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# A computational strategy suitable for multiphysics problems

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**Abstract**
Multiphysics phenomena and coupled-field problems usually lead to computationally intensive structural analyses. Strategies to keep these problems computationally affordable are of special interest. For coupled fluid-structure problems for instance, partitioned procedures and staggered algorithms are often preferred to direct analysis (also called the monolithic approach), from a computational efficiency point of view.
Recently, a mixed domain decomposition method has been designed for parallel computing environments, and a multi-level approach embedding a homogenization procedure makes it suitable for highly-heterogeneous problems. From the generalization of the concept of geometric interfaces between substructures to an interface between different physics, the Large Time INcrement method (LATIN) allows building an approach suited for solving coupled multiphysics problems.
The proposed application concerns the consolidation of porous saturated soil, i.e. a coupled fluid-solid problem in the domain. The feasability of the method and its performance comparison with a standard partitioning scheme (the so-called ISPP) has been presented in a previous paper.
As an improvement, the further step is to take into account different time scales arising from multiphysics problem. Thus, the present paper proposes a time multiscale strategy.

## 1 Introduction – reference problem

For coupled multiphysics problems such as fluid-structure interactions, partitioned procedures and staggered algorithms [1, 2, 3, 4, 5, 6, 7] are often preferred to direct analysis (also called the monolithic approach), from a computational efficiency point of view. Moreover, partitioning strategies allow use of different analyzers for different subsystems and help keep the software manageable.
Recently, a mixed domain decomposition strategy based on the Large Time INcrement method (LATIN, see [12]) has been designed for parallel computing environments [8], and a multi-level approach embedding a homogenization procedure makes it suitable for highly-heterogeneous problems [9]. Within the same framework, an approach suited to multiphysics problems has been designed and applied to the consolidation of porous saturated soils, which is a typical example of strong coupled fluid-structure interaction problem.
The feasability of the method and its comparison with a standard partitioning scheme (ISPP, see [11]) has been presented in the previous paper [10]. The proposed strategy has then appeared to be competitive from a computational efficiency point of view. As an improvement, the further step is to take into account different time scales arising from multiphysics problem. Thus, the present paper proposes a time multiscale strategy.
We consider a structure $\Omega$ composed of saturated porous material, under small perturbations and isothermal evolution over the studied time interval $[0,T]$. The loadings are: a prescribed displacement $\underline{U}_d$ on an initial part of the boundary $\partial_1\Omega$, a traction force $\underline{F}_d$ on the complementary part $\partial_2\Omega$, a fluid flux $w_d$ on another part $\partial_3\Omega$, and finally, a prescribed pore fluid pressure $p_d$ on the complementary part $\partial_4\Omega$ of $\partial_3\Omega$. For the sake of simplicity, no body forces are considered.
For solid quantities, the strain is denoted with $\boldsymbol{\varepsilon}$, the stress with $\boldsymbol{\sigma}$. For fluid quantities, the pore pressure gradient is denoted with $\underline{Z}$, the opposite of the Darcy's velocity is $\underline{W}$; finally, $q$ denotes the rate of fluid mass accumulation in each representative elementary volume.

The state of the structure is given with the set of fields $\mathbf{s} = (\boldsymbol{\varepsilon}, p, \underline{Z}, \boldsymbol{\sigma}, q, \underline{W})$ defined on the whole structure $\Omega$ and the studied time interval $[0,T]$. The problem is to find $\mathbf{s}$ in its corresponding space $\mathbf{S}^{[0,T]}$ that satisfies at each time step the following:

- for the solid, the equilibrium of stress $\boldsymbol{\sigma}$ and the compatibility of strain $\boldsymbol{\varepsilon}$:

$$\begin{aligned} \underline{\operatorname{div}}\,\boldsymbol{\sigma} = \underline{0} \quad \text{on } \Omega \quad \text{and} \quad \boldsymbol{\sigma}\underline{n} = \underline{F}_d \quad \text{on } \partial_2\Omega \\ \underline{U} \in \mathcal{U}^{[0,T]} \quad \text{and} \quad \boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\underline{U}) \end{aligned} \tag{1}$$

  $\mathcal{U}^{[0,T]}$ is the set of finite-energy displacement fields on $\Omega \times [0,T]$, which equal $\underline{U}_d$ on $\partial_1\Omega$.

- for the fluid, the flow conservation for Darcy's velocity $-\underline{W}$:

$$\begin{aligned} q = \operatorname{div}\underline{W} \quad \text{on } \Omega \quad \text{and} \quad \underline{W}\cdot\underline{n} = w_d \quad \text{on } \partial_4\Omega \\ \underline{Z} = \underline{\operatorname{grad}}\, p \quad \text{on } \Omega \quad \text{and} \quad p \in \mathcal{P}^{[0,T]} \end{aligned} \tag{2}$$

  $\mathcal{P}^{[0,T]}$ is the set of finite-energy pressure fields on $\Omega \times [0,T]$, which equal $p_d$ on $\partial_3\Omega$.

- the constitutive relations:

  – Hooke's law: the macroscopic stress $\boldsymbol{\sigma}$ is related to the strain $\boldsymbol{\varepsilon}$ and coupled with the pore pressure $p$ such as

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon} - bp\mathbf{1} \tag{3}$$

– Darcy's law relates Darcy's velocity to the pore pressure gradient

$$\underline{W} = \frac{K}{\mu_w}\underline{Z} \tag{4}$$

– the influence of compressibility relates the fluid accumulation rate to the pressure rate and couples it with the rate of volume modification

$$q = \frac{1}{Q}\dot{p} + b\,\mathrm{Tr}\,\dot{\boldsymbol{\varepsilon}} \tag{5}$$

$\mathbf{D}$ is Hooke's tensor of the drained skeleton, $b$ is Biot's coefficient, $K$ is the intrinsic macroscopic permeability and $\mu_w$ is the dynamic viscosity of the saturation fluid. In all of the following, the operator $\frac{K}{\mu_w}\mathbf{1}$ will be denoted as $\mathbf{H}$. Finally, $Q$ is Biot's modulus.

## 2 LATIN computational strategy

The LATIN method is a non-incremental iterative approach originally designed for non-linear time-dependant problems. Nevertheless, its framework has been successfully applied to dynamic problems, post-buckling, etc. At each iteration, this method produces an approximation of the solution on the whole domain and over the entire studied time interval. It is based on three principles:

- The first principle consists of splitting the difficulties. For coupled-field problems, an initial set of equations, $\mathbf{A_d}$, contains the so-called admissibility conditions (1), (2). To avoid the simultaneous treatment of both a global and a coupled problem, the remaining equations are combined into a second set of equations $\mathbf{\Gamma}$; these equations are local in space variables ; they are the constitutive relations (3), (4), (5). In order to find the solution, i.e. the set of fields belonging to both $\mathbf{A_d}$ and $\mathbf{\Gamma}$, an iterative procedure is used.

- The second principle of the method consists of using search directions to build approximated solutions of $\mathbf{A_d}$ and $\mathbf{\Gamma}$ alternatively, until a sufficient level of convergence has been reached. Each iteration contains 2 stages.

  The *local stage* at iteration $n+1$ provides an element $\hat{\mathbf{s}}_{n+1/2} \in \mathbf{\Gamma}$ once an element $\mathbf{s}_n \in \mathbf{A_d}$ is known, using an initial search direction:

$$\begin{aligned}(\hat{\boldsymbol{\sigma}}_{n+1/2} - \boldsymbol{\sigma}_n) + \mathbf{L}(\hat{\dot{\boldsymbol{\varepsilon}}}_{n+1/2} - \dot{\boldsymbol{\varepsilon}}_n) &= 0\\ (\hat{\underline{W}}_{n+1/2} - \underline{W}_n) + \mathbf{H}(\hat{\underline{Z}}_{n+1/2} - \underline{Z}_n) &= 0\\ (\hat{q}_{n+1/2} - q_n) + r(\hat{p}_{n+1/2} - p_n) &= 0\end{aligned} \tag{6}$$

  $\mathbf{L}$ and $r$ are two parameters of the method; they do not modify the solution when convergence is reached. Nevertheless, their values modify the convergence rate of the algorithm. In a dimensional analysis, they can be chosen of the form $\mathbf{L} = t_m\mathbf{D}$ and $r = \frac{1}{Qt_h}$ where $t_m$ and $t_h$ are two characteristic times to be chosen.

  The *linear stage* provides an element $\mathbf{s}_{n+1} \in \mathbf{A_d}$ once an element $\hat{\mathbf{s}}_{n+1/2} \in \mathbf{\Gamma}$ is known. $\mathbf{s}_{n+1}$ must satisfy admissibility relations, and is defined with a search direction conjugated with the

previous one, such that both the mechanical and hydraulic problems remain decoupled:

$$\begin{aligned}(\boldsymbol{\sigma}_{n+1} - \hat{\boldsymbol{\sigma}}_{n+1/2}) - \mathbf{L}(\dot{\boldsymbol{\varepsilon}}_{n+1} - \hat{\dot{\boldsymbol{\varepsilon}}}_{n+1/2}) = 0 \\ (\underline{W}_{n+1} - \underline{\hat{W}}_{n+1/2}) - \mathbf{H}(\underline{Z}_{n+1} - \underline{\hat{Z}}_{n+1/2}) = 0 \\ (q_{n+1} - \hat{q}_{n+1/2}) - r(p_{n+1} - \hat{p}_{n+1/2}) = 0\end{aligned} \tag{7}$$

- A third principle calls for to taking into account the feature whereby successive approximations are defined over both on the entire domain and the entire time interval, in order to represent the solution on a radial loading basis.

### 2.1 Linear stage with representation at iteration $n+1$

The unknowns of $\mathbf{A_d}$ are searched of the following form: a sum of products of a time scalar function with a field that depends only on the space variables. For instance, the strain rate is approximated with: $\dot{\boldsymbol{\varepsilon}}(M,t) = v_0(t)\boldsymbol{\varepsilon}(\underline{V}_0) + \sum_j v_j(t)\boldsymbol{\varepsilon}(\underline{V}_j)$, where the couples $(v_k(t), \underline{V}_k(M))$ have to be automatically determined by the algorithm. At the iteration $n+1$, a correction to the previous approximation has to be determined, i.e. $\mathbf{s}_{n+1} = \mathbf{s}_n + \Delta\mathbf{s}$.

A first version of this principle is described in the following. It consists in using a representation for the admissible kinematic corrections, while the static corrections are computed with the search direction. As a consequence, the static quantities are not strictly admissible unless the convergence is reached. Computing admissible static quantities would require an additional numerical effort [12].

**Solid part.** The *strain correction* is searched of the approximated form: $\Delta\dot{\boldsymbol{\varepsilon}} = v\boldsymbol{\varepsilon}(\underline{V})$, where $v(t)$ is a time scalar function, and $\underline{V}(M) \in \mathcal{U}_0$. The stress is computed with the search direction:

$$\boldsymbol{\sigma}_{n+1} = \mathbf{L}(\dot{\boldsymbol{\varepsilon}}_{n+1} - \hat{\dot{\boldsymbol{\varepsilon}}}_{n+1/2}) + \hat{\boldsymbol{\sigma}}_{n+1/2} = \mathbf{L}v\boldsymbol{\varepsilon}(\underline{V}) + \underbrace{\mathbf{L}(\dot{\boldsymbol{\varepsilon}}_n - \hat{\dot{\boldsymbol{\varepsilon}}}_{n+1/2}) + \hat{\boldsymbol{\sigma}}_{n+1/2}}_{\text{known at iteration } n+1}$$

In the present version of the algorithm, the unknown couple $(v, \underline{V})$ is searched in such a way that the stress is weakly admissible, i.e.

$$\forall \underline{U}^\star = v^\star \underline{V} + v\underline{V}^\star \in \mathcal{U}_0^{[0,T]}, \quad \int_0^T\!\!\int_\Omega \mathrm{Tr}[\boldsymbol{\sigma}_{n+1}\boldsymbol{\varepsilon}(\underline{U}^\star)]d\Omega dt = \int_0^T\!\!\int_{\partial_2\Omega} \underline{F}_d \cdot \underline{U}^\star dS dt$$

**Fluid part.** The previous scheme for the solid part of the multiphysics problem has to be extended on the same way for the hydraulic components $(p, \underline{Z})$ and $(q, \underline{W})$. For instance, the *pressure correction* is expressed with the following approximation: $\Delta p = \pi P$ and $\Delta\underline{Z} = \pi\, \underline{\mathrm{grad}}\, P$, where $\pi(t)$ is a scalar time function and $P(M) \in \mathcal{P}_0$.

The static counterpart unknowns are obtained with the search direction:

$$\begin{aligned}\underline{W}_{n+1} = (\underline{\hat{W}}_{n+1/2} - \mathbf{H}\underline{\hat{Z}}_{n+1/2}) + \mathbf{H}\underline{Z}_{n+1} = \mathbf{H}\underline{Z}_{n+1} = \mathbf{H}\pi\, \underline{\mathrm{grad}}\, P + \mathbf{H}\underline{Z}_n \\ q_{n+1} = r(p_{n+1} - \hat{p}_{n+1/2}) + \hat{q}_{n+1/2} = r\pi P + \underbrace{r(p_n - \hat{p}_{n+1/2}) + \hat{q}_{n+1/2}}_{\text{known at iteration } n+1}\end{aligned}$$

The best couple $(\pi, P)$ is obtained to get a weak static admissibility:

$$\forall p^\star = \pi^\star P + \pi P^\star \in \mathcal{P}_0^{[0,T]}, \quad \int_0^T\!\!\int_\Omega (\underline{W}_{n+1} \cdot \underline{\mathrm{grad}}\, p^\star + q_{n+1}p^\star)d\Omega dt = \int_0^T\!\!\int_{\partial_4\Omega} w_d p^\star dS dt$$

The fluid and solid problems are decoupled at the linear stage, so there is no difficulty to use a different time discretization for each.

### 2.2 Local stage with time multiscale approach at iteration $n+1$

With the constitutive relations, the local stage leads to solving a small local-in-space-variable ODE system at each integration point:

$$\begin{aligned} &\mathbf{L}\hat{\dot{\boldsymbol{\varepsilon}}} + \mathbf{D}\hat{\boldsymbol{\varepsilon}} - b\hat{p}\mathbf{1} = \mathbf{A}_n(t) \\ &\frac{1}{Q}\hat{\dot{p}} + r\hat{p} + b\,\mathrm{Tr}\,\hat{\dot{\boldsymbol{\varepsilon}}} = \alpha_n(t) \end{aligned} \tag{8}$$

where $\mathbf{A}_n = \boldsymbol{\sigma}_n + \mathbf{L}\dot{\boldsymbol{\varepsilon}}_n$ and $\alpha_n = q_n + rp_n$ are known quantities at local stage $n+1$, and with the initial conditions on the pressure and the strain fields. As the the solid and the fluid unknowns interact with each other, this subsection presents a strategy to deal with different time scales. This method is an adaptation of the micro-macro strategy proposed in [13, 14] in order to solve space multiscale problems.

The idea is to split the unknowns into the form $s = s^M + s^m$ where $s^M$ is the set of the "macroscopic" quantities and $s^m$ is the additive "micro" complement. Herein, we choose to express the state variables of the solid part with only a macro time scale (for instance: $\boldsymbol{\sigma} = \boldsymbol{\sigma}^M$ and $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^M$). The fluid part possesses both a macro and a micro scale (for instance: $p = p^M + p^m$ and $q = q^M + q^m$). The superscripts $m$ and $M$ denote respectively *time* micro and macro scales.

The macro part of a time function $f \in \mathcal{F}_{[0,T]}$ is given with the projector $\boldsymbol{\pi}$ as $f^M = \boldsymbol{\pi} f \in \mathcal{F}^M_{[0,T]}$. Then, the micro part is the complement $f^m = (\mathbf{id} - \boldsymbol{\pi})f \in \mathcal{F}^m_{[0,T]}$. $\mathcal{F}^M_{[0,T]}$ and $\mathcal{F}^m_{[0,T]}$ denote the micro and macro time functions spaces.

Let $\mathbf{e}^M = \{e^M_0, \ldots, e^M_{n_M}\}$ be a basis of $\mathcal{F}^M_{[0,T]}$ which is orthonormalized with respect to the scalar product $\langle f, g\rangle = \int_0^T fg\,dt$. $\boldsymbol{\pi}$ can be explicited with

$$f^M = \boldsymbol{\pi} f = \sum_{k=0}^{n_M} \langle f, e^M_k\rangle e^M_k \quad \text{and} \quad f^m = f - f^M$$

The micro-macro splitting is performed at the continuum level and involves *a priori* no discretization.

Let us consider a partition of the time interval $[0,T]$ into $n_M$ subintervals $\{I_1, \ldots, I_{n_M}\}$ given by $I_k = [T_{k-1}, T_k]$ with nodes $0 = T_0 < T_1 < \cdots < T_{n_M} = T$. We define, for instance, $\mathcal{F}^M_{[0,T]}$ as the space of the functions which are continuous and linear over each $I_k$. An orthonormal basis $\mathbf{e}^M$ can be built by a Schmidt procedure on the $n_M + 1$ usual basis functions of $\mathcal{F}^M_{[0,T]}$. Figure 1 shows an example of the function with its macro part. As opposite to a hierarchical approach, $f^M(T_k)$ does not have to be equal to $f(T_k)$.

For the multiphysic problem we are dealing with, we define 2 time discretizations:

- for the **fluid part**, $n$ subintervals $i_k = [t_{k-1}, t_k]$ with nodes $0 = t_0 < \cdots < t_n = T$;
- for the **solid part**, $n_M$ subintervals $I_k = [T_{k-1}, T_k]$ with nodes $0 = T_0 < \cdots < T_{n_M} = T$.

We choose to set $n_M \leqslant n$ and, in order to simplify, the solid time discretization is assumed to be embedded in the fluid one as in Figure 2.

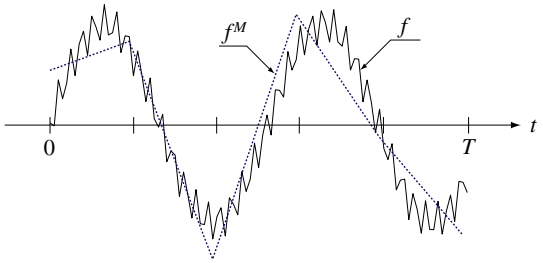



Figure 1: The macro part $f^M$ of a function $f$


Figure 2: An example of solid and fluid time discretizations

With $(\hat{\dot{\boldsymbol{\varepsilon}}}, \hat{\boldsymbol{\varepsilon}}, \mathbf{A}_n)$ described on the solid mesh (macro scale) and $(\hat{\dot{p}}, \hat{p}, \alpha_n)$ on the fluid mesh (both macro and micro scale), the local stage (8) is transformed into:

$$\mathbf{L}\hat{\dot{\boldsymbol{\varepsilon}}} + \mathbf{D}\hat{\boldsymbol{\varepsilon}} = \mathbf{A}_n(t) + b\boldsymbol{\pi}\hat{p}\mathbf{1} \tag{9a}$$

$$\frac{1}{Q}\hat{\dot{p}} + r\hat{p} = \alpha_n(t) - b\,\mathrm{Tr}\,\mathbf{P}\hat{\dot{\boldsymbol{\varepsilon}}} \tag{9b}$$

where $\boldsymbol{\pi}$ is the previous projector, $\mathbf{P}$ a linear interpolation. (9a) can be solved on the solid time discretization whereas (9b) can be solved on the fluid one.

Such a local stage is a simplification of (8) when the fluid part is expected to have a rapid variation evolution (micro part) that does not influence the solid behaviour. Effectively, when the search directions are optimized for the LATIN approach without time multiscale feature, the ODE to solve in (9a) possesses a characteristic time $t_m = 0.128$ s, whereas the macro time step will be $0.03$ s and the micro time step will vary between $0.002$ s and $0.03$ s for the further numerical simulations. As the micro time scale is far less than $t_m$, the previous approximation is expected to be valid.

This new local stage is performed with a fixed point method. The initial guess for the pression is chosen as $\hat{p} = p_n$. Once $\hat{p}$ is known, $\hat{\dot{\boldsymbol{\varepsilon}}}$ is found with (9a), then $\hat{p}$ is computed with (9b). The number of subiterations will be discussed in a following section.

### 2.3 The error indicators

Let us introduce the following energy norm:

$$e^2(\mathbf{s}) = \underbrace{\frac{1}{2}\int_0^T\int_\Omega \mathrm{Tr}[\boldsymbol{\varepsilon}\mathbf{D}\boldsymbol{\varepsilon}]d\Omega dt}_{e_S^2(\mathbf{s})} + \underbrace{\frac{1}{2}\int_0^T\int_\Omega pQ^{-1}p\;d\Omega dt}_{e_F^2(\mathbf{s})}$$

The contribution of each "physics" on the total error is $\eta_S = \dfrac{e_S(\mathbf{s}-\mathbf{s}_{ex})}{e(\mathbf{s}_{ex})}$ for the solid part and $\eta_F = \dfrac{e_F(\mathbf{s}-\mathbf{s}_{ex})}{e(\mathbf{s}_{ex})}$ for the fluid part. The total error is $\eta = \sqrt{\eta_S^2 + \eta_F^2}$.
The reference solution $\mathbf{s}_{ex}$ is produced with a direct monolithic scheme (see [15] for a brief description).

# 3 Test case

## 3.1 Problem description

The proposed test case is the consolidation of a Berea sandstone soil. The material characteristics in Table 1 have been identified in [16]. The uniaxial strain 1D and the plain strain 2D problems have been treated (see Figure 3). The time interval is $T = 1$ s and the pressures are $p_d = 0.1$ MPa and $p_1 = 0.1$ MPa with a time $t_1 = 0.5$ s; the initial condition is $p(t = 0) = p_d$.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Porosity | $n$ | = | 0.19 | Biot modulus | $Q$ | = | 13.5 GPa |
| Young modulus | $E$ | = | 14.4 GPa | Biot coefficient | $b$ | = | 0.78 |
| Poisson coefficient | $\nu$ | = | 0.2 | Permeability | $\frac{K}{\mu_w}$ | = | $2\ 10^{-10}$ m$^3$.s.kg$^{-1}$ |

Table 1: Water-saturated Berea sandstone poro-elastic material characteristics

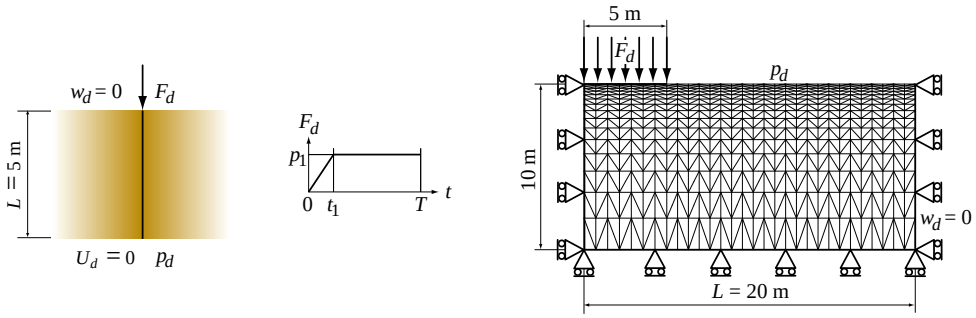



Figure 3: 1D and 2D test problems

The space discretization features P2 elements (6-node triangles for the 2D case) for the displacement and a P1 linear interpolation (also continuous throughout the elements) for the pore pressure, see [17, 18]. Concerning the time discretization, the $\theta$-method with a linear evolution of variables in time is used. [19, 20] give an accuracy condition: $\frac{\Delta t}{\Delta \ell^2} \geqslant \frac{1}{6\theta c}$ where $\Delta t$ is the length of a time step, $\Delta \ell$ the size of a space element and $c = E\frac{K}{\mu_w}\frac{3-2\nu}{3(1+\nu)(1-2\nu)}$. For 1D case, this condition leads to a minimum $\Delta t$ of $10^{-4}$ s.

## 3.2 Numerical results

### 3.2.1 Error contributions with the time multiscale strategy

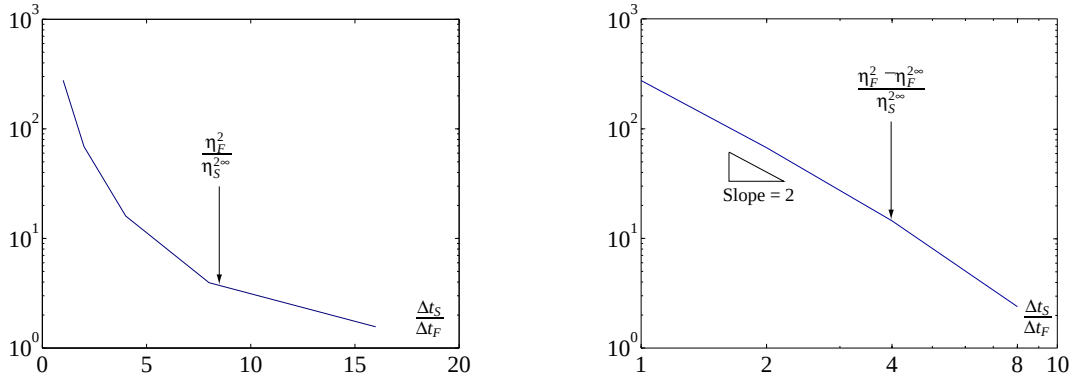
The computation of the 1D problem is performed. The solid part is expected to possess only a macro time scale with $n_M = 32$ time steps and the fluid part is discretized with $n$ time steps. The influence of $n$ on the fluid contribution to global error is tested. The ratio between the time step lengths is $\Delta t_S/\Delta t_F = n/n_M$.

Table 2 shows the evolution of the contributions $\eta_S$ and $\eta_F$ on the total error $\eta$ versus $\Delta t_S/\Delta t_F$. As the problem is coupled, a refinement of the fluid time discretization decreased both of the contributions. Nevertheless, the fluid contribution decreases faster than the solid one. When the fluid part is sufficiently refined, it can be considered as exact in time. $\eta_S^{2\infty}$ and $\eta_F^{2\infty}$ are the corresponding contributions. None of them is null as the coarse discretization of the solid part contributes to the errors. Figure 4(a) shows the evolution of $\eta_F^2/\eta_S^{2\infty}$ versus $\Delta t_S/\Delta t_F$.

To study the fluid error due to the fluid discretization only, we consider $\eta_F^2 - \eta_F^{2\infty}$. With the numerical convergence of $\eta_F^2 - \eta_F^{2\infty}$ obtained on Figure 4(b), one can notice that, in order to get a similar contribution to the error for the fluid part ($\eta_F^{2\infty}$) and for the solid part ($\eta_S^{2\infty}$), the relative time steps are $\Delta t_S/\Delta t_F \sim 16$.

| $\Delta t_S/\Delta t_F$ | $\eta_S$ | $\eta_F$ | $\eta$ | $\eta_F/\eta_S$ |
|---|---|---|---|---|
| 1 | $150\ 10^{-5}$ | $310\ 10^{-5}$ | $347\ 10^{-5}$ | 2.08 |
| 2 | $75\ 10^{-5}$ | $156\ 10^{-5}$ | $173\ 10^{-5}$ | 2.07 |
| 4 | $38\ 10^{-5}$ | $75\ 10^{-5}$ | $84\ 10^{-5}$ | 2.00 |
| 8 | $28\ 10^{-5}$ | $37\ 10^{-5}$ | $47\ 10^{-5}$ | 1.32 |
| 16 | $19\ 10^{-5}$ | $23\ 10^{-5}$ | $30\ 10^{-5}$ | 1.25 |

Table 2: Error contributions with the time multiscale strategy




(a) $\eta_F^2/\eta_S^{2\infty}$ versus $\Delta t_S/\Delta t_F$ (b) $(\eta_F^2 - \eta_F^{2\infty})/\eta_S^{2\infty}$ versus $\Delta t_S/\Delta t_F$

Figure 4: Error contributions with the time multiscale strategy

### 3.2.2 Results for 2D simulation

Figure 5 shows the influence on the total error $\eta$ of the number of subiterations in the local stage when the solid time discretization contains 60 time steps and the fluid one contains 120. The local stage needs at least 2 subiterations to converge.

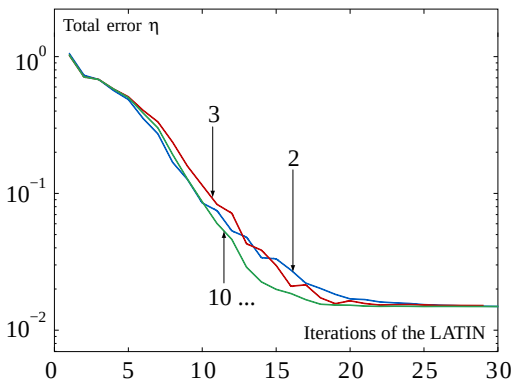



Figure 5: Influence of the number of subiterations in the local stage

Figure 6 reports the evolution of the maximum pore pressure during the time interval $[0, T]$ at several iterations of the method. The studied time interval effectively corresponds to the transient part of the soil response.

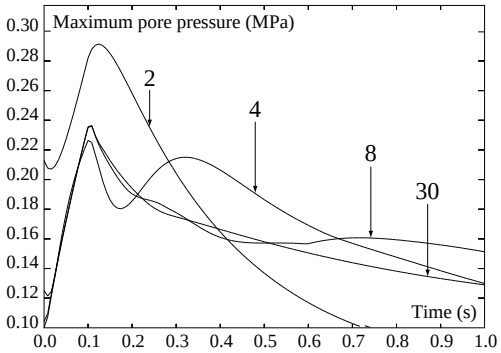



Figure 6: Maximum pore pressure at several iterations

## 4 Conclusion

In this paper, we have described a partitioned strategy based on the LATIN approach that allow to use different time steps for the solid and the fluid parts of a consolidation problem. In particular, one needs a shorter time step for the fluid in order to perform an iso-quality simulation (same contributions to the global error).

To improve the efficiency of this approach, the third principle of the method must be used (representation of the unknowns, see [10]). The first tests with the time multiscale strategy show that the convergence rate is still the same with or without this representation.

In the present numerical tests, uniform time discretizations were used. An interesting feature will be the automatic adaptivity of the time discretizations. Further work concerning the use of a time discontinuous Galerkin formulation is on development.

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