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Modeling of Passive Electronic Circuits with Sensitivity Analysis Dedicated to the Sizing by Optimization

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Abstract. The paper deals with an approach that automates the computation of frequential characteristics of passive electronic circuit and their associated sensitivities according to all the components of the circuit. This method enables the designer to focus on the physical behavior of the circuit since the modeling and computing tasks are automatically performed without any computer science skills. It is useful to size circuits with many constraints by using optimization based on gradient algorithm.

Keywords: Passive filter, modeling, simulation, sensitivity analysis, optimization.

1 Introduction

Sizing a device consists in finding values leading to an expected behavior. This task can be seen as a classical constrained optimization problem with criteria \([C]\) depending on parameters \([P]\). Obviously, in a passive electronic circuit, the parameters are the frequency, the passive components (e.g. resistors, capacitors and inductors). Classical criteria in such a circuit are usually voltage ratio, impedances or current ratio valued for numerous frequencies. Several types of algorithms are able to deal with this optimization of such criteria. Among them, there are gradient based algorithms \([1]\). This kind of algorithms requires the derivative value of every criterion according to every parameter. This paper proposes an approach to compute these derivative values.

In the sizing area, some work has already been completed \([2]\). However, the originality of the paper is to show methods and tools easy to use in an industrial area.

The sizing step aims to constraint the values of such criteria according to the frequency, mainly for the resonance and antiresonance frequencies \([3][4]\), by respecting curve envelop (see Fig.1).
2 The Modeling Principle of a Passive Electronic Circuit

Before valuing the needed derivative values, the passive electronic circuit is modeled with a set of implicit analytical equations. In circuit theory and using Kirchhoff current law, a linear independent equation system is obtained. In our method, the chosen unknown variables are the voltage potentials of every node: so one node has to be selected as a reference. The leitmotiv of this modeling step is rather simple: for a circuit with \( n \) nodes, \( n \) independent expressions using the \( n \) voltage potentials at these \( n \) nodes have to be found.

2.1 Modeling Simple Circuit

Modeling circuit composed of one voltage source and passive components consists in splitting nodes in two sets (see Fig. 2).

The first set gathers nodes connected to the unique voltage source: there are two nodes: one used for the voltage source and the other for the circuit reference (i.e. the ground). The second set gathers all the other \( n-2 \) nodes of the circuit.

For the two nodes of the first set, two equations are provided. The first one refers to the definition of the voltage source, and the second expresses the location of the circuit reference.

For each node of this second set, an equation describing the current preservation (Kirchhoff’s law) is obtained. This equation only requires the type of passive components that are connected to the selected node. Then, the sum of all the currents through these components is cancelled. Equation of node 6 in Fig. 2 is

\[
0 = \frac{V0 - V6}{R1} + \frac{V2 - V6}{R2} + \frac{V9 - V6}{L1*p} + (V3 - V6)*Cl*p
\]

(1)

Where \( VX \) is the voltage potential of the node \( X \) and \( p \) is the Laplace variable.
Hence, n-2 equations associated with the n-2 nodes of the second set are written. Finally a system, with \( n \) equations and \( n \) unknown voltage potentials, is obtained.

### 2.2 Modeling Circuit with Multi Voltage Sources

The modeling of circuits made of several voltage sources has also been investigated. Once again, nodes are split in two sets. The first set gathers \( s \) nodes connected to the \( s \) voltage sources. The second set has all the others (\( n-s \)) nodes of the circuit.

\( s \) equations referring to the definition of a voltage source are associated with nodes from the first set.

Then, a node is removed from the second set to express the location of the circuit reference.

Like the modeling of the previous circuit, Kirchhoff's law is applied on each node of the second set which currently contains \( n-s-1 \) elements. As shown on Fig. 3, either passive components or voltage sources are connected to these nodes. These two kinds of elements require a modification to use Kirchhoff's law in the right way. On the first side, currents (I1, I2, I3 and I4) passing through passive component are already

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**Fig. 2.** A simple circuit

**Fig. 3.** Kirchhoff's law on a node gathering passive component and voltage source
known because they rely only on the passive component type. On the second side, current \(IX\) through a voltage source is not given by with the voltage source itself. So the solution consists in valuing the current coming from the other node of the voltage source.

At this end of the process \((s + 1 + (n - s - 1)) = n\) equations are written linking the \(n\) unknown voltage potential of the circuit.

### 3 The Equation System Solving and Its Associated Sensitivities

#### 3.1 Solving the Equation System Modeling the Circuit

In both cases, the equation system obtained after the modeling step is linear with the voltage potentials of the circuit (see Eq. (1)).

So, for a circuit using \(m\) parameters \([P] = \{P_j, 1 \leq j \leq m\}\) and \(n\) voltage potentials \([V] = \{V_i, 1 \leq i \leq n\}\), Every linear equation of the system can be written as followed:

\[
\alpha_{i,j}([P]) \times V_j + \ldots + \alpha_{n,i}([P]) \times V_n = \beta_i([P])
\]

Where \(\{\alpha_{i,j}\}\) and \(\{\beta_i\}\) depend only on the parameters. The linear equation system is:

\[
[A][V] = [\beta], \text{where } [A] = [\alpha_{i,j}, 1 \leq i \leq n, 1 \leq j \leq n]
\]

The paper proposes to value the solution of Eq.3, called \([V_{sol}]\) by numerical methods [5]. It represents the real voltage potential of the circuit.

#### 3.2 Calculation of the Sensitivities

The adjoint variable method [6] is classically used to extract the sensitivities of linear systems. However, another one relying on the theorem of implicit functions [7] is also available. This one has already been successfully used [2] before and is reused in this approach.

From every line of Eq. (2), an implicit function \(F_i([V],[P])\), is defined:

\[
F_i([V],[P]) = \alpha_{i,j}([P]) \times V_j + \ldots + \alpha_{n,i}([P]) \times V_n - \beta_i([P])
\]

By definition of \([V_{sol}]\), each function verifies:

\[
F_i([V_{sol}],[P]) = 0 \, \forall i \in [1, n]
\]

Thanks to the theorem of implicit functions [7], the sensitivities (gradients) are valued by Eq. (6):

\[
\left[ \frac{\partial V_{sol_j}}{\partial P_k} \right] = \left( \left[ \frac{\partial F_i}{\partial V_j} \right]_{V_j = V_{sol_j}} \right)^{-1} \left[ \frac{\partial F_i}{\partial P_k} \right]
\]

Moreover, the derivative of every function \(F_i\), according to a voltage \(V_j\) is \(\alpha_{j,i}\). So, by using Eq. (3), Eq. (6) becomes:
Note that \([A]^{-1}\) is required to solve both equations Eq. (3) and Eq. (7), what enables to get the voltage values and their sensitivities.

4 Calculation of Sizing Criteria of the Circuit and Their Sensitivities

Once the linear equation system is obtained, the voltage potentials at every node, \([V_{sol}] = \{V_{sol_i}, 1 \leq i \leq n\}\), and the associated sensitivities according to each parameter \(P_k\):

\[
\left\{ \frac{\partial V_{sol_i}}{\partial P_k} \right\}_{1 \leq i \leq n, 1 \leq k \leq m}
\]

are known.

Therefore, sizing criteria and their associated sensitivities according to the parameters \([P]\) have to be valued.

4.1 Calculation of a Sizing Criterion

A criterion is usually the modulus of such a mathematical expression for several values of frequencies. Whatever, the mathematical expression complexity is, the computation is always simple because each value is known since mathematical library dealing with complex number is used [5].

Eq. (8) is an example of a sizing criterion. In such an equation, \(V\) prefixes voltage potentials, \(R\) and \(C\) are circuit parameters, \(p\) is the Laplace variable:

\[
RL = \frac{V_{12}}{V_{11} - V_{12}} \frac{R}{1 + R \cdot C \cdot p} + \frac{V_{19}}{V_{19} - V_{20}} \frac{L \cdot p}{1 + R \cdot C \cdot p}
\]

To get \(RL\), \(p\) is substituted by \(2 \cdot j \cdot \pi \cdot \text{freq}\) where \(\text{freq}\) is a specific frequency and \(j\) is the imaginary complex.

4.2 Calculation of the Sensitivities of a Sizing Criterion

To obtain the sensitivity of a sizing criterion, mathematical expressions like Eq. (9) have to be valued:

\[
\frac{\partial RL}{\partial C}([V_{sol_{11}}, V_{sol_{12}}, V_{sol_{19}}, V_{sol_{20}}], [R, C, L])
\]

Generally, the problem is: how to express \(S(p_j) = \frac{\partial g([V_{sol}], [P])}{\partial P_j}\), where \(g\) is a function used to calculate a criterion.
The derivative of a complex variable modulus according to its real part is:

\[
\frac{\partial |z|}{\partial \text{Re}(z)} = \frac{\partial \sqrt{\text{Re}(z)^2 + \text{Im}(z)^2}}{\partial \text{Re}(z)} = \frac{2\text{Re}(z)}{2\sqrt{\text{Re}(z)^2 + \text{Im}(z)^2}} = \frac{\text{Re}(z)}{|z|}
\]  

(10)

An equivalent expression gives the derivative according to its imaginary part. Hence, another expression for \(S(P_j)\) is obtained:

\[
S(P_j) = \frac{1}{g([V_{\text{sol}}],[P])} \left( \text{Re}(X) * \frac{\partial \text{Re}(X)}{\partial P_j} + \text{Im}(X) * \frac{\partial \text{Im}(X)}{\partial P_j} \right)
\]  

(11)

Where \(X\) stands for \(g([V_{\text{sol}}],[P])\). Because \([P]\) is a set of real variables and thanks to simple properties on complex number, Eq. (11) becomes:

\[
S(P_j) = \frac{1}{g([V_{\text{sol}}],[P])} \text{Re}\left( g([V_{\text{sol}}],[P]) * \frac{\partial g([V_{\text{sol}}],[P])}{\partial P_j} \right)
\]  

(12)

Where \(\overline{X}\) is the conjugate of the complex number \(X\).

5 Software Implementation

Fig. 4 shows the workflow of the process that performs the modeling and the generation of executable code that computes each criterion and its sensitivity.

5.1 The Modeling Step (blocks 1 and 2)

First, a netlist from Pspice is provided. After an analysis of this netlist, a representation tree is built thanks to the XML technology [8]. Two successive transformations (thanks to XSLT technology [9]) lead to an analytical equation.
system describing the studied circuit as presented in part 2 (block 1 - Fig. 4). The modeling step relies on a component library (block 2 - Fig. 4) describing the behavior of each passive component (resistor, capacitor, inductor).

5.2 Automated Generation of Criterion Computation (blocks 3 to 6)

The computation of each criterion relies on the analytical equation system, in the frequency domain, provided by the modeling step. First of all, thanks to a Graphical Interface User (GUI), the designer specifies all the sizing criteria of its circuit (block 3 - Fig. 4), e.g. as Eq. (8). The symbolic expressions given by the designer are checked 'on-line' meaning that tests check mathematical syntax and names of variables and parameters that are used ('error' window – left part of Fig. 4). Then, the code generation (block 4 - Fig. 4) begins. A "Core" file solves the system and gives the voltage potentials and their sensitivities (as described in part 3). This code uses a Java library (block 5 - Fig. 4) dedicated to matrix computations with complex variables. After, another executable code (block 6 - Fig. 4), named "External" is generated to solve sizing criteria given by the designer. "External" code uses methodology shown in part 4 to perform expected computations. This final code can be used for simulation and optimization, as in [3][4].

6 Example

This workflow has been used to model an industrial device: an Asymmetrical Digital Subscriber Line (ADSL) splitter. This device allows simultaneous use of high speed internet and phones services. Basically, it is an electronic device composed of resistors, capacitors and inductors that must fulfill dedicated telecommunication specifications.

Defining all criteria for the sizing of the ADSL splitter required ten different circuits (a topology of such a circuit is shown on Fig. 5).
Once all criteria are defined by the designer (nearly a hundred criteria using fifty parameters), the presented workflow is performed within few minutes on a current personal computer.

Once this workflow is completed, software from the CADES framework [10], dedicated to the design and the sizing of engineering systems are used (Fig. 6). Among them, Component Generator creates a dedicated software component and Component Optimizer proposes methods for the sizing of the splitter.

![Fig. 6. ElectronicalCircuitModeler and CADES Framework](image)

The Component Generator builds the software component computing all criteria needed for the sizing of the splitter. This software component is plugged in the Component Optimizer to size this device. Several kinds of strategies are available to reach an efficient sizing. For example there are a deterministic algorithm VF13 (available at www.cse.clrc.ac.uk/nag/hsl/hsl.shtml) and other none deterministic one based on genetic approach [10].

### 7 Conclusion

Other approaches have been proposed [3] [4], but the method proposed in the paper, allows to treat greater applications than [3], and it is more generic than [4]. Moreover, it is more accurate for the sizing step than simulation tools like Saber, Pspice, Simplorer (where finite differences are used to value the gradients), because the gradients are valued from analytical expressions. At the present time, the generated models are used to size circuits with some hundreds constraints by using optimization based on gradient algorithm.

### References


