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SIMPLE MODELING AND CONTROL OF PLASMA CURRENT PROFILE

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Abstract: The purpose of this paper is to present a simplified model and control law of the current and temperature profile in a tokamak plasma. Based on a description of the plasma as a magnetised fluid, the model is expressed in the form of coupled one dimensional transport-diffusion equations. A simple feedback is used to obtain a given stationary profile. The numerical simulations are done in the Scilab/Scicos environment.

Keywords: plasma modeling, plasma control, tokamak

1. INTRODUCTION

A tokamak is a facility in which a plasma is magnetically confined and heated in order to produce nuclear fusion reactions. The magnetic confinement of the plasma particles in the vacuum vessel torus is obtained through the combination of toroidal and poloidal fields produced by external coils (see Figure 1) with the additional field produced by an electrical current flowing along the plasma ring. This plasma current is generally firstly generated by induction (the plasma ring can then be considered as the secondary loop of a transformer whose primary loop is the ohmic field coil (see Figure 1)). It allows to heat up the plasma, which behaves as a resistive conductor. But, in practice, ohmic heating and current drive do not allow to reach the adequate plasma temperature and duration required for future fusion reactors. Indeed the plasma resistivity decreases with temperature and technology limits the ohmic field coil current. Non inductive heating and current drive methods were thus developed to take it over, namely high power microwave or fast neutral beams injection.

![Fig. 1. Tokamak.](image)
• control of the current, temperature and density profiles.

We are here concerned with the latter problem, which has been studied more recently in (Walker and al, 2006; Laborde, 2005; Moreau and al, 2006). The goal is to provide the operating conditions (in terms of profiles shapes) that are necessary to achieve advanced confinement schemes able to increase the fusion power production efficiency.

The profile control approaches cited above are mainly based on black box linear models where plasma physics is only used to select the set of relevant variables and the way they are coupled. These approaches require the identification of a MIMO system approximating the distributed physical system that is highly dependent on the operating conditions, which makes them costly in terms of experimentations. The first aim of this paper is to provide a simple PDE control-oriented model based on:

• the evolution of the resistive equation averaged on the magnetic surface as explained in (Blum, 1989),
• the experimental identification of some diffusion coefficients.

The second aim is to provide a simple control policy depending on the temperature providing a given current profile improving the fusion reaction. The feedback determines the energy profile that the set of antennas has to produce or to approach but does not give explicitly the effective control to apply.

Numerical experimentation have been performed in the Scilab-Scicos environment. Based on some experimental data, from the Tore Supra Tokamak the simulation results are compared with the outputs obtained by a more complex physics oriented code namely the Cronos software (Basiuk and al, 2003). Cronos is one of the main plasma integrated modeling codes, but it cannot be easily used in real-time or for control purposes.

In the second section, we recall the plasma physics background and the assumptions made to derive the distributed control-oriented model. The form used for the diffusion coefficients and the sources terms of the PDEs are detailed in the third section. They have been obtained by simplifying (Witrant and al, 2007) after a sensibility analysis.

In the last section, the current profile control problem is set and solved.

2. TOKAMAK PLASMA PHYSICS

We recall here some basic physics background about the plasma macroscopic description on which the model is based.

2.1 Plasma magnetohydrodynamics

The dynamics of a plasma is governed by (see (Blum, 1989; Wesson, 2004)) the MHD equations:

\[
\nabla \times E = -\partial_t B, \quad \text{Faraday’s law,}
\]
\[
E + \zeta j_n + u \times B = \zeta j, \quad \text{Ohm’s law,}
\]
\[
\nabla \cdot B = 0, \quad \text{B conservation,}
\]
\[
\nabla \times B = \mu_0 j, \quad \text{Ampère’s law,}
\]
\[
\partial_t n + \nabla \cdot (nu) = n_s, \quad \text{particles conservation,}
\]
\[
m_n \dot{u} + \nabla p = j \times B, \quad \text{momentum conservation,}
\]
\[
\frac{3}{2} \dot{p} + \frac{5}{2} p \nabla \cdot u + \nabla Q = p_s, \quad \text{energy conservation,}
\]
\[
p = k n T, \quad \text{perfect gases law,}
\]

where \(\dot{\varepsilon} \triangleq \partial_t v + v \cdot \nabla v\), \(E\) is the electric field, \(B\) is the magnetic field, \(u\) is the mean particles velocity, \(j\) is the current density, \(j_n\) is the non inductive current density, \(n\) is the particles density, \(p\) is the plasma pressure, \(T\) is the temperature, \(Q\) is the heat flux, \(m\) is the particle mass, \(\mu\) is the magnetic permeability, \(\zeta\) is the resistivity tensor, \(k\) is the Boltzmann constant, \(n_s\) is the particle source and \(p_s\) is the energy source.

2.2 Time Constants

In order to provide a model appropriate for plasma control studies, it is important to understand the time constants associated with the different physical phenomena at stake. We can bring out four time constants:

• The Alfvén time \(\tau_A = a/(\mu_0 m n)^{1/2}/B_0\), where \(a\) is the minor radius of the plasma ring and \(B_0\) the toroidal magnetic field at the vacuum vessel center, is of the order of \(10^{-6}\) s in present tokamaks.
• The density diffusion time \(\tau_n = a^2/D\), where \(D\) is the particle diffusion coefficient, for modern tokamaks, is in the range 0.1s-1s;
• The heat diffusion time \(\tau = na^2/K\), where \(K\) is the thermal conductivity coefficient is also in the range 0.1s-1s (3.4s for ITER);
• The resistive diffusion time constant \(\tau_r = \mu_0 a^2/\zeta\) is of few seconds (100s-3000s for ITER).

The Alfvén time scale is used to describe the MHD instabilities phenomena, which are not considered here. Our model is focused on the dynamics of the resistive behavior of the plasma but a dynamical temperature species (electrons and ions) model is given. Due to the bad knowledge of the dynamics of species densities and the fact that online measurements are available, we suppose that they are given explicitly (in practice we use a filtered version of the ones given by the Cronos software).
To sum up, therefore the magnetic field lines and the current flowing at the point \( M \).

Denoting by \( D \) the density profile, i.e., phenomena at the resistive wall time scale. At this time scale, we can consider that density profile is independent of the toroidal angle \( \phi \) exists and the following formula applied to \( B \).

From the conservation of \( B \), it follows that there exists \( A \) such that \( B = \nabla \times A \) with \( A = (A_r, A_\varphi, A_z) \). Due to the axis symmetry, \( A \) is independent of the toroidal angle \( \varphi \). Therefore \( B_r = -(1/r)\partial_r(rA_\varphi) \) and \( B_z = (1/r)\partial_r(rA_z) \).

In the following \( rA_\varphi \) will be denoted \( \Psi \).

Denoting by \( D(r, z) \) the horizontal disk centered on the \( z \)-axis with its boundary passing through the point \( M \) (with coordinates \((r, z)\)) the Stoke’s formula applied to \( D \) and the field \( B \):

\[
2\pi \Psi = 2\pi r A_\varphi = \int_{\partial D} A = \int_D \nabla \times A = \int_D B ,
\]
gives the interpretation of \( \Psi \) as the poloidal magnetic flux. Similarly, applying Stoke’s to \( D \) and the field \( j \), using Ampère’s law and denoting \( rB_\varphi \) by \( f \) we have:

\[
2\pi f = 2\pi r B_\varphi = \int_D \partial B = \int_D \nabla \times B = \mu_0 \int_D j .
\]

From Ampère’s law and the definition of \( \Psi \) we have:

\[
j_\varphi = \frac{\partial_r B_r - \partial_z B_z}{\mu_0} = L \Psi \equiv -\frac{1}{\mu_0} \left[ \frac{\partial_r \Psi}{r} + \frac{\partial_z \Psi}{r} \right] 
\]

To sum up,

\[
B = \left( -\frac{\partial_z \Psi}{r}, \frac{f}{r}, \frac{\partial_r \Psi}{r} \right) , \quad (1)
\]

\[
j = \left( -\frac{\partial_r f}{\mu_0 r}, L \Psi, \frac{\partial_z f}{\mu_0 r} \right) . \quad (2)
\]

2.4 Magnetic Surfaces and Associated Quantities

We are interested in the dynamics of the current density profile, i.e., phenomena at the resistive wall time scale. At this time scale, we can consider that the momentum equation is at the equilibrium i.e.:

\[
\nabla p = j \times B .
\]

This equation yields \( B \cdot \nabla p = 0 \) and \( j \cdot \nabla p = 0 \), and therefore the magnetic field lines and the current lines lie in the so called magnetic surface which are surfaces of constant pressure. The magnetic surfaces form a set of nested toroids as in Figure 2.

The torus symmetry implies that \( \partial_\varphi p = 0 \). The magnetic field \( B \) being orthogonal to \( \nabla p \) we have \(-\partial_r p \partial_\varphi \Psi + \partial_\varphi p \partial_r \Psi = 0 \) which means that \( \nabla p \) is proportional to \( \nabla \Psi \), thus \( \Psi \) is constant on a magnetic surface. We can show using the orthogonality of \( j \) with \( \nabla p \) that \( f \) is constant on the magnetic surfaces, as it has been done for \( \Psi \).

Using (1),(2) and the colinearity of \( \nabla \Psi, \nabla j \), and \( \nabla p \)

\[
\nabla p = j \times B = \frac{L \Psi}{r} \nabla \Psi - \frac{f}{\mu_0 r^2} \nabla f ,
\]

which gives the plasma equilibrium Grad-Shafranov equation:

\[
L \Psi = r \partial_r p + \frac{1}{2\mu_0 r^2} \partial_r (f^2) .
\]

Given that many physics quantities are constant over magnetic surfaces, it was found very useful to define an averaging method of any quantity over a magnetic surface in order to finally get 1D quantities. In the following of this section the magnetic surface will be indexed by the parameter \( \rho \).

We define (see (Blum, 1989)) \( \langle A \rangle \equiv \partial_\varphi \int_V A dV \) with \( V \) the volume inside the magnetic surface. Denoting \( v' = \partial_\varphi V \), it can be shown that:

\[
\langle \nabla A \rangle \equiv \partial_r \langle A \nabla V \rangle = \frac{1}{v'} \partial_\rho \langle v'(A \nabla \rho) \rangle ,
\]

A particular indexing choice of the magnetic surface corresponds to the following definition of \( \rho \):

\[
\rho \equiv \sqrt{\frac{\Phi}{\pi B_0}} , \quad (3)
\]

where \( B_0 \) is the toroidal magnetic field at the vacuum vessel center (which is assumed to be constant) and

\[
\Phi \equiv \int_S B dS = \frac{1}{2\pi} \int_V B r^2 dV = \frac{1}{2\pi} \int_V f r^2 dV ,
\]

where \( S \) denotes a poloidal section of a magnetic surface and \( V \) the volume enclosed by this magnetic surface.

It has the dimension of a length. In a first approximation, it can be seen as the mean geometrical radius of the magnetic surface since the toroidal field applied from outside the plasma is very large compared with the toroidal field (produced by the plasma by the diamagnetic and/or paramagnetic effects).

The security factor is defined by

\[
q \equiv -\frac{1}{2\pi} \partial_\varphi \Phi .
\]
Higher values of \( q \) lead to greater plasma stability, and specific \( q \) profiles can be related to higher core confinement thus it is an important output plasma variable.

### 2.5 Resistive Diffusion Equation

Note that since \( \Psi \) and \( f \) are constant on each magnetic surface there exists \( \Psi \) and \( f \) such that
\[
\Psi(r,z) = \Psi(\rho(r,z)) \quad \text{and} \quad f(r,z) = f(\rho(r,z)).
\]

It can be shown after some calculation (see (Blum, 1989; Imbeaux and al, 2006)) that
\[
\frac{\partial \bar{\Psi}}{\partial t} = -\frac{(E.B)}{f(1/r^2)}.
\]

Now, using Ohm’s law we have \( (E.B) = \eta(j - j_n).B) \)

Therefore we obtain (see (Blum, 1989; Imbeaux and al, 2006)) :
\[
\frac{\partial \bar{\Psi}}{\partial t} = \frac{\eta f}{\mu_0 c_3} \frac{\partial}{\partial \rho} \left( \frac{c_2}{\rho} \frac{\partial \bar{\Psi}}{\partial \rho} \right) + \frac{\eta(j_n.B)}{f(1/r^2)}, \quad (5)
\]

with
\[
c_2(\rho) = v'((\nabla \rho)^2/r^2), \quad c_3(\rho) = v'(1/r^2).
\]

Using (4) and (3) we have :
\[
\frac{\partial \rho \Phi}{\partial t} = \frac{\bar{f} v}{2\pi}(1/r^2) = 2\pi \rho B_0,
\]

and therefore
\[
\bar{f} = \frac{4\pi^2 \rho B_0}{c_3}.
\]

Substituting \( \bar{f} \) by its value in (5) we obtain the resistive equation :
\[
\frac{\partial \bar{\Psi}}{\partial t} = \frac{\eta f}{\mu_0 c_3} \partial \rho \left( \frac{c_2 \rho}{\rho} \partial \rho \bar{\Psi} \right) + \frac{\eta(j_n.B)}{4\pi^2 \rho B_0}. \quad (6)
\]

By symmetry, the boundary condition at \( \rho = 0 \) is
\[
\partial \rho \bar{\Psi}(0) = 0. \quad (7)
\]

The boundary condition at \( \rho = \rho_{\text{max}} \) is obtained by computing \( I \), the total toroidal plasma current (see (Blum, 1989; Imbeaux and al, 2006)) which gives :
\[
\partial \rho \bar{\Psi}(\rho_{\text{max}}) = -\frac{2\pi \mu_0 I}{c_2}. \quad (8)
\]

Note that \( V = \bar{\Psi} \) can also be used as a boundary condition.

In the sequel, we will assume that the magnetic surfaces are time constant, that \( S \) is a disk, and that \( \varepsilon \equiv \rho/R \) (where \( R \) is the major radius) is small.

### 3. Resolution of the Diffusion Resistive Model

In this section we specify the resistive equation by making some assumption on magnetic surface shapes and by giving empirical formula for the resistivity \( \eta \), bootstrap current and lower hybrid current which drive the non inductive profile. Only two sources of non inductive current are considered here the bootstrap and lower hybrid currents. We solve the corresponding resistive equation using the ODE solver of Scilab-Scicos and compare the results obtained with those computed by Cronos. The model proposed here is a simplified version of the model introduced in (Witrant and al, 2007). The simplification has been obtained empirically by studying the sensibility of the parameters introduced and studying their contribution to error with results obtained by Cronos on three shots.

#### 3.1 Geometric Hypotheses

In the following we write the diffusive equation under the cylindrical hypothesis. We consequently assume that \( \varepsilon = \rho/R \) is small and that the corresponding poloidal sections are circular. Under these assumptions we have :
\[
\bar{v} = 4\pi^2 R \rho, \quad c_2 = c_3 = \frac{4\pi^2 \rho B_0}{R}, \quad (|\nabla \rho|)^2 = 1,
\]

\[
\eta v'(j_n.B) \frac{4\pi^2 \rho B_0}{4\pi^2 \rho B_0} = \eta R j_n.
\]

#### Table 1. Primitive Constants

| \( \varepsilon \) | electric electron charge |
| \( f \) | ion electron charge |
| \( m_e \) | electron mass |
| \( m_i \) | average ion mass (kg) |
| \( Z \) | effective ion electron charge ratio |
| \( \mu_0 \) | permeability of free space (H/m) |
| \( \varepsilon_0 \) | permittivity of free space (F/m) |
| \( R \) | major radius of the plasma (m) |
| \( a \) | minor radius of the plasma |
| \( B_0 \) | toroidal magnetic field at the plasma center |
| \( I \) | total plasma current (A) |
| \( V \) | loop voltage |

During the heating process, antennas are used (here the numerical experiments will be done only with the lower hybrid antenna (LHCD)) and the dynamic equation of the magnetic flux is :

\[
\begin{align*}
\partial_t \bar{\Psi} &= R \eta(T_x) (j_b(\partial_x P_x, \partial_x \bar{\Psi}) + j_b + c_j \frac{1}{x} \partial_x (x \partial_x \bar{\Psi})), \\
\partial_y \bar{\Psi}(t,0) &= 0, \quad \partial_y \bar{\Psi}(t,1) = -c_f I(t),
\end{align*}
\]

where :

- \(-c_j \frac{1}{x} \partial_x (x \partial_x \bar{\Psi})\), denoted \( j_\rho(\bar{\Psi},t) \), is the toroidal current plasma profile.
### 3.3 Temperature Profile

To determine the resistivity we need a temperature model. We use the following simplified dynamics for describing the electronic temperature $T_e$ and ion temperature $T_i$ evolutions:

\[
\frac{3}{2} \frac{\partial t}{\partial t_e} (n_e T_e) = \frac{1}{(a^2 x^2)} \partial_x (x \chi_{n_e} \partial_x T_e) - \frac{c_D n_e (T_e - T_i)}{\tau_{ei}},
\]

\[
\frac{3}{2} \frac{\partial t}{\partial t_i} (n_i T_i) = \frac{1}{(a^2 x^2)} \partial_x (x \chi_{n_i} \partial_x T_i) - \frac{c_D n_e (T_i - T_e)}{\tau_{ei}},
\]

with

\[
\tau_{ei} = c_e T_e^{1.5} / n_i,
\]

\[
\chi_e = c_e^3 n_e q_e^2 \partial_x T_e, \quad \chi_i = c_i^3 n_i q_i^2 \partial_x T_i.
\]

where the species densities $n_e$ and $n_i$ are supposed to be given (in fact provided by Cronos software).

To improve this model a term of radiating loss will be added in future numerical experiments.

### 3.4 Bootstrap Current

The bootstrap current comes from a complex mechanism where some particles do not follow the magnetic field but are trapped in a plasma zone. The contribution of the electron (only considered here) to the induced current is given by:

\[
j_b(\partial_x P_e, \partial_x \bar{\Psi}) = c_b \frac{\partial_x P_e}{\partial_x \bar{\Psi}}.
\]

In (Wesson, 2004) Section 14.12 we can find more precise formulas where the bootstrap dependence not only of $P_e$ but also $P_i$, $T_e$ and $T_i$ in the future we will look more precisely to the influence of the other terms.

### 3.5 Lower Hybrid Current Drive

To be able to evaluate the quality of the proposed model for the plasma control, we need to compare the result obtained with experiments. For that the lower hybrid heating antenna deposit used is a simplified version of the one given in (Witrant and al., 2007). In the control part we will not use this particular current deposit but will consider an arbitrary coupled current-heating deposit that the set of antennas available will have to approximate.

### 3.6 Security Factor

The security factor can be rewritten as:

\[
q(t,x) = -c_q x \frac{\partial_x P_e}{\partial_x \bar{\Psi}(t,x)}.
\]

Typically, we control the plasma current density profile using the heating antenna source $j_b$ and the
flux variation at the plasma edge $I$. We would like to obtain and stabilize a specified security factor profile $q^*$ appropriate for fusion conditions, under the constraint of securing the plasma stability.

3.7 Scilab/Scicos Implementation

This partial differential equation model has been solved numerically using the free software Scilab. The equation is solved using the default ode solver of Scilab. The state derivatives are approximated by appropriate finite difference matrices. The simulation is done by a script function or implemented using the Scicos block-diagram editor (see Figure 3). The ode solver uses multistep formulas and the numerical results are obtained within a few seconds.

3.8 Numerical Results

The numerical results obtained are compared to those obtained by the Cronos software from experimental datas coming from a typical Tore Supra shot. Cronos is a set of Matlab programs dedicated to the simulation and the experimental data processing of the plasma transport phenomenon. It contains the description of the actuator interaction with the plasma. The simulation obtained by the simplified model described here uses the Cronos data for the unmodeled states (the species densities).

Figure-4-5 compares the Cronos and the proposed model simulation results. We see that the model seems to give enough information to have an overview of the tracking control quality.

4. CURRENT DENSITY PROFILE CONTROL

In this section we describe how to stabilize the security profile around a specific value which corresponds to an improved temperature profile for the fusion purpose. For that we can use several heating and current deposit antennas. Each antenna has its specific profile deposit. We suppose that with all these means we are able obtain any current shape deposit. Clearly, this is a strong hypothesis only suitable for a preliminary study. This hypothesis means that any shape can be approximated with the available antennas.

4.1 Remarks on tracking control

Let us consider the following linear system
\[
\begin{align*}
\dot{x} &= Ax + By + u \\
\dot{y} &= Cx + Dy + Eu
\end{align*}
\]
and let us suppose that we want that the state $x$ tracks asymptotically $x^*$ that is:
\[
\lim_{t \to \infty} x(t) = x^*
\]
The feedback
\[
u = -(Ax^* + By),
\]
leads to the closed loop system:
\[
\begin{align*}
\dot{x} &= A(x - x^*) \\
\dot{y} &= Cx - EAx^* + (D - EB)y
\end{align*}
\]
which achieves the desired tracking. Of course the above control is valid only if both $A$ and $D - EB$ are asymptotically stable.

Necessary and sufficient condition for tracking control models with more natural assumptions...
are given in (Francis, 1977) and (Isidori and Byrnes, 1990) but are difficult to implement in the plasma model.

4.2 Security factor profile tracking

We can apply the tracking feedback of the previous section to the plasma magnetic flux in (9) where $\Psi$ play the role of $x$ and $(T_e, P_e)$ the role of $y$ and $j_h$ the role of $u$. Moreover, let us suppose that we want a specific security profile $q*$ which corresponds to $\Psi^*$ defined up to an additive constant.

Let us take for example:

$$q^* = c_q \max \left[ 1, e^{3(x-0.3)} \right],$$

where $c_q$ is chosen in such way that the magnetic flux boundary condition at $x = 1$ is satisfied.

Since asymptotically in time $\partial_t \Psi = V$ where $V$ is the loop voltage, we obtain a $q^*$ tracking control called $j_h$ as a feedback on the pressure from (9):

$$j_h = -j_b \left[ \partial_x P_e, \frac{-c_q x}{q^*} \right] - c_j \frac{1}{x} \partial_x \left[ x \partial_x \left[ \frac{-c_q x}{q^*} \right] \right] + \frac{V}{R \eta \eta(T_e)}.$$

This control is easy to implement because it does not need a model for the temperature and the pressure. This point is very useful since, for the time being, there does not exist good model for these quantities.

To slow down the influence of the tracking feedback we use a time varying gain $G(t)$ starting from zero at time 0 and converging to one with time going to infinity.

Governed by this feedback the magnetic flux admits the dynamics:

$$\partial_t \Psi = R \eta(T_e) \left( j_b + G(t) j_h + c_j \frac{1}{x} \partial_x (x \partial_x \Psi) \right).$$

We observe numerically (with the simplify temperature and densities model) that the desired equilibrium is stable see Figure-6. The current profile obtained is not smooth because the target is not smooth. More smooth current profile can be obtained. The sensibility of the current and the pressure with the security factor target is high and will be explore in future work.

5. CONCLUSION

Based on a resistive model of the current in the plasma a simple tracking of a desired security factor profile has been obtained. The simulation results are encouraging. The next step is to generate the current deposit determined by the tracking feedback using the available antennas. The sensibility analysis of the security factor with respect to the control shape deposit will be an important issue since the available shape space obtained using the existing antenna set is limited. Other validations of the proposed feedback with better models will be also an important issue. Finally, a proof of stability of the closed loop nonlinear PDE system would be very useful.

6. REFERENCES


