# SOLVING OF WAITING LINES MODELS IN THE AIRPORT USING QUEUING THEORY MODEL AND LINEAR PROGRAMMING THE PRACTICE CASE: A.I.M.H.B

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# Solving of waiting lines models in the airport using queuing theory model and linear programming The practice case : A.I.M.H.B

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**Abstract –** Waiting lines and service systems are important parts of the business world .In this article we describe several common queuing situations and present mathematical models for analysing waiting lines following certain assumptions. Those assumptions are that (1) arrivals come from an infinite or very large population, (2) arrivals are Poisson distributed,(3) arrivals are treated on a FIFO basis and do not balk or renege,(4) service times follow the negative exponential distribution or are constant, and (5) the average service rate is faster than the average arrival rate.

The model illustrated in this airport for passengers on a level with reservation is the multiple-channel queuing model with Poisson Arrival and Exponential Service Times (M/M/S).After a series of operating characteristics are computed, total expected costs are studied, total costs is the sum of the cost of providing service plus the cost of waiting time.

We also study the Tunisian aerial transport, while analysing the situation of the latter, we have privately chose to develop the waiting line's problem on a level with landing on a runway. Among the different cause of this waiting, we were limited in which that in relation to the weakness on a level with the programming of flights by the civil aviation direction, as far as that goes we have been enumerated the main studies which have been consecrated in this reason. Finally, one way to resolve the waiting problem, a good linear programming is taken into consideration. On the basis of this analysis, we have chosen to apply the developed model in to real case in order to value the contribution which is able to generate the settling in such models in Tunisian airport.

**Keywords:** Service; FIFO; M/M/s; Poisson distribution; Queue ; Service cost; Unlimited or Infinite Population ; Utilisation Factor; Waiting cost ; Waiting Time, Landing, agents , the approach control, estimated time of arrival , linear programming ,optimisation; waiting in air, Congestion.

## HISTORY:

Queuing theory had its beginning in the research work of a Danish engineer named A. K. .Erlang. In 1909 Erlang experimented with fluctuating demand in telephone traffic. Eight years later he published a report addressing the delays in automatic dialling equipment. At the end of World War II, Erlang's early work was extended to more general problems and to business applications of waiting lines.

## 1-Introduction:

The study of waiting lines, called queuing theory, is one of the oldest and most widely used quantitative analysis techniques. Waiting lines are an everyday occurrence, affecting people shopping for groceries buying gasoline, making a bank deposit, or waiting on the telephone for the first available airline reservationists to answer. Queues, another term for waiting lines, may also take the form of machines waiting to be repaired, trucks in line to be unloaded, or airplanes lined up on a runway waiting for permission to take off. The three basic components of a queuing process are arrivals, service facilities, and the actual waiting line.

The linear programming (LP) models-seem to be particularly suitable for the queuing theory because the solution time required to solve some of that may be excessive even on the fastest computer. We demonstrate how each the formulation of (LP) can be used on the following example of waiting lines of the aeroplanes. The numerous applications and the rapidly growing interest in the solution of (LP) take this fact into account.

We briefly study characteristics and exploit advantages and disadvantages of some of the most relevant queuing theory models in section 2 and 3 .In section 2, we discuss how analytical models of waiting lines can help managers evaluate the cost and effectiveness of service systems, we begin with a look at waiting line costs and then describe the characteristics of waiting lines and the underlying mathematical assumptions used to develop queuing models ; we also provide the equation needed to compute the operating characteristics of a service system and show in our case how they are used .Section 3, presents the (LP) and discussions the empirical results of the specific implementation. Later in these sections, you will see how to save computational time by applying queuing tables and by running waiting line computer programs .Finally, we conclude the paper with some remarks and future research directions.

### 2- Application The Basic Queuing System Configurations And Discussion The Waiting line Costs On A Level With Passengers Checking-in: Case Study Tunisair Company at A.I.M.H.B (At Rush Hour)

#### 2- 1 CHARACTERISTICS OF A QUEUING SYSTEM:

We take a look at the three part of a queuing system (1) the arrival or inputs to the system (sometimes referred to as the *calling population*),(2) the queue* or the waiting line itself, and (3) the service facility. These three components have certain characteristics that must be examined before mathematical queuing models can be developed.

**Arrival Characteristics**

The input source that generates arrivals or passengers for the service system has three major characteristics. It is important to consider the size of the calling population, the pattern of arrivals at the queuing system, and the *behavior* of the arrivals.

***Size of the Calling Population*** **:** Population sizes are considered to be either *unlimited* (essentially *infinite) or* limited *(finite).*When the number of passengers or arrivals on hand at any given moment is just a small portion of potential arrivals, the calling population is considered unlimited. For practical purpose, in our examples the unlimited passengers arriving to check-in for traveller at airport (an independent relationship between the length of the queue and the arrival rate moreover the arrival rate of passengers lower). Most queuing models assume such an infinite calling population. When this is not the case, modelling becomes much more complex. An example of a finite population is a shop with only eight machines that might break down and require service.

***Pattern of arrivals at the System*****:** Passengers either arrive at a service facility according to some known schedule (in our case, 13.361 regular passengers every 95 minutes or 0.140

Passenger every one minute) or else they arrive ***randomly.*** Arrivals are considered random when they are independent of one another and their occurrence cannot be predicted exactly. Frequently in queuing problems, the number of arrivals per unit of time can be estimated by a probability distribution known as the *Poisson distribution* **.**For any given arrival rate, such as two passengers per hour, or four airplanes per minute , a discrete Poisson distribution can be established by using the formula

$$P(n\,;t)=\frac{(\lambda t)^n}{n!}e^{-\lambda t} \qquad \text{for } n=0,1,2,3,4,\ldots$$

Where

* The word queue is pronounced like the letter Q, that is, "kew".

P(n ;t) = probability of n arrivals
$\lambda$ = average arrival rate
e = 2, 7183
n = number of arrivals per unit of time

***Behavior of the Arrival*** **:** Most queuing models assume that an arriving passenger is a patient traveller .Patient customer is people or machines that wait in the queue until they are served and do not switch between lines. Unfortunately, life and quantitative analysis are complicated by the fact that people have been known to balk or renege .*Balking* refers to passengers who refuse to join the waiting lines because it is to suit their needs or interests. *Reneging* passengers are those who enter the queue but then become impatient and leave the need for queuing theory and waiting line analysis. How many times have you seen a shopper with a basket full of groceries, including perishables such as milk, frozen food, or meats, simply abandon the shopping cart before checking out because the line was too long? This expensive occurrence for the store makes managers acutely aware of the importance of service-level decisions.

**Waiting Line Characteristics**

The waiting line itself is the second component of a queuing system. The length of a line can be either *limited* or *unlimited*. A queue is limited when it cannot, by law of physical restrictions, increase to an infinite length. Analytic queuing models are treated in this article under an assumption of *unlimited* queue length. A queue is unlimited when its size is unrestricted, as in the case of the tollbooth serving arriving automobiles.

A second waiting line characteristic deals with *queue discipline*. This refers to the rule by which passengers in the line are to receive service. Most systems use a queue discipline known as the *first-in*, *first-out rule* (FIFO)¤.This is obviously not appropriate in all service systems, especially those dealing with emergencies.
In most large companies, when computer-produced pay checks are due out on a specific date, the payroll program has highest priority over other runs.

**Service Facility Characteristics**

The third part of any queuing system is the service facility. It is important to examine two basic properties :(1)the configuration of the service system and (2)the pattern of service times.
***Basic Queuing System Configurations*** **:** Service systems are usually classified in terms of their number of channels, or number of servers, and number of phases, or number of service stops, that must be made.

**Identifying Models Using Kendall Notation**

D.G. Kendall developed a notation that has been widely accepted for specifying the pattern of arrivals, the service time distribution, and the number of channels in a queuing model. This notation is often seen in software for queuing model. The basic three-symbol Kendall notation is in the form: arrival distribution/service time distribution/number of service channels open.

Where specific letters are used to represent probability distributions. An abridged version of this convention is based on the format A/B/C/D/E/F. These letters represent the following system characteristics:

**A** = represents the interarrival-time distribution.
**B** = represents the service-time distribution.

[Common symbols for **A** and **B** include **M** (exponential), **D** (constant or deterministic), **Ek** ( Erlang of order k), and **G**(arbitrary or general)]

**C** = represents the number of parallel servers.
**D** = represents the queue discipline.
**E** = represents the system capacity.
**F** = represents the size of the population.

## 2-2 SINGLE-CHANNEL QUEUING MODEL WITH POISSON ARRIVALS AND EXPONENTIAL SERVICE TIMES (M/M/1) :

We present an analytical approach to determine important measures of performance in a typical service system. After these numeric measures have been computed, it will be possible to add in cost data and begin to make decisions that balance desirable service levels with waiting line service costs.

***Assumptions of the Model***

The single-channel, single-phase model considered here is one of the most widely used and simplest queuing models. It involves assuming that seven conditions exist:

1. Arrivals are served on a FIFO basis.

¤ The term FIFS (*first in, first served*) is often used in place of FIFO .Another discipline, LIFS (*last in, first served*),is common when material is stacked or piled and the items on top are used first.

2. Arrivals are described by a Poisson probability distribution and come from an infinite or very large population.
3. Service times also vary from one passenger to the next and are independent of one another, but their average rate is known.
4. Service times occur according to the negative exponential probability distribution.
5. The average service rate is greater than the average arrival rate.

When these seven conditions are met, we can develop a series of equations that define the queue's *operating characteristics*.

***Queuing Equations***

$\lambda =$ mean number of arrivals per time period (for example, per hour)

$\mu$ = mean number of people or items served per time period

When determining the arrival rate ( $\lambda$ ) and the service rate ( $\mu$ ), the same time period must be Used. For example, if the $\lambda$ is the average number of arrivals per hour, then $\mu$ must indicate the average number that could be served per hour.

**The queuing equations follow:**

**1**. The average number of passengers or units in the system, Ls, that is, the number in line plus the number being served:

$$Ls = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$

**2**. The average time a passenger spends in the system, Ws, that is, the time spent in line plus the time spent being served:

$$Ws = \boxed{\frac{1}{\mu-\lambda}}$$

**3**. The average number of passengers in the queue, Lq:

$$Lq = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

**4**. The average time a passenger spends waiting in the queue, Wq:

$$Wq = \boxed{\frac{\lambda}{\mu(\mu-\lambda)}}$$

**5**. The utilisation factor for the system, $\rho$ , that is, the probability that the service facility is being used:

$$\rho = \frac{\lambda}{\mu}$$

**6**. The present idle time, Po, that is, the probability that no one is in the system:

$$Po = 1 - \frac{\lambda}{\mu}$$

**7**. The probability that the number of passengers in the system is greater than k, P n >k :

$$P\ n{>}k = \left(\frac{\lambda}{\mu}\right)^{k+1}$$

## 2-3 MULTIPLE-CHANNEL QUEUING MODEL WITH POISSON ARRIVALS AND EXPONENTIAL SERVICE TIMES (M/M/S):

The next logical step is to look at a multiple-channel queuing system, in which two or more servers or channels are available to handle arriving passengers. Let us still assume that travellers awaiting service form one single line and then proceed to the first available server. Each of these channels has an independent and identical exponential service time distribution with mean $1/\mu$ . The arrival process is Poisson with rate $\lambda$ . Arrivals will join a single queue and enter the first available service channel.

The multiple-channel system presented here again assumes that arrivals follow a Poisson probability distribution and that service times are distributed exponentially. Service is first come, first served, and all servers are assumed to perform at the same rate. Other assumptions listed earlier for the single-channel model apply as well.

### *Equations for the Multichannel queuing Model:*

If we let

S = number of channels open,

$\lambda$ = average arrival rate, and

$\mu$ = average service rate at each channel.

The following formulas may be used in the waiting line analysis:

**1.** The probability that there are zero passengers or units in the system:

$$Po = \frac{1}{\sum_{n=0}^{S-1}\frac{(\rho)^n}{n!} + \frac{(\rho)^S}{S!(1-\frac{\rho}{S})}}$$

**2.** The average number of passengers or units in the system:

$$L_s = \left[\frac{(\rho)^S}{\mu S! S(1-\frac{\rho}{S})^2} P_0 + \frac{1}{\mu}\right] \lambda$$

**3.** The average time a unit spends in the waiting line or being serviced(namely, in the system):

$$\boxed{Ws = \frac{(\rho)^S}{\mu S! S(1-\frac{\rho}{S})^2} P_0 + \frac{1}{\mu}} \quad \text{Or} \quad W_s = L_s / \lambda$$

**4.** The average number of passengers or units in line waiting for service:

$$L_q = \frac{(\rho)^{S+1}}{S! S(1-\frac{\rho}{S})^2} P_0 \quad \text{Or} \quad L_q = L_s - \rho$$

**5.** The average time a passenger or unit spends in the queue waiting for service:

$$W_q = \frac{(\rho)^S}{\mu S! S(1-\frac{\rho}{S})^2} P_0 \quad \text{Or} \quad W_q = W_s - \frac{1}{\mu}$$

**6.** Utilisation rate:

$$\rho = \frac{\lambda}{s\mu}$$

These equations are obviously more complex than the ones used in the single-channel model, yet they are used in exactly the same fashion and provide the same type of information as did the simpler model.

#### *SOME GENERAL OPERATING CHARACTERISTIC RELATIONSHIP :*

Certain relationships exist among specific operating characteristics for any queuing system in a *steady state.* A steady state condition exists when a queuing system is in its normal stabilized operating condition, usually after an initial or *transient state* that may occur. John D.C. Little is credited with the first of these relationships, and hence they are called *little*'s *Flow Equations.*

$$W_q = L_q / \lambda \quad (\text{or} \quad L_q = \lambda W_q)$$

$$W_s = L_s / \lambda \quad (\text{or} \quad L_s = \lambda W_s)$$

A third condition that must always be met is:

Average time in system = average time in queue + average time receiving service

$$W_s = W_q + \frac{1}{\mu}$$

The advantage of these formulas is that once one of these four characteristics is known, the other characteristics can easily be found. This is important because for certain queuing models, one of these may be much easier to determine than the other. There are applicable to all of the queuing systems except the finite population model.

## 2-4 WAITING LINE COSTS:

One of the goals of queuing analysis is finding the best level of service for an organization. When *TUNISAIR* Company does have control, its objective is usually to find a happy medium between two extremes. On the one hand, a firm can retain a large staff and provide many service facilities. This, however, can become expensive.

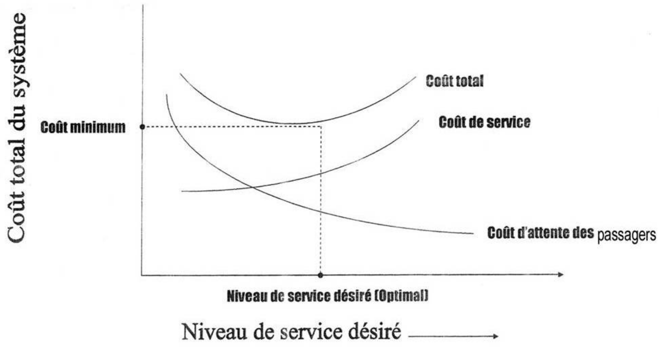
The other extreme is to have the *minimum* possible number of checkout lines, gas pumps, or teller windows open. This keeps the service cost down but may result in customer dissatisfaction. How many times would you return to a large discount department store that had only one cash register open during the day you shop? As the average length of the queue increases and poor service results, customers and goodwill may be lost.

Managers must deal with the trade-off between the cost of providing good service and the cost of customer waiting time. The latter may be hard to quantify.

One means of evaluating a service facility is thus to look at a *total expected cost;* is the sum of expected service costs plus expected waiting costs. As service improves in speed, however, the cost of time spent waiting in lines decreases. This waiting cost may reflect lost productivity of workers while their tools or machines are awaiting repairs or may simply be an estimate of the costs of customers lost because of poor service and long queues.

The objective is to minimize total expected costs. This analysis is summarized in table **II**. To minimize the sum of service costs and waiting costs, the company makes the decision to employ ten travelling agents for registration at rush hours.

**FIGURE:**
***Queuing Costs and Service Levels***




## 2-5 NUMERICAL EXAMPLE AND DISCUSSION:

As an illustration, let's look at the case of *TUNISAIR* company at rush hours (To 20/09/03 from 07/10/03) all flights are charters flights.

***Characteristics of Queuing Systems:***

**1**-Arrivals are served on a FIFO basis.

**2**- The Calling Population size is considered infinite.

**3**- The system is a multiple-server operating in parallel (S=10).

So, we can test if the arrivals are described by a Poisson probability distribution and if the service times occur according to the negative exponential probability distribution (as well if the service times are a Poisson probability distribution).

***The estimated Arrivals' Characteristic at the System:***

**(a)** Average number of travellers arriving at reservation counter per 95 minutes :

$\overline{X} = 13.361$

**(b)** The variance of the distribution from the passengers number is :

V(X) = 26.453

**(c)** The Standard Deviation is :

$\sigma_x = \sqrt{26.453} = 5.1430$

***Arrival's validation with Karl-Pearson's Test:***

$$\chi_c^2 = \frac{\sum_{i=1}^{n} (F_{ith} - F_i)^2}{\sum_{i=1}^{n} F_{ith}}$$

If ( $\chi_c^2 < \chi_{(\alpha)}^2$ ): We accept that the arrivals distribution is Poisson (**Ho).**

If ( $\chi_c^2 > \chi_{(\alpha)}^2$ ): We reject the hypothesis **(Ho).**

For our case, we have:

$\chi_c^2 = 2.2641$

**d** = 5- 2 = 3

The decision's rule:

For all α< 66, 28%, we assert that the distribution of arrivals at a single counter follows a Poisson distribution of expected value λi = 13.361

***The estimated Characteristic of Service System:***

**(a)** Average number of passengers checked in / 95 minutes:

$\overline{X} = 17.083$

**(b)** The variance of the distribution from the service number is :

V(X) = 43.032

**(c)** The Standard Deviation is:

$\sigma_x = \sqrt{43.032}$ = 6.56.

**(Ho):** The observed distribution of service times is a Poisson probability

**(H1):** It is not be a question of a Poisson probability distribution for the service times.

***Service Times Validation with Karl-Pearson's Test:***

$\chi_c^2 = 2.9902$

**d** =5 – 1 – 1 = 3

The decision's rule:

We accept the hypothesis Ho for all α < 56.032%.

So; for all α < 56.032% the service times are described by a Poisson probability distribution of average μ=17.083 called the mean number of passengers served per time period (Service rate).

***The estimated Characteristic of the System:***

Average number of travellers arriving at the system λ = 133.61 = (13.361x10) ;

The mean number of passengers served per 95 minutes μ = 17.083;

The utilisation factor for the system ρ=λ/μ = 7.821;

The traffic intensity for the system $\psi = \rho / S$ = 7.821/10 = 0.7821 < 1 ;

Probability that there are zero travellers in the system Po = 0.00034;

The average number of passengers in line waiting for service (check in) Lq = 1.3298;

The average number of travellers in the system $L_s$ = 9.1508;

The average time a passenger spends in the queue waiting for service $W_q$ = 0.01;

The average time a traveller spends in the waiting line or being serviced (namely, in the system): $W_s$ = 0.0685;

The number of unoccupied stations $\overline{\rho}$ = 10(1-0.7821) = 2.179 (nearly two counters are inactive) ;

The probability that the passenger wait before to be served P(>o) ≈ 36.82% ;

The inactivity factor per agent $K_i$ = 21.8%.

*The probability that the system is empty (Po) increases;
*The average length of the queue (Lq) and in the system (Ls) decline;
*The average waiting time in the queue (Wq) and the average time in the system (Ws) decrease also;

## *Our problematical:*

What is the optimal number of station's *Tunisair* Company to install, in order to minimise the sum of service costs and waiting costs? We let the arrival rate ($\lambda$) and that of service ($\mu$) are fixed.

***The objective Function:***

Min {E (TC) = E (SC) + E (WC)}

*Where*

**TC**: Total Expected Cost;
**SC**: Cost of Providing Service;
**WC**: Cost of Waiting Time.

**E (TC) = Co + S.Cs + Ca.Ls**

Co: The fixed cost of operation system per unit of time;
Cs: The marginal cost of a registration agent per unit of time (or Total hourly service cost);
Ca: The cost of waiting based on time in the queue and in the system.

* **Note** : For this problem , an analytical solution does not exist , and it be necessary to solve the problem by groping especially that to be question of convex function in other words whether her curve is in form of U , therefore it just takes to estimate E(TC) for the values of S growing up until the cost cease to decrease .

***Cost of Waiting Time (WC :)***

*One unit of waiting time of a traveller was estimated on the basic wage of the latter.
*The mean waiting time cost per passenger is: Ca = Σ WiCi .

*Where*

Ci: Hour-wage of a passenger belong to the socio-professional category i.
Wi: Weight of the category i is extracted from the total of the sample.

***Empirical Result:***

Size of the sample: N = 1000 passengers after a questionnaire accomplished by *Tunisair* company .As mentioned inside of this sample , *Tunisair* was found among these categories : students, children ,handicapped person and the unemployed.
Our effective sample is of size N =897 travellers .
This analysis is summarized in Table **I** :

| **socio-professional category** | **Number of passengers** | **Theoretical Frequency (in %)** |
|---|---|---|
| [700 ; 1000[ | 332 | 37 |
| [1000 ; 1300[ | 251 | 28 |
| [1300 ; 1600[ | 170 | 19 |
| [1600 ; 1900[ | 90 | 10 |
| [1900 ; et + [ | 54 | 6 |
| Total | 897 | 100 |

**Remark:** We have cancel the socio-professional category [0; 700 [because the lack into gain for this class is low.

$$\begin{cases} \text{Ho: The old distribution (observed by the airport's responsible) is still valid .} \\ H_1\text{: The old distribution is not valid.} \end{cases}$$

$\chi^2_C$ =9.235

$\chi^2_{(\alpha\, ;\, k\text{-}1)} = \chi^2_{(0.05\, ;\, 4)} = 9.488$

$$☞\ \chi^2_C < \chi^2_{(\alpha;k-1)}$$

Therefore, we have accepted the hypothesis Ho.

**$C_1$** = 850 / 192 = 4.4270 DT/H ; **$W_1$** = 30% = 0.3
**$C_2$** = 1150/192 = 5.9896 DT /H ; **$W_2$** = 21% = 0.21
**$C_3$** = 1450/192 = 7.5520 DT/H ; **$W_3$** = 25% = 0.25
**$C_4$** = 1750/192 = 9.1146 DT/H ; **$W_4$** = 14% = 0.14sw
**$C_5$** = 2050/192 = 10.6770 DT/H ; W5 = 10% = 0.10

In this way, $C_a$ = 6.818 DT/ hour: on average, a passenger lost 6.818 DT per hour of time spent waiting in line.

**Total waiting cost:**

E (WC) = Ca..Ls = 62.3916 DT/H

The passengers do lose (on average) 62.3916 DT/H per hour of time spent waiting in the system.

***Analysis The Cost of Providing Service***

The wage of travelling agent per unit of time :
$S_h$ = 3.41 DT/H
The wage of packer man per unit of time :
$S_{Eh}$ = 2.16 DT/H
The amortized of electronic weighing machine :
$A_{Bh}$ = 0.123 DT/H
The amortized of reservation system (**Gaetan)** :
$A_{Gh}$ = 1.462 DT / H
The cost of occupied surface:
$L_h$ = 0.146 x 1.70 = 0.248 DT/H.

**Cost analysis for service is then: $C_s = S_h + SE_h + A_{Bh} + A_{Gh} + L_h + F$**

*Where:*

**F**: It is the imputation factor of indirect charges to total cost (F= 5% from the indirect charge by hypothesis)

**Cs = 7.773 DT/H**

Total hourly service cost : E(SC)= S.Cs
E (SC) = 10 x 7.773 = 77.73 DT

On average, one hour of service costs 77.73 DT by the company.

***Total Expected Cost***

**E(TC) = E(WC) + E(SC)**
= 140.1216 DT/H

One hour of time spent waiting in system costs on average 140.1216 DT.
55.5% of this cost is endured by *Tunisair* company, whereas the rest (44.5 %) by the passengers.

| **Number of travelling agents S** | **Total Hourly Service Cost E(SC)=S.CS** | **The Average Number of Passengers In The System Ls =Lq +ρ** | **Total Hourly Service Cost E(WC) =Ca Ls** | **Total Expected E(TC)=E(SC)+** |
|---|---|---|---|---|
| 8 | 62.184 | 48.39 | 329.923 | 392.107 |
| 9 | 69.957 | 11.79 | 80.3842 | 150.3412 |
| **10** | **77.73** | **9.1508** | **62.3916** | **140.1216** |
| 11 | 85.503 | 8.3495 | 56.926 | 142.429 |
| 12 | 93.276 | 8.0502 | 54.886 | 148.162 |
| 13 | 101.049 | 7.9198 | 53.997 | 155.046 |
| 14 | 108.822 | 7.863 | 53.609 | 162.431 |
| 15 | 116.595 | 7.838 | 53.439 | 170.034 |

| | | | | |
|---|---|---|---|---|
| 16 | • 124.368 | • 7.828 | • 53.371 | • 177.739 |
| 17 | • 132.141 | • 7.823 | • 53.337 | • 185.478 |
| 18 | • 139.914 | • 7.822 | • 53.330 | • 193.244 |
| 19 | • 147.687 | • 7.8213 | • 53.325 | • 201.012 |
| 20 | • 155.46 | • 7.8211 | • 53.324 | • 208.784 |

➥ **S* = 10**

**Table II: The Optimal Configurations**

## 2-6 ENHANCING THE QUEUING ENVIRONMENT:

Although reducing the waiting time is an important factor in reducing the waiting time cost of a queuing system, *Tunisair* Company might find other ways to reduce this cost. The total waiting time cost is based on the total amount of time spent waiting and the cost of waiting. Reducing either of these will reduce the overall cost of waiting. Enhancing the queuing environment by making the wait less unpleasant may reduce (WC) as travellers will not be as upset by having to wait .There are magazines in the waiting room of the airport A.I.M.H.B for passengers to read while waiting. Music is often played while telephone callers are placed on hold. At major amusement air terminal there are video screens and televisions in some of the queue lines to make the wait more interesting .For some of these, the waiting line is so entertaining that it is almost an attraction itself.

All of these things are designed to keep the passenger busy and to enhance the conditions surrounding the waiting so that it appears that time is passing more quickly than it actually is. Consequently, the cost of waiting becomes lower and the total cost of the queuing system is reduced. Sometimes, reducing the total cost in this way in this way is easier than reducing the total cost by lowering Ws or Wq. In this case of *Monastir'* s airport , *Tunisair* might consider putting a television in the waiting room and remodelling the air terminal so passengers feel more comfortable while waiting for their aircraft to be travelled.

## 3-Analysis The Waiting Lines Theory In A.I.M.H.B For The movement's Traffic On A Level With Landing ( In Arrival At Rush Hour) :

The purpose of this section is to discuss the concept and approach of the waiting in airspace on the level with landing of aeroplane on the runway. Among the different causes of this waiting, we are limited to the weakness of flight programming by the civil aviation's office. In this way, we are enumerated the main researches that been consecrated at this problem.

## 3-1 PREVIOUS WORKS:

### 3-1.1-The « flow control »:

Until to eighty year, the only method of the « flow control » has been used for palliate the congestion's problem on the level with landing.

The basic principle:

This method principle consists in convert the estimated waiting in area into linear waiting. This operation consists in estimate, in step with the runway's capacity, the eve of the concerned day, The expected approach time of programmed flight for this day; and if a congestion is expected on the level with landing traffic, in that case the different controls centres will be contacted in order to compel the aeroplanes affected by the congestion to reduce their speed whole of the track flight.

Calculation of the expected approach time:

The expected approach time is the time to which it is anticipated that the aeroplane at its arrival will be authorized for begins its approach for the landing. In other words the time to which the aircraft will be able to leave the waiting point .This time lies with the elapsed time that we desire to maintain between the arrivals of the aircrafts above the aerodrome. The calculation principle of the expected approach time (HAP) is done of the following manner*:

HAP= QRE+ D

Where QRE designates the time of which the aeroplane will wait the admission point in the
Delegated zone at the approach; whereas D appointed to the calculated delay in step with the runway's capacity and the spacing between two successive arrivals in the same waiting point.

**Distribution the waiting between the area control and the approach control:**

Consider that the forecast of crossing on the point is not always easy, only for the some part of the total waiting anticipated is absorbed in advance, the rest of this waiting , called "buffer", is absorbed in the level of waiting approach way.
As an illustration, let's look at this simple example: it is estimated that an aeroplane:

- To leave the beginning approach point at 12H07mn;
- To leave at 12H00 the airspace under the responsibility of the area control service;
- To be at the runway at 12H17mn;

If the calculation does indicate a waiting of 11 minutes and if the value of the "buffer" is fixed at 5 Minutes then an aeroplane ought to undergo in advance a waiting of 6 minutes. The supervisor will indicate to the aeroplane that ought to:

-Leave the controlled airspace by the area control service at 12H06 mn

12H00mn + 6mn = 12H06 mn

- Leave the waiting point for begin the approach at 12H18 mn

12H06 mn + 5mn + (12H07mn-12H00 mn)= 12H06mn + 5mn + 7mn= 12H18mn

* E. Legrand : "L'approche", Annexe VIII, Ecole Nationale de l'Aviation Civile, Toulouse.

- And to settle on the runway at 12H28mn

12H18mn + (12H17 – 12H07 mn)= 12H28mn

**Advantages and Disadvantages:**

The method of the « flow control »permits to reduce a part of the waiting area on the level with the approach .The rest of the waiting is effected in the removing air traffic .This method allows to reduce the congestion in the area, however it don't permit to reduce the total waiting and the delay undergo by a flight during its execution.

**3-1.2- The Application Of N-Job One-Machine Scheduling Programming Models:**

In 1981, BIANCO and RICCIARDELL [1] had developed a study concerning the automation's problem of landing on the runway.

**The basic principle:**

The approach base's principle is to determine the landing order permitting to reduce the waiting in area. They have used the programming models of N jobs for one machine (N – job, one – machine scheduling).

**The Model:**

The landing of N flights on the runway, have been considered like being N jobs differences to achieve through one machine (runway).They had supposing that every job i employ the machine during the elapsed-time (meantime) equal to $P_i$ .If $t_i$ is the beginning expected time of the job i and if i the beginning actual time of this machine in that case the relative delay to the $d_i=t_i - i$. It is necessary to determine the succession order of the jobs which reduce the total delay, $D_i$ , N jobs:

$$D_i=\sum_{i=1}^{N} d_i$$

($H_1$): All the delay are possible.

($H_2$): The discipline of the first in, first served (FIFO) could be replaced by anything other discipline.

**Resolution:**

For the resolution of this model BIANCO and RICCIADELLI have used the concept of« Branch and bound ». This concept is an algorithm for solving all- integer and mixed-integer linear programs and assignment problems. It divides the set of feasible solutions into subsets that are examined systematically. It consists in found the whole branch of the algorithm choice, in which all nodes of the level k stand for a partial succession $S_k$ of k jobs ($k \leq N$).

The divided feasible region results in sub-problems that are then solved .Bound on the value of the objective function are found and used to help determine which sub-problems can be eliminated from consideration and when the optimal solution has been found. If the solution to a sub-problem does not yield an optimal solution, a new sub-problem is selected and branching continues.

The specific steps involved when dealing with the N-Job One-Machine Scheduling Programming Models:

(a) If a branch yields a solution to the $S_k$ (the partial succession for the N jobs) that is not feasible, terminate the branch.

(b) If a branch yields a solution to the $S_k$ that is feasible, but not optimal solution, go to the last step.

(c) If the branch yields a feasible $S_k$ solution, examine the value of the objective function. If this value equals the upper bound, an optimal solution has been reached. If it is not equal to the upper bound, but exceeds the lower bound, set it as the new lower bound and go to the last step. Finally, if it is less than the lower bound, terminate this branch.

(d) Examine both branches again and set the upper bound equal to the maximum value of the objective function at all final nodes. If the upper bound equals the lower bound, stop. If not, go back to step (a).

**Advantages and Disadvantages:**

This approach allows to reduction of the waiting in the air in acting simply on the landing order .Yet, it not be able applied for a high number of landing. Indeed, starting from 15 landings, the necessary time for the model resolution become enormous.

[1] L. Bianco and S. Ricciadelli : "Automated Management in the terminal Area", Scientific Management of transport systems, N.K Jaiswal, pp.9-18.

**3-1. 3 The Application From Process Of Decision Markov Analysis:**

In 1985, RUE and ROSENSHINE [(2)] have developed a new approach have had for objective the programming of day's flight so as to eliminate the expected waiting in air.

**The basic principle:**

The principle of this approach is to limit the queue's length of aeroplanes waiting for landing on the runway during the rush period. The restricted aeroplane's number in the queue for different category describes the rejection points of the system formed by the runway and the airspace understand as for the waiting. In this way, RUE and ROSENSHINE have used the issues from two process of Markov decision with type (M/M/1) and (M/ $E_k$/1). It is necessary to determine the optimal decision of system which permits to maximize the profit. Two kind of solution have been enumerated, the individual solution (optimal on the level with every classes) and the global solution (optimal for all system).

**The Optimal Solution with The M/M/1 Process:**

In their article, RUE and ROSENSHINE have used the result from the M/M/1 model that they have already expanded in 1981.

a/ The optimal individual solution :

If an aeroplane from the m classes comes and finds i aeroplanes in front it, one's net profit anticipated if it joins the system is equal to: $\frac{R_m-(i+1)C_m}{\mu}$

An airplane can to join the system if the number of the aeroplane be situated there is inferior to $n_{sm}$ defined by:

$$\frac{R_m-(n_{sm}+1)C_m}{\mu} < 0 \le \frac{R_m-n_{sm}C_m}{\mu}$$

$n_{sm}$ represents the limit number of aeroplanes that which any aircraft, from the m class, will undergo a loss while meeting to the system, the aeroplane are supposed join the system when the net profit is zero.

So, $n_{sm}$ be resolved by the entire part of : $\frac{R_m\mu}{C_m}$

$n_{sm}=\left[\frac{R_m\mu}{C_m}\right]$ The brackets indicates the entire part of $\frac{R_m\mu}{C_m}$

b/ The optimal solution of the system :

In this case, the decision-maker decides if an aeroplane from the m classes can to join the system whether the i aeroplanes that had before proceeded. Then, he determines the vector $n_0$= ( $n_{01}$, $n_{02}$,…$n_{0n}$) of the rejection point that maximizes the total system's profit. In other words the maximum of profit is generated in the system when the member of the class m is not authorized to join the system whether the aeroplane number that we had proceeded is inferior than $n_{0m}$ .If n = ( $n_1$, $n_2$,…. $n_m$) represent any vector of rejection point ,then the total net profit of the system tally with n is :

$$g(n)=\sum_{m=1}^{M}\lambda'_m(n)R_m-C_mL_m(n)$$

*Where* $\lambda'_m$ (n) represents the arrival real rate from the class m and Lm (n) represents the contribution from the m classes to aeroplane's number in the system when the n vector of rejection point is used. RUE and ROSENSHINE have showed that the total net profit of the system can be computed by means of formula:

$$g(n)=\sum_{i=1}^{n^*}\Phi_i(n)\sum_{m=1}^{M}\Delta_m(i)\lambda_m[R_m-(i+1)C_m/\mu]$$

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2 R.C. RUE and M. Rosenshine : "The application of Semi-markov decision process to queueing of aircraft at an airport", Transportation science, vol. 19, N°2, pp.154-172.

*Where*

- $n^* = \text{Max}\,[n_m]$
- $\Phi_i$ (n) is the probability that i aeroplanes being in the system knowledge that the rejection point of the system are described by the n vector :

$$\Delta_m(i)=\begin{cases}1 & si\ i<n_m\\ 0 & si\ \ i\geq n_m\end{cases}$$

For each m classes the research of the optimal rejection point of system has been limited between 0 and $n_{sm}$. Be learning that the two authors had already showed for all m varied between 1 and M, $n_{0m} \leq n_{sm}$

**The Optimal Solution Of K-Erlang Model :**

RUE and ROSENSHINE [(3)] had used in this case the results of the model $M/E_k/1$ which they had developed in them article published in 1983:

a/ The individual optimal solution :

If an aeroplane from m class comes and finds i aeroplanes in front it, its anticipated net profit to join the system is equal to:

$$R_m - iC_m/\mu - ((k+1)/2k)/\mu)C_m$$

An aeroplane joins to system whether the number of aeroplane that we had preceded is inferior to $n_{sm}$ such as $n_{sm}$ defined by:

$$R_m - C_m[(n_{sm}/\mu)+((k+1)/2k\mu)]<0\leq R_m - C_m\,[((n_{sm}-1)/\mu)+((k+1)/2k\mu)]$$

In this way, $n_{sm}$ is computed by:

$$n_{sm}=[\mu R_m/C_m+(k-1)/2k]$$

b/ The optimal solution of system :

If v represents the any vector of rejection point, at that the total net profit of the system correspond with v is calculated by:

$$g(v)=\sum_{i=1}^{n^*}\Phi_i(v)\sum_{m=1}^{M}\Delta_m(i)\,\lambda_m[R_m-(i+k)C_m/k\mu]$$

*Where*

$v^*=\max_m (v_m)$

$\Phi_i$ and $\Delta_m$ are defined in the same manner that in the case of the exponential model.

This formulation is resolved by the iterative algorithm, the optimal solution is gave in words of service's phase in the system, since the system's states are defined into the number of service's phases. The service corresponds to k phases. So, if i aeroplanes are in the system then the state of the system could be from anything what phase between (i -1) k +1 and ik.

**Advantages and Disadvantages:**

This approach allows reducing the aeroplane's number while waiting for the landing by a good flight's programming enough in advance. She also permits to reduce the lateness generated during the execution's flights , however it presents some disadvantages, the hypothesis concerning the discipline's arrivals and service didn't been justified. Otherwise in the calculation

[3] R.C. RUE and M. Rosenshine : "Optimal control for entry of many classes customers to an $M/E_k/1$ queue", Naval Research Logistics Quarterly, vol. 30, pp.217-228.

the rejection point of the system, the authors didn't take from consideration the problem of availability the aircraft parking .In this way of an airport had been a limited space for the aeroplane's parking, a like approach don't permit to lead to an optimal decision.

**3-1.4- Application Of Dynamic Programming Modeling:**

In 1987, ANDREATTA and ROMANIN – JACUR [(4)] have developed an other approach.

The basic principle :

The principle of this approach is the transformation of the waiting in air's part into ground waiting. The choice of the flights that it is necessary to delay at the departure airport is based on the reduction of the total waiting cost's criterion. They have considered for that the following hypothesis:

**(H1)** It is anticipated that N aeroplanes take off from different airport at different timetables $t_i$ (supposed negative) and land at a same moment t=0 in a same airport.

(**H2)** Be given a lot of factor can affect the airport's capacity (specially the meteorological circumstance), the estimation of an airport's capacity at t=0 is defined. K is supposed taken the values 0, 1, 2,…, n with the respectively probability P(0), P(1), P(2),…P(n).

**(H3)** The whole retarded aeroplanes will been able to land at the moment t=1.

The mathematical modelling:

It is necessary to decide for each aeroplane i (i=1, 2,…, n)if it is preferable to inflict a delay to the ground at the departure airport or to it authorize the takeoff at the expected time $\mathbf{t_i}$ **.**Two kind of costs are defined for the delayed aeroplane i: The cost $\mathbf{g_i}$ for the delay to the ground and the cost $\mathbf{a_i}$ for the waiting in air. Otherwise, the other model's variable is:

The probability which the capacity K of an airport don't exceed k (k=0, 1,…, n) is defined by :

$$P(K)=\sum_{h=0}^{k}P(h)=P_{prob}[K\leq k]$$

For each aeroplane i a delay decision variable $\mathbf{d_i}$ is defined by:

$$\mathbf{d_i} \begin{cases} 1 & \text{if i is delayed on the ground} \\ 0 & \text{elsewhere} \end{cases}$$

$$d=(d_1, d2,\dots d_n)$$

The aeroplanes are classed by the priority order of landing according to one landing priority $\pi$ defined by the index $\pi_i$ for every aeroplane i .So if $\pi_i > \pi_j$ then the aeroplane i is having priority upon the aeroplane j .The number of aeroplanes having priority to landing upon the aeroplane i are defined by :

$$S_i=\sum_{h=1}^{n}S(h,i)$$

*Where*

$$S\,(h,i)=\begin{cases} 1 & if \quad \pi_h > \pi_i \\ 0 & elsewhere \end{cases}$$

The number of aeroplanes having a priority of landing on the aeroplane i and that will undergo a waiting on the ground is $D_i$

[4] G. Andretta and G. Romanin Jacur : « Aircraft flow management under congestion », transportation science, vol 21, N°4, pp.249-253.

$$D_i=\sum_{h=1}^{n} d_h S(h,i)$$

The probability which aeroplanes didn't been authorized to landing at moment t=0, namely that $D_i$ aeroplane having priority on i be came under a delay on the ground is:

$$P(i/S_i, D_i)=P(S_i-D_i)$$

The delay cost for each aeroplane i is computed by:

$$C_i(d_i, D_i) = \begin{cases} C_i(d_i, D_i) = g_i & \text{if } d_i = 1 \\ a_i P(i/s_i, D_i) & \text{if } d_i = 0 \end{cases}$$

This is equivalently:
$C_i(d_i, D_i) = d_i g_i + (1-d_i)a_i P(i/S_i, D_i)$
Thus the total cost to minimize is:

$$C(d) = \sum_{h=1}^{n} [d_i g_i+(1-d_i)a_i P(i/S_i, D_i)]$$

$$= \sum_{h=1}^{n} C_i(d_i, D_i)$$

Solution :

To determine the optimal delay decision, the aeroplanes are firstly in the increasing order of priority. In other words an aeroplane i be having priority on an aeroplane j if i>j. In that case we have had:
The number of aeroplanes having priority on the aeroplane j is calculating by:

$$S_i=n-i$$

The number of aeroplanes having priority on the aeroplane i, and they do undergo a delay to ground is computed by:

$$D_i=\sum_{h=j+1}^{n} d_h = d_{i+1}+D_{i+1}$$

So, the optimal vector d*= $(d_1^*, d_2^*, \cdots, d_n^*)$ satisfied the following recursive equation :

$$C_i(D_i) = \underset{di}{Min}[C_i(d_i, D_i)+C_{i-1}(d_i+D_i)]$$

With the initial condition:

$$C_0(d_1+D_1) = 0 \quad \forall d_1, D_1$$

The amount $C_i(D_i)$ is interpreted like being the optimal value of the delay cost in i first aeroplanes knowledge that Di aeroplanes having priority than i had undergo a lateness in to ground. In this way the optimal total cost is given by: $C^* = C(d^*) = C_n(0)$.

In basing on this recursive relation, ANDREATTA and ROMANIN-JACUR had expanded an algorithm of dynamic programming making it possible to lead to the optimal solution. **Advantages and Disadvantages:**

This study can be considered like being an initial development of a new approach, be given that ANDREATTA and ROMANIN-JACUR have used the hypothesis which do simplify enormously the reality. They have supposed that the congestion in the waiting level didn't last whether one unit of time; in other words whether the all aeroplanes will be able to land at the extremity of the one unit of times. In the same way they have supposed that the distribution of the probability relative to an airport's capacity don't change in time. Otherwise, seeing that the case for the others researches, the problem of assignment the aircrafts parking is not taken by consideration.

**3-1.5- An Assignment Problem From The Aircrafts Parking:**

On the contrary with the latter, an other researches have been limited at the assignment from the aircrafts parking. We enumerate in this frame the developed model on 1985 by MANGOUBI and MATHAISEL. [5]

## The basic principle:

The principle of this assignment is to reduction of the distance travelled through the passengers between the aircrafts parking and the air terminal. Two approaches had been developed; the first uses the linear programming with integer values whereas the second uses the heuristic formula's problem.

**The linear formulation :**

**a/ Notations :**

A separate 0-1 variable can be defined for each position to assignment of the flight i into the aircraft parking j.

$$X_{ij} = \begin{cases} 1 & \text{if the flight i is assigned to the parking j .} \\ O & \text{if not} \end{cases}$$

The passengers' flight are classified by three category: The arrivals passengers, $P_i^a$, the departures passengers, $P_i^d$, and the transits passengers, $P_i^t$.The distance travelled through by these three typical passengers from the parking j is as follow $d_j^a$, $d_j^d$ and $d_j^t$.The total distance covered with all passengers is written down Z.
The number of flights to assign into an aircraft parking is written down M.
The number of the aircrafts parking is written down N.

**b/ The objective function :**

The objective of this formulation consists in reducing the total distance covered with all passengers, it is formulated as:

$$MinZ = \sum_{i=1}^{M} \sum_{j=1}^{N} (P_i^a d_j^a + P_i^d d_j^d + P_i^t d_j^t) X_{ij}$$

**c/ The constraints :**

Two types of constraints are possible: those which are own to an assignment problem and those that depend on the airport or else the airline company using the airport.

The constraints of the first type are:
-Every flight ought to be associated in one and only aircraft parking.

$$X_{ij} = 1 \quad \forall i \in \{1 \cdots M\}$$

The number of these typical constraints is equal to the flights number, M.

- Two aeroplanes didn't should to be associated in a same aircraft parking at the same time :

$$X_{hj} + X_{ij} \leq 1 \qquad \begin{matrix} \forall h \in L(i) \\ \forall i \in \{1 \cdots M\} \\ \forall j \in \{1 \cdots N\} \end{matrix}$$

L(i) designates the set of flights have which landed before the i flight and they which still in the parking when the flight i comes. So whether the flights are indicated in the order of their arrivals, L (i) can be defined as follow:

$$L(i) = \{h / t_h^d > t_i^a, h \in \{1 \cdots i-1\}\}$$

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5 R.S. Mangoubi and D.F.X. Mathaisel : « Optimizing gate assignements at airport terminals », transportation science, vol 19, pp173-188.

Where $t_h^d$ is the departure time of the flight h and $t_i^a$ is the arrival time of the flight i.

The second type of constraints depends on the airport in question. Two constraints of this type have been developed:

- Subdivision the aircrafts parking on independent areas for each company: Some airports are subdivided on the reserved areas for exclusive use by the airline company. If S represents the number of company using the airport, in that case the flights will be divided into S groups $I_1, I_2, \dots, I_s$ .In the same way the aircrafts parking will be also divided into S groups $J_1, J_2, \dots, J_s$. S sub-terminals are also obtained for which S under integer linear programming similar to the previous formulation are developed.
- Some parking are too smaller for some aeroplanes. If I represents the biggest size of set aeroplanes and J represents the set parking don't be able to accept in such aeroplanes, then:

$$X_{ij} = \quad \begin{array}{l} \forall \quad i \in I \\ \forall \quad j \in J \end{array}$$

### The Heuristic Formulation :

This approach consists in to assign the aeroplanes that have had the greatest number of passengers into parking tally with the smallest distance covered. The specific steps involved in this algorithm are as follows:

**Step 1**: Classify the flights into decreasing order from the passenger's number;

**Step 2**: Consider the first flight in the list and note F ;

**Step 3**: Be reassemble on the set S all the aircrafts parking being able to accept the flight F and which were available at the arrival of the flight F;

**Step 4**: Assign the flight F at parking $s \in S$, that tally with the minimal distance covered with the passengers of flight F:

$$P_F^a d_S^a + P_F^d d_S^d + P_F^t d_S^t = \underset{i \in S}{Min} \left[ P_F^a d_i^a + P_F^d d_j^d + P_F^t d_j^t \right]$$

**Step 5:** If the all flights are affected at the aircrafts parking, the procedure is then terminated. Otherwise, it be necessary to consider the following flight and go back to step (3).

Those both approach have been applied at an international airport of Toronto in Canada. The improvement has been noticed in relation to the initial assignment from the aircrafts parking. Indeed, the application of the first approach has allowed the reduction of covered distance with the passengers of 32% in relation to initial assignment, at a time when the application of the second approach has reduced this same distance of 28, 1%.

**Advantages and Disadvantages:**

The application of these both approaches has supplied with considerable improvements concerning the covered distance. However the objective used in the assignment don't be the first importance, particularly what the transport's means between the air terminal and the aircrafts parking are became more and more available at the airports.

## 3-2 A PROBLEM FORMULATION FROM THE PROGRAMMING OF DAY'S FLIGHTS:

The landing of aeroplanes at an airport generally became according to the discipline known as the first-in, first-out rule. The authorizations of landing were agreed by the approach control office particularly basis on the both following parameters:

- At the rate of the using of the runway.
- And the availability of an aircraft parking for the aeroplane in question.

It follows then, that for avoid the expected waiting in air by aeroplanes, it is necessary to take into account of these both parameters in the programming of day's flights.

### 3-2-1- Notations and Abbreviations:

| | |
|---|---|
| N | The number of aircrafts parking |
| M | The number of programmed flights for the designed day |
| $t_i^a$ | Estimated time of arrival flight i |
| $t_i^d$ | Estimated off-block time flight i |
| $t_{ij}$ | Landing time proposed by civil aviation centre if the aeroplane i will be assigned into parking j ($t_{ij}$ is supposed not null in this case). |
| C | The runway's rate |
| T | An upper bound for $t_{ij}$ variables |
| $W_i$ | The weight of the flight i in relation to others flights' day |
| $t_i$ | Landing time proposed by the civil aviation centre for the aeroplane i |
| $\underline{t}_i$ | Takeoff time flight i whether it did land at time $t_i$. |
| $y_i$ | The assigned aircraft parking to the flight i |

| | |
|---|---|
| $y_{ij}$ | An integer variable with Zero-one values |
| DAC | Civil aviation direction |
| Card | The cardinal of the set |

**3-2-2- The Model:**

We proposed to formulate the programming problem from flights' day in the form of linear programming.

The objective function:

The objective of this formulation is to maintain or to delay as minimum as possible the timetables asked for the airlines company .The objective function will express then the total reduction of estimated waiting.
It is necessary to decide the proposed timetables by the airlines company or to suggest of again timetables for the M flights at the designed day.

Hence, we do consider the integer decision variables $t_{ij}$ bounded is defined as follows:

- If the flight i will be assigned in to parking j in that case $t_{ij}$ will take a not null value lower or equally than T. T is the upper bound for the variables $t_{ij}$.
- If the flight i won't be assigned in to parking j then the variable $t_{ij}$ will take a null value.

The variable $t_{ij}$ (not null) represents the landing time that will propose the civil aviation direction (DAC) in to flight i if it will be assigned to the parking j, known that the airline company had organized this flight asks for the landing time $t_i^a$ , and the takeoff time $t_i^d$ .

$t_{ij}$ can be also interpreted as being the minimal time to which the flight i be able to start her procedure of landing known that it will be in to approach point at time $t_i^a$ .

We suppose that the airlines company don't ask for the zero time as the landing time. The minimal time asked for is equal to one minute. We suppose as well as the DAC won't suggest the zero time like the landing time. The minimal time proposed is supposed equal to one minute. Thus, if $t_{ij}$ will take a zero value, that won't mean what the DAC will propose in to flight i the parking j , and the zero time for his landing; but that will mean what the flight i won't be assigned in to parking j after his landing.

Note that for each flight i, an only one variable $t_{ij}$ will be not null, since it will be assigned in to single parking j.

The number of variables $t_{ij}$ so defined is equal to N.M.

In this way if the landing time asked for the airlines company will be kept then the estimated waiting for every flight i before its landing, be calculate by:

$$\left(\sum_{j=1}^{N} t_{ij}\right) - t_i^a \qquad (1)$$

The expected total waiting for the M flights is expressed by:

$$\sum_{i=1}^{M}\left[\left(\sum_{j=1}^{N} t_{ij}\right) - t_i^a\right] \qquad (2)$$

In the same way it be sometimes finds that some airport use the discriminately policy between the flights or between the airlines company, such as favour some company by supplying its minimum timetable's gap.

As it were of the same cases, we do propose to introduce a positive factor $W_i \leq 1$ for every flight i expressing the weight of the flight i in relation to others flights. Note that if $W_i > W_{i'}$ in that case the flight is more favour than the flight i'.

We do thus lead to following objective function:

$$Min\, Z = \sum_{i=1}^{M} w_i \left[ \left( \sum_{j=1}^{N} t_{ij} \right) - t_i^a \right] \qquad (3)$$

Constraints :

We have classified the constraints in two groups. The constraints which are inherent in problem and the constraints that depend on the airport in question or on the used rule by the decision maker.

We do distinguish, in the first group, four different constraints ((C1),(C2),(C3) and (C4)). When into second group, we have chosen to formulate an only constraint (C5), being gave that it is commonly in the whole Tunisian airports. This constraint does express the assignment problem of aircrafts parking from different model of aeroplanes.

a/ Constraints(C1):

The time which have to suggest the DAC for each flight i, is at least equal to the wanted time for this flight, by the correspondent airline company. This constraint results from the objective problem which consists in decide to keep or to delay, for every flight, the wanted landing time.

We obtain then:

$$\sum_{j=1}^{N} t_{ij} \geq t_i^a \qquad \forall i \in \{1 \cdots M\} \qquad (4)$$

The number of these constraints is equal to the number of flights programming for a day, M.

b/ Constraints ($C_2$) :
If the runway can't accept so an only landing at a gave instant, any aeroplane be found at the approach point can't land only when the runway will be free.
So whether i+1 is the aeroplane coming after the aeroplane i at that time the landing time of the aircraft i+1 ought to be at least equal to the leaving time on the runway. Being gave that the medium spacing between two landings correspond to the rate C of the runway.
The constraints (C2) are doing express by:

$$\sum_{j=1}^{N} t_{i+1\,j} \geq \sum_{j=1}^{N} t_{ij} + C \qquad \forall i \in \{1 \cdots M-1\} \qquad (5)$$

The number of these constraints is equal to M-1

c/ Constraints (C3) :

An only aircraft parking is assigned at every aeroplane i after its landing. It follows that if $t_{ij}$ takes a not null value then anybody else variables $t_{il}$ (l being a parking different to j)will do necessary take the null value.

Mathematically, the constraints (C3) take the following form:

$$\text{If } t_{ij} \neq 0 \text{ then } t_{il} = 0 \quad \begin{array}{l} \forall \quad i \in \{1 \ldots M\} \\ \forall \quad l \in \{1 \ldots N\} \quad \text{and} \quad l \neq j \\ \forall \quad j \in \{1 \ldots N\} \end{array} \tag{6}$$

**Lemma 1 :**

For all $i \in \{1 \ldots M\}$ and $j \in \{1 \ldots N\}$, let $y_{ij}$ an integer variable with 0-1 values defined by the both equations (7) and (8)

$y_{ij} \leq t_{il}$ **(7)**

$t_{ij} \leq T\, y_{ij}$ **(8)**

$$\text{The equation } t_{il} \leq T(1-y_{ij}) \quad \begin{array}{l} \forall i \in \{1 \ldots M\} \\ \forall l \in \{1 \ldots N\} \quad et\ l \neq j \\ \forall j \in \{1 \ldots N\} \end{array} \tag{9}$$

Is equivalent to the expression (6).

**Proof:**

Let $i \in \{1 \ldots M\}$ and let $j \in \{1, \ldots, N\}$.

* If $t_{ij} \neq 0$, then it follows according to the equation (8) that $y_{ij} \neq 0$; $y_{ij}$ being not null at that time $y_{ij} = 1$

The equation (9) be wrote then $t_{il} \leq 0$ which proved that $t_{il} = 0$ (being gave that $t_{il}$ is a variable) for all $l \neq j$ and $l \in \{1 \ldots N\}$.

Thus, whether the equation (9) is confirmed then the expression (6) is it also.

*Let 1 an element of $\{1 \ldots N\}$ such as $l \neq j$

If $y_{ij} = 0$ then $t_{il} \leq T(1-y_{ij})$ is confirmed since $t_{il} \leq T$ for all $l \in \{1, \ldots, N\}$ *and* $l \neq j$ If $y_{ij} = 1$ then we obtain according to equation (7), $t_{ij} \neq 0$ ;it follows on (6) that, for all $l \in \{1 \ldots N\}$ and $l \neq j$ , $t_{il} = 0$ (which be also wrote too $t_{il} \leq 0$ ) which proved that $t_{il} \leq T(1-y_{ij})$

Thus if the expression (6) is confirmed then the equation (9) is it also. The constraints (C3) will be expressed by the equations in (7),(8) and (9). The number of constraints which are given from (7),(8) and (9) is equal to :

$N.M + N.M + N.M.(N-1) = N.M + N^2.M$

These constraints do introduce more and more N.M integer variables $y_{ij}$

d/ Constraints (C4) :

Two aeroplanes don't should be assigned at a same aircraft parking at a same time. In other words if an aeroplane h be found in a parking j just when the aeroplane i is arrived in that case the latter doesn't ought to be assigned in the parking j.

From the definitions of this type of constraints we do consider for each flight i the set of aeroplanes which they went to land and which they also will were, in the parking at its arrival.

We take again the notation, L (i), from MANGOUBI and MATHAISEL, for this set :

$L(i) = \{h \in \{1 \ldots, i-1\} / t_h^d > t_i^a\}$

Mathematically these constraints are defined as follows:

$$\text{If } t_{hj} \neq 0 \text{ then } t_{ij} = 0 \quad \begin{array}{l} \forall h \in L(i) \\ \forall i \in \{1 \ldots M\} \\ \forall j \in \{1 \ldots N\} \end{array} \tag{10}$$

**Lemma. 2 :**

Let $y_{ij}$ an integer variable confirmed the equations (7) and (8).

$$\text{The inequality } t_{ij} \leq T(1-y_{ij}) \quad \begin{array}{l} \forall h \in L(i) \\ \forall i \in \{1 \ldots M\} \\ \forall j \in \{1 \ldots N\} \end{array} \tag{11}$$

Is equivalent to the expression (10).

**Proof. :**

Let $i \in \{1 \ldots M\}$ and let $j \in \{1 \ldots N\}$

* Let $h \in L(i)$ such as $t_{hj} \neq 0$ then it follows, according to (11), that $1 - y_{ij} \neq 0$ in other words that $y_{ij} = 0$

$y_{ij}$ Being null, then according to (8) we deduce that $t_{ij}=0$
In this way, if the inequality (11) is verified then the expression (10) is it too.
*If $y_{ij}=0$ at that time the inequality (11) is confirmed as:

$t_{hj} \leq T$ For all $h \in L(i)$ and $j \in \{1 \ldots N\}$

If $y_{ij}=1$ then it follows, according to (7), that $t_{ij} \neq 0$ which proved, according to (10), that $t_{hj}=0$ for all $h \in L(i)$

Thus if the inequality (10) is checked, in that case the inequality (11) is it also.
The constraints ($C_4$) will be expressed by the equations (7), (8) and (11) for every flight i, the number of constraints defined in (11) is equal to N. card (L (i)) which give a total number of constraints (for all flights)

$$\sum_{i=1}^{M} N.card(L(i)) = N.\sum_{i=1}^{M} card(L(i))$$

Where (L (i)) designates the element's number of set L (i).

e/ Constraints (C5) :

A second group of constraints depends on the airport in question or on decision maker. In the way of we have it already mentioned; we have chose to develop the constraint which does actually expressed that some aeroplanes type can't be parked in some aircrafts parking. In be basing on the distribution ,the aircrafts parking and the flights we obtain thus, for all k varying from 1 to q :

$$t_{ij}=0 \quad \forall i \in I_k \text{ and } j \in \bigcup_{p=k+1}^{q} J_p \qquad \textbf{(12)}$$

At each value k correspond, to (12), a number of constraints are equal to: $card\,(I_k).\left(\sum_{p=k+1}^{q} card(J_P)\right)$

By definition, the total number of constraints is expressed in (12) equal to: $\sum_{k=1}^{q} card\,(I_k).\left(\sum_{p=k+1}^{q} card(J_P)\right)$

**The complete formulation problem :**

$$Min\,Z=\sum_{i=1}^{M} w_i \left[\left(\sum_{j=1}^{N} t_{ij}\right) - t_i^a\right]$$

- $\sum_{j=1}^{N} t_{ij} \geq t_i^a \qquad \forall\, i \in \{1 \ldots M\}$
- $\sum_{j=1}^{N} t_{i+1j} \geq \sum_{j=1}^{N} t_{ij} + C \quad \forall\, i \in \{1 \ldots M-1\}$
- $y_{ij} \leq t_{ij} \qquad \forall i \in \{1 \ldots M-1\},\ \forall j \in \{1 \ldots N\}$
- $t_{ij} \leq T y_{ij} \qquad \forall i \in \{1 \ldots M-1\},\ \forall j \in \{1 \ldots N\}$

$$t_{il} \leq T(1-y_{ij}) \quad \begin{matrix} \forall i \in \{1 \ldots M\} \\ \forall j \in \{1 \ldots N\} \end{matrix}$$

$$\forall \quad l \in \{1 \ldots N\} \quad such \quad as \quad l \neq j$$

**Subject to:**

$$t_{hi} \leq T(1-y_{ij}) \quad \begin{matrix} \forall i \in \{1 \ldots M\} \\ \forall h \in L(i) \\ \forall j \in \{1 \ldots N\} \end{matrix}$$

$$t_{ij} = 0 \qquad \begin{matrix} \forall i \in I_k \\ \forall j \in \bigcup_{p=k+1}^{q} J_P \end{matrix}$$

This linear programming defined 2.M.N decision variables and M + (M-1) + M .N + M . N + M. N. (N-1)+N. $\sum_{i=1}^{M} card(L(i)) + \sum_{k=1}^{q} card(I_k).\left(\sum_{p=k+1}^{q} card(J_P)\right)$ constraints.

Furthermore:

2M-1 +M .N + M. $N^2$ + N. $\sum_{i=1}^{M} card(L(i)) + \sum_{k=1}^{q} card(I_k).\left(\sum_{p=k+1}^{q} card(J_P)\right)$ Constraints.

## CONCLUSION:

Queuing models have found widespread use in the analysis of service facilities, production and many other situations where congestion or competition for scarce resources may occur. This paper has introduced the basic concepts of queuing models, and shown how linear programming, and in some cases a mathematical analysis, can be used to estimate the performance measures of a system. The key operating characteristics for a system are shown to be (1) utilization rate,(2) percent idle time, (3) average time spent waiting in the system and in the queue, (4) average number of travellers in the system and in the queue , and (5) probabilities of various numbers of passengers in the system.

We have just seen a lot of special types of programming models for instance assignment models, flow control method, dynamic programming…etc; that were handled by making certain modifications to the general approach. This article presents a series of other important mathematical programming models that arise when some of the basic assumptions of LP are made more or less restrictive. The latter can, of course, be applied only to cases in which the constraints and objective function are linear.

In our case study with using the software Hyper-Lindo the optimal solution is obtained after 801 iterations and 16 branches. The objective function does take an optimal value equal to 45176. Be given that the factor $W_i$ has been supposed equal to 1 then this value tally with the amount of landing hours proposed by the DAC for the 46 flights. while deducting from this value the whole minutes of landing hours asking for the airlines company for these flights (45966) we obtain the total lateness which ought to inflict the DAC on these flights in relation to wanted hours, in order to avoid the waiting in air of aeroplanes in the course of the designed day (15 July 2003). This waiting is estimated to 790 minutes in other words 13H and10min, the analysis of results indicates that the linear modelling gives to a minimum shifting timetable. Only, be given its enormous cost of execution (in time) the setting up in such model at airport less interesting.

Unlike the discrete and continuous probability distribution, simulation is often used in the analysis of queuing models (can be used to generate one or more artificial histories of a complex system). As mentioned earlier, the assumptions required for solving waiting lines problems analytically are quite restrictive. For most realistic queuing systems, simulation may actually be the only approach available. Models to handle have been developed by operations researchers. The computations for the resulting mathematical formulations are somewhat more complex than the ones covered in our case, and many real-world queuing applications are too complex to be modelled analytically at all. When this happens, quantitative analysts usually turn to *computer simulation*.

ACKNOWLEDGEMENTS:
I am very grateful to Professor Kammoun Hichem of G.I.A.D laboratory of University of Sfax in Tunisie, where a part of this research has been conducted for his interest and guidance in conducting this research.

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