the Earth’s plasma sheet, in any case, spectral steepening with the scaling index α ~ 2.6 is robustly observed from both spacecraft below the time scale of a few seconds, representing an interesting challenging question for theorists. The experimental investigation of fluctuations below the ion gyroperiod and the corresponding spectral steepening should be complemented by the analysis of higher order statistics. Non-gaussian distributions in the Earth’s plasma sheet can be interpreted in terms of turbulence models (Vörös et al., 2007), but alternatively e.g. through Lévy statistics (Consolini et al., 2005). We also mention the possibility that scaling indices near α ~ 2 can be observed simply because of the occurrence of jumps in the data. In the solar wind these jumps can be e.g. shock fronts. The jumps in the data should lead to non-stationary estimations in sliding window analysis. In our analysis, however, quasi-stationarity of estimations was required.

Let us consider now the case of multiple flows. These may contain several MSQS intervals, groups of bursty flows and quiescent intervals between them. Observations of several hour long flow intervals are rare in the plasma sheet. Figure 4 shows the logscale diagram for the whole seven hour long interval on 25 January 2000 (interval A in Fig. 2) together with a similar curve corresponding to four hour long interval of multiple flows on 10 December 1996. In both cases α ~ 2.6 below ~10 s which means that the scaling over the smallest time scales is not significantly affected by the occurrence of multiple flows.
Despite the larger error bars over $\sim 100 \text{s}$ a new scaling region with $\alpha \sim 1$ can be identified. In the solar wind, the interplanetary magnetic field at 1 AU (astronomical unit) exhibits frequency spectra with a $1/f^\alpha$ ($\alpha \sim 1$) dependence in the energy containing range from $3 \times 10^{-6}$ to $8 \times 10^{-5}$ Hz. The $1/f$ spectrum results from the superposition of uncorrelated samples of solar surface turbulence that have log-normal distributions of correlation lengths (Matthaeus and Goldstein, 1986). In the plasma sheet, using the same sort of reasoning, the energy containing large-scales exhibiting $1/f$ scaling due to the superposition of uncorrelated multiple flows occurs over the time scales from $\sim 100 \text{s}$ to $\sim 3000 \text{s}$ (Fig. 4). Naturally, the largest scales involved in the $1/f$ scaling region cannot be determined from finite length measurements. It is rather easy to notice that any spectral index between 1 and 2.6 can be observed between the time scales of a few seconds and hundreds of seconds. The inertial range characteristic for MSQS intervals (Fig. 3) is smeared out by the mixing and superposition of multiple flows (Fig. 4). This is the physical reason why MSQS intervals have to be considered for proper identification of the rather narrow inertial range of transient turbulence in the plasma sheet where the bulk flow changes from zero to several hundreds of km/s or more during multiple flow intervals. Although there might be other reasons, like the geometry and position of the observed flows or specific local features of the interaction of the flows with the magnetic field or currents, we believe that the main reason for discrepancies between previous estimates of scaling indices mentioned in the introduction is in the physical difference of origins between the individual flows (MSQS intervals) and multiple flows.

5 Conclusions

Using proper selection criteria for finding localized turbulence data intervals we succeeded for the first time in identifying the inertial range scaling from magnetic field fluctuations driven by bursty bulk flows in the Earth's plasma sheet. The observable inertial range is rather narrow spanning over the time scale of $\sim$ tens of seconds. Taking bulk flow values between 250 and 1000 km/s and supposing the validity of Taylor’s hypothesis the corresponding spatial scales along flow directions are roughly between $\sim 0.5$ and $15 R_E$. We have to suppose, however, that the bulk speed of the plasma flows is strongly decreasing in the near-Earth region because of their breaking in dipolar magnetic field. Therefore, the large-scale of the flows are of the order of a few $R_E$, instead of $15 R_E$, which still means that bulk flow driven magnetic turbulence can occupy rather large parts of the near-Earth’s magnetotail. Below the time scale of a few seconds another bulk flow driven scaling region appears. The corresponding spatial scales along flow directions are less than $\sim 0.5 R_E$ possibly down to $\sim 100 \text{km}$. The physical mechanisms involved in the generation of this small-scale scaling region can be Hall physics related, but are not fully understood. As a possibility in magnetohydrodynamics, in contrast to the hydrodynamic case, magnetic structures can evolve towards smaller scales when the resistivity is initially negligible below the viscous cut-off. Anyhow, the associated scaling is robustly observed with error bars typically smaller than within the inertial range. The comparison of individual and multiple flow statistics shows that the latter originates in the superposition of uncorrelated multiple flows and reflects the dynamics of multiple driving sources (e.g. magnetic reconnection, instabilities) in the magnetotail. When individual flow intervals are not selected properly the mixing of multiple flows with changing bulk speeds and scaling characteristics smears out the true scaling within the narrow inertial range. In summary, the identification of scaling regions with differing physics heavily depends on the interval selection criteria. It can explain the discrepancies between the values of previously estimated scaling indices by different authors.

Acknowledgements. We thank T. Takada, V. Carbone, O. Alexandrova, G. Zimbardo, and L. Sorriso-Valvo for helpful discussions and comments. The authors are grateful to H. U. Eichelberger for helping in the Cluster data analysis, to S. Kokubun and T. Mukai for providing Geotail data. The work by Z. Vörös was partially supported by the Czech Science Foundation under project B300420509. The work by M. Volwerk was financially supported by the German Bundesministerium für Bildung und Forschung and the Zentrum für Luft- und Raumfahrt under contract 50 OC 0104.

Edited by: B. Tsurutani
Reviewed by: two anonymous referees

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