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Heating of the solar wind in the outer heliosphere

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Abstract. Small-scale Alfvénic turbulence in the outer heliosphere is mainly determined by a source which is connected with the instability of the initially highly anisotropic velocity distribution of interstellar pick-up protons. The main portion of the generated turbulent energy is subsequently absorbed by the pick-up protons themselves due to the cyclotron-resonant interaction between waves and particles. A small fraction of this energy can be transferred to solar wind protons resulting in their heating. The heating is more efficient in the high-speed solar wind.

1 Introduction

The measurements of the solar wind parameters by the Voyager-2 spacecraft (e.g. Richardson and Smith, 2003) show that the spatial behaviour of the proton temperature differs essentially from adiabatic cooling. Moreover, the proton temperature even increases beginning with 25–30 AU. According to the present view the main mechanism of the solar wind heating in the outer heliosphere is connected with pick-up protons originating in the solar wind as a result of ionization of interstellar hydrogen atoms (Williams et al., 1995; Zank et al., 1996; Matthaeus et al., 1999; Smith et al., 2001; Fahr and Chashei, 2002; Chalov et al., 2005). In the solar wind rest frame the speed of newly created protons is only approximately equal to the local wind speed due to the fact that the neutrals move into the heliosphere at about 20 km/s. The velocity distribution of the protons is highly anisotropic. This distribution is unstable (Wu and Davidson, 1972) and for a short time as compared with the time of convective transport evolves to a nearly isotropic distribution. The energy density of the isotropic distribution is smaller than the energy density of the initial anisotropic distribution stable on shortest timescales. The free energy is realized in the form of Alfvén waves. Dissipation of these waves due to the cyclotron resonant interaction with solar wind protons is considered as the principal mechanism of the solar wind heating in the outer heliosphere. This mechanism is efficient beyond 10 AU where the number density of atoms is sufficiently large.

The free energy of the anisotropic distribution constitutes approximately $v_A/V_{SW}$ of the total energy of pick-up ions, where $v_A$ and $V_{SW}$ are the Alfvénic and solar wind speed, respectively. However, as it follows from the results by Williams et al. (1995) and Smith et al. (2001) only about 5% of this free energy is enough to explain the spatial distribution of the proton temperature in the outer heliosphere. It was unclear for a long time why the other 95% do not contribute to the proton heating. Isenberg et al. (2003) and Isenberg (2005) have shown that the amount of turbulent energy transferred to protons can be reduced if the dispersion effects are taken into account. On the other hand, Chalov et al. (2004, 2005) have shown that turbulent energy generated by pick-up protons during the process of their isotropization can be reabsorbed by the pick-up protons due to cyclotron resonant interaction. It follows from Chalov et al. (2005) that pick-up protons absorb the major portion of the turbulent energy (so that the problem of 95% is absent), but, nevertheless, a small fraction of this energy can be transferred to solar wind protons. This amount of energy is, however, sufficiently large to heat the solar wind up to the measured temperatures. In the present paper we extend the results by Chalov et al. (2005) by taking into account the influence the solar wind speed on the heating rate.

2 Closed system of governing equations for protons and waves

The effect of pick-up protons on the thermodynamical properties of solar wind plasma becomes pronounced at $\sim 10–20$ AU from the Sun and it essentially increases further with the heliocentric distance due to relative increase of the pick-up proton number density as compared with the number density of solar protons in the expanding solar wind.
At such large distances the interplanetary magnetic field (IMF) can be considered as azimuthal on average almost everywhere in the supersonic solar wind with the exception of the polar regions. Thus, at these distances velocities of newly created pick-up ions are almost perpendicular to the magnetic field lines and their velocity distribution is close to a ring distribution. Such kind of the distribution is unstable and results in generation of a broad spectrum of waves. However, Alfvénic waves propagating along the magnetic field lines have largest increments (e.g. Brinca , 1991), and this is the main reason why just these waves are considered in the majority of studies concerning the instability of the anisotropic velocity distribution of pick-up ions. At the azimuthal configuration of the IMF, the generated Alfvén waves propagating parallel \((k > 0)\) and antiparallel \((k < 0)\) to the magnetic field have equal intensities to the first-order accuracy, ignoring terms of order \(v_A/V_{SW}\), where \(v_A\) is the Alfvén speed and \(V_{SW}\) is the solar wind speed. Within this approximation and in the case of pick-up protons constituting the most abundant population among pick-up ions, the total energy density injected per unit of time into waves propagating in both directions due to instability of the ring distribution of pick-up protons is (Huddleston and Jonstone, 1992)

\[
Q_T = \beta n_H m_p v_A V_{SW} / 2 ,
\] (1)

where \(\beta \) is the ionization frequency of and \(n_H\) is the number density of hydrogen atoms, and \(m_p\) is the proton mass.

Resonant interaction of pick-up ions with the Alfvén waves results in pitch-angle scattering of the particles and, therefore, in isotropization of their velocity distribution (in the solar wind rest frame). At the same time the spectral density of the self-generated waves evolves to some stationary distribution in wave number space. Estimates show that the isotropization time is substantially smaller than the characteristic times of such processes as convection with the solar wind speed, adiabatic cooling, energy diffusion and so on (Bogdan et al., 1991). Since just these latter processes are of most interest in the present paper, we consider isotropization as instantaneous and will consider only the isotropic velocity distribution of pick-up protons.

The initial speed of newly created pick-up protons (in the solar wind frame) are equal approximately to the local solar wind speed. Therefore, the injection of the energy into the turbulent wave field occurs around the wave number \(k_{ij} = \Omega_p / V_{SW}\), where \(\Omega_p\) is the proton gyrofrequency. The initial distribution of the wave intensity evolves in space and time due to convection, nonlinear interaction of waves, and damping connected with the wave-particle interaction. In the frame of a spherically-symmetric approximation with \(V_{SW} = \text{const}\) the closed system of equations for the velocity distribution function of pick-up protons \(f(t, r, v)\) and for the spectral density of Alfvén waves \(W_k(t, r, k)\) can be written as

\[
\frac{\partial f}{\partial t} + V_{SW} \frac{\partial f}{\partial r} = \frac{1}{v^2} \frac{\partial}{\partial v} \left( v^2 D_{vv} \frac{\partial f}{\partial v} \right) + 2 V_{SW} \frac{\partial f}{\partial v} + S(r, v) ,
\] (2)

\[
\frac{\partial W_k}{\partial t} + \frac{V_{SW}}{r^3} \frac{\partial}{\partial r} (r^3 W_k) = \frac{\partial}{\partial k} \left( D_{kk} \frac{\partial W_k}{\partial k} \right) + \gamma_k W_k + Q_k(r, k) .
\] (3)

In Eq. (2) \(D_{vv}(r, v)\) denotes the energy diffusion coefficient, \(v\) is the speed of particles in the solar wind rest frame, and \(S(r, v)\) is the source of pick-up protons connected with ionization of interstellar hydrogen atoms. The energy diffusion coefficient depends on the spectral density of Alfvénic turbulence, \(W_k\), and, in the frame of the quasi-linear approximation, is given by (see, e.g., Miller and Roberts, 1995)

\[
D_{vv} = 2 \frac{\pi^2 \beta}{m_p} \left( \frac{v_A}{c} \right)^2 \frac{1}{v} \int_{k_{\text{min}}}^{k_B} \left[ 1 - \left( \frac{k_{\text{min}}}{k} \right)^2 \right] \frac{W_k}{k} \, dk ,
\] (4)

where \(k_{\text{min}} = \Omega_p / v\) is the minimum resonant wave number for a pick-up proton of speed \(v\). The source term in Eq. (2) describing production of pick-up protons due to ionization of interstellar hydrogen atoms is

\[
S(r, v) = \frac{\beta n_H}{4 \pi V_{SW}^3} \delta (v - V_{SW}) .
\] (5)

Here Dirac’s \(\delta\)-function implies that the initial isotropic distribution of pick-up protons is shell-like. Along the upward direction the number density of interstellar hydrogen atoms is sufficiently well given by (Fahr, 1968, 1971)

\[
n_H(r) = n_{H\infty} \exp \left( - \frac{\beta E V_k}{V_{H\infty} r} \right) .
\] (6)

In Eq. (6) \(n_{H\infty}\) and \(V_{H\infty}\) are the number density and bulk velocity of interstellar hydrogen at a sufficiently large distance from the Sun (more precisely, near the solar wind termination shock).

Equation (3) describes evolution of the wave power in the expanding solar wind taking into account diffusion in wave number space connected with nonlinear wave-wave interactions and resonant damping due to wave-particle interactions. The diffusion theory of hydrodynamical turbulence has initially been developed by Leith (1967) and latter it was applied to solar wind turbulence by Zhou and Matthaeus (1990). We assume here that the spectral density is symmetric with respect to the wave number \(k = 0\), i.e. \(W_k(k) = W_k(-k)\). In the frame of the Kolmogorov phenomenology, the diffusion coefficient according to Zhou and Matthaeus (1990) has the following form:

\[
D_{kk}(r, k) = \frac{v_A |k|^{7/2}}{2} \sqrt{\frac{W_k}{B^2 / 4 \pi}} ,
\] (7)

where \(B\) is the magnetic field magnitude. We prefer to use the Kolmogorov form of the diffusion coefficient, since it describes solar wind turbulence more adequately than the Kraichnan phenomenology in the case when the normalized cross-helicity is small (Miller and Roberts, 1995). The second term in the right-hand side of Eq. (3) describes the damping of Alfvén waves due to resonant interaction with pick-up protons. The characteristic time-scale of this process is larger
than that of the evolution of the ring velocity distribution to the isotropic distribution and it results in stochastic acceleration of the pick-up protons. The associated damping rate \( \gamma_k \) has the following form (e.g. Bogdan et al., 1991; Miller and Roberts, 1995):

\[
\gamma_k(r, k) = -2\pi^2 e^2 \left( \frac{v_A}{c} \right)^2 \frac{1}{|k|} \int_{\min}^{\infty} 4\pi v f(r, v) \, dv ,
\]

(8)

In Eq. (8) \( v_{\min} = \Omega_p/|k| \) is the minimal speed of pick-up protons at which the resonant interaction between the particles and Alfvén waves with wave number \( k \) is possible. Finally, the source term \( Q_k \) is the energy input in the wave field from pick-up protons during the process of isotropization. In Chalov et al. (2004) it was assumed for the sake of simplicity that injection of the wave energy occurs exactly at \( k = \pm k_{\text{inj}} \) (\( k_{\text{inj}} = \Omega_p/V_{\text{SW}} \)). Here as well as in the recent paper by Chalov et al. (2005), the more realistic expression for the source term obtained by Huddleston and Jonstone (1992) taking into account a spread of the injected energy in wave number space is used. If as above we ignore terms of order \( v_A/V_{\text{SW}} \) this expression is

\[
Q_k(r, k) = \frac{1}{2} Q_T(r) \frac{\Omega_p}{V_{\text{SW}}} \left( 1 - \frac{k_{\text{inj}}}{|k|} \right) \frac{1}{k^2} .
\]

(9)

The value of \( Q_k \) is given by Eq. (1) and \( k_{\text{inj}} \leq |k| \leq \Omega_p/v_A \). Note that the dispersion effects which can be important at \( k \approx \Omega_p/v_A \) are not taken into account here. Note that the instability of the ring distribution of pick-up ions leads to the generation of left hand polarized waves. However, the expression (4) for the energy diffusion coefficient is valid in this case too as follows from the general quasi-linear theory taking into account arbitrary wave’s polarization states and propagation directions (Schlickeiser, 1989). It is only important for our analysis that the powers of waves propagating parallel and antiparallel to the magnetic field are equal.

In derivation of Eq. (9) the effects of wave-wave interactions are ignored, although these effects can influence the spectral properties of the generated waves. To treat these effects a more complicated model taking into account the pitch-angle evolution of the velocity distribution function of pick-up ions is needed. In the present paper we restrict our consideration to more simple model.

In addition to the local source (9), we take into account Alfvénic turbulence generated near the Sun and convected into the outer heliosphere. We include the effect of this energy input assuming that at \( r = r_E \) (e.g. Toptygin, 1983)

\[
W_k = \frac{\Gamma(\nu/2 + 1)}{2\pi^{3/2}\Gamma(\nu/2 - 1/2)} \frac{\langle \delta B_E^2 \rangle / k^2 / k_{E}^3}{\left[ 1 + (k / k_{E})^2 \right]^{\nu/2+1}} ,
\]

(10)

where \( \Gamma(x) \) is the gamma-function, \( \langle \delta B_E^2 \rangle \) is the mean-squared value of magnetic field fluctuations, \( k_{E} = 2\pi/k_{E} \) is the correlation length of turbulence, and \( \nu \) is the spectral index (= 5/3). Note that according to observations in the inner heliosphere, Alfvén waves in the fast solar wind are mainly outward propagating, while our consideration is restricted to the case when forward and backward propagating waves have equal intensities. However, nonlinear interactions of the waves leads to decrease the anisotropy at large distances.

The velocity distribution function and spectral energy density are normalized such that \( \int_{0}^{\infty} 4\pi v^2 f(r, v) \, dv \) is the number density of pick-up protons and \( \int_{-\infty}^{+\infty} W_k(r, k) \, dk \) is the total energy density of waves propagating parallel and antiparallel to the magnetic field.

### 3 Boundary conditions and basic parameters of the problem

Equations (2) and (3) are a closed system of integro-differential equations with respect to the isotropic velocity distribution function of pick-up protons \( f(r, v) \) and the spectral density of Alfvénic turbulence \( W_k(r, k) \). These equations (and, therefore, \( f \) and \( W_k \)) are coupled through the energy diffusion coefficient (4) and damping rate of Alfvénic fluctuations (8). To solve Eq. (2) the boundary condition \( f(r_E, v) = 0 \) is used. In addition we assume that \( f(r, v) \) is finite at all values of \( r \) and \( v \). As the boundary conditions for Eq. (3) we adopt that \( W_k(k = 0, r) = 0 \) and \( W_k(k = k_{\text{max}}, r) = 0 \). The largest value of the wave number, \( k_{\text{max}} \), has been varied in the range \((10^5 - 10^6) k_{\text{inj}} \) to make sure that the obtained solutions of Eq. (3) do not depend on \( k_{\text{max}} \) in a vicinity of the solar wind proton dissipation scale \( k_{\text{dis}} = \Omega_p/v_A \) (e.g. Leamon et al., 1998). The knowledge of \( W_k \) near \( k_{\text{dis}} \) will allow to estimate the diffusive flux of wave energy through \( k_{\text{dis}} \) which can be absorbed by solar wind protons.

At the inner boundary of the spatial calculation region \((r = r_E)\) the condition (10) is used. It means that we ignore a possible contribution of the self-generated wave energy in \( W_k \) at 1 AU. This approach is justified since the number density of hydrogen atoms is low in the inner heliosphere.

To solve Eq. (2), the method of stochastic differential equations is used. Unlike for Eq. (2), the finite-difference method, based on a splitting of physical processes, is used to solve Eq. (3). We emphasize that only steady state solutions of the equations are considered in the present paper. However, the full time-dependent version of Eq. (3) is used to find these steady solutions. For an extended description of numerical methods which we apply to solve Eqs. (2) and (3) see Chalov et al. (2004, 2005).

In the present calculations we consider two sets of the solar wind parameters (e.g. Toptygin, 1983; Fahr, 1989):

1. \( V_{\text{SW}} = 450 \text{ km/s}, \ n_{pE} = 6.4 \text{ cm}^{-3}, \ \beta_{E} = 0.63 \cdot 10^{-6} \) (ordinary solar wind in the ecliptic plane);
2. \( V_{\text{SW}} = 750 \text{ km/s}, \ n_{pE} = 3 \text{ cm}^{-3}, \ \beta_{E} = 0.41 \cdot 10^{-6} \) (high-speed solar wind).

In addition we adopt that \( B_E = 5.2 \cdot 10^{-5} \text{ G}, \ \lambda_{E} = 0.023 \text{ AU} \), \( \langle \delta B_E^2 \rangle / B_E^2 = 0.05 \). The parameters of interstellar neutral hydrogen are \( V_{H\infty} = 20 \text{ km s}^{-1} \) and \( n_{H\infty} = 0.1 \text{ cm}^{-3} \) (more precisely, these values are at the TS position).

Calculated differential fluxes of pick-up protons (in the solar wind rest frame) at the high-speed solar wind condi-
tions are presented in Fig. (1). Solid lines correspond to self-consistent solutions of Eqs. (2) and (3), while dashed lines show fluxes in the no-damping case when $\gamma_k = 0$ in Eq. (3), that is when absorption of the turbulent energy by pick-up protons is not taken into account. The main conclusion which can be made from these calculations coincides completely with the results obtained by Chalov et al. (2004, 2005). Namely, stochastic acceleration of pick-up protons by self-generated turbulence in the outer heliosphere can not produce extended suprathermal tails in their velocity distributions if absorption of the turbulent energy is taken into account. One can also see in this figure that the relative contribution of low-energy particles increases with distance from the Sun due to adiabatic cooling in the expanding solar wind.

Figure (2) shows normalized spectral wave energy densities, $W^*_k = \left( \frac{e^2 V_{SW}^2}{m_p c} \right)^2 W_k$, at different distances from the Sun as functions of the dimensionless wave number $k^* = k V_{SW}/\Omega_{pE}$. The solid lines are the self-consistent solutions of Eqs. (2) and (3). The solutions clearly show the formation of a second maximum (at large $k$) in the spectral distributions as the distance from the Sun increases. The existence of this second maximum is connected with the source term $Q_k$ in Eq. (3) describing generation of Alfvénic turbulence by pick-up protons during the process of their isotropization. The dashed lines show the spectral energy density in a non-self-consistent case, when generation of turbulent energy and stochastic acceleration of pick-up protons are taken into account, while the damping term in Eq. (3) is absent, that is $\gamma_k = 0$. It follows from the results presented in Fig. (2) that the main portion of the self-generated energy in the outer heliosphere is reabsorbed by pick-up protons themselves and only a small portion of this energy can be transferred to solar wind protons. As we show below, this small portion nevertheless is sufficiently large to heat the protons up to the observed temperature. The vertical dashed lines show the solar wind proton dissipation wave numbers at 10, 30, and 100 AU.

The self-consistent evolution of the pick-up ion velocity distribution and solar wind turbulence has been considered before by le Roux and Ptuskin (1998). The main difference between our and le Roux and Ptuskin (1998) models is the absence of pick-up ion generated wave energy in the latter model. As we show, however, contribution from this source prevails at medium and small wave numbers at large heliocentric distances and essentially modifies the spectral shape of wave turbulence.

4 Heating of the outer solar wind

In the short-wavelength part of turbulence the wave energy can be absorbed by solar wind protons, and this process results in heating of the solar wind in the outer heliosphere. The extent of this heating can be easily estimated on the base of the present results. The estimates are based on the calculation of the diffusive energy flux transferred through $k = \pm k_{dis}(r)$ (see Fig. (2)) to larger wave numbers. The equation for the solar proton temperature in the case when
In Eq. (11) corresponding to the adiabatically expanding flow and to the non-self-consistent solution for $W_k$ when $\gamma_k = 0$ in Eq. (3). The solid and dashed lines show the temperatures at different values of $k_{\text{dis}}$ in the case when $W_k$ is the self-consistent solution of Eqs. (2) and (3).

\[ V_{SW} = \text{const} \] and $n_p = (r_{PE}/r)^2 n_{PE}$ can then be written as (see e.g. Fahr and Chashei, 2002)

\[ \frac{dT}{dr} + \frac{4}{3} \frac{T}{r} = \frac{2}{3} \left( \frac{r}{r_{PE}} \right)^2 \frac{\Phi(r)}{k_B V_{SW} n_{PE}}. \]  

(11)

In Eq. (11) $k_B$ is the Boltzmann constant and the diffusion energy flux is

\[ \Phi(r) = -2D_{kk} \left( r, k_{\text{dis}} \right) \frac{\partial W_k \left( r, k_{\text{dis}} \right)}{\partial k}. \]  

(12)

Figures (3) and (4) show numerical solutions of Eq. (11) corresponding to the ecliptic wind, $V_{SW} = 450 \text{ km/s}$, $n_{PE} = 6.4 \text{ cm}^{-3}$, $\beta_{PE} = 0.63 \cdot 10^{-6}$, and to the high-speed wind, $V_{SW} = 750 \text{ km/s}$, $n_{PE} = 3 \text{ cm}^{-3}$, $\beta_{PE} = 0.41 \cdot 10^{-6}$. In both cases $T_E = 70000 \text{ K}$ is adopted. The dotted lines are the adiabatic behaviour of the temperature of solar wind protons with the distance ($\Phi = 0$ in Eq. (11)) and the spatial distribution of the temperature in the non-self-consistent case when $\gamma_k = 0$. In the latter case the temperature increases extremely as compared with the Voyager-2 measurements. The solid lines show the calculated spatial distributions of the temperature in the self-consistent case. In order to estimate the sensitivity of the temperature to the value of $k_{\text{dis}}$ we consider two additional cases: $k_{\text{dis}} = 2 \Omega_p/v_A$ and $k_{\text{dis}} = \Omega_p/2v_A$ (dashed lines). We make this estimation since the equation $k_{\text{dis}} = \Omega_p/v_A$ only approximately defines the onset of the dissipative range.

It is evident from Figs. (3) and (4) that in the case when reabsorption of the self-generated turbulent energy by pick-up protons is taken into account, the temperature of solar wind protons is essentially lower than it is in the frame of the non-self-consistent approach with $\gamma_k = 0$. The next interesting feature is the factor of 2–3 difference in the proton temperatures in the high-speed and low-speed flows. The difference is connected with the fact that the amount of energy transferred from pick-up ions to Alfvénic turbulence (the source term (1)) is larger in the high-speed wind. Besides that, the number density of protons in the high-speed flows is lower. This effect can explain the large variations of the solar wind temperature observed by Voyager 2 (see also Chashei et al., 2003).

We point out, however, that the temperature measured in the outer parts of the heliosphere ($\sim 10^4 \text{ K}$, see Richardson and Smith, 2003) is somewhat higher than in our calculations. Chalov et al. (2005) proposed several explanations of this difference. First of them is a strong heating of the solar wind observed inside 10 AU which is not connected with pick-up ions and more likely is due to interplanetary shock waves and dissipation of turbulent fluctuations in regions of an interaction between high- and low-speed flows. It follows, however, from Figs. (5) and (6) that the distributions of the solar wind temperature inside 10 AU have rather small influence on the temperature in the outer heliosphere. These figures show the same as Figs. (3) and (4), but the boundary condition for the temperature is posed at 10 AU. In accordance with the observations we adopted that $T \left( 10 \text{ AU} \right) = 20000 \text{ K}$.

The main reason of the relatively low theoretical temperature compared to observational values is connected with the argumentation concerning the diffusive flux (12) needed to describe the transfer of energy from Alfvénic turbulence to solar wind protons. This approximation is rather crude. In
reality the absorption of energy by solar protons results in a more steep spectral distribution near \( k_{\text{dis}} \) and, therefore, in an increase of the diffusive flux. Strictly speaking, in order to calculate the solar wind temperature, we must introduce in Eq. (3) an additional term describing absorption of the turbulent energy by protons and solve Eqs. (2) and (3) together with the energy equation for the protons.

5 Conclusions

We have assumed that small-scale Alfvénic turbulence in the outer heliosphere is generated mainly by pick-up protons during their isotropization. The main portion of this energy is then reabsorbed by the pick-up protons themselves as a result of the cyclotron resonant interaction between particles and waves, and only a small fraction can be transferred to solar wind protons. This amount of energy is, however, sufficiently large to heat the solar wind up to the measured temperatures. The heating is more pronounced in the high-speed solar wind.

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