Response of wall hot-film gages with longitudinal diffusion and conduction to the substrate

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The present investigation studies the combined effects of axial diffusion through the trailing and leading edges of the HFG and conduction to the substrate on the frequency response of the wall

shear stress sensors. The full 2D formulation of the conjugate heat transfer problem is presented here, and the results are analyzed both in the steady and unsteady regimes.

The dimensionless equations in the fluid F , in the solid S , and at the fluid-solid (substrate) $F-S$ interface are respectively

$$S: \quad \frac{\alpha_F}{\alpha_S} \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \quad (1b)$$

$$F-S: \quad \frac{k_F}{k_S} \frac{\partial \theta_F}{\partial y} = \frac{\partial \theta_S}{\partial y} \quad (1c)$$

The boundary conditions are $\partial\theta/\partial x \rightarrow 0$, and $\partial\theta/\partial y \rightarrow 0$, as $x, y \rightarrow \infty$, in the substrate and $\partial\theta/\partial x \rightarrow 0$ for $x \rightarrow \pm\infty$, and $\theta \rightarrow 0$ when $y \rightarrow \infty$ in the fluid. The velocity distribution in the forced convection heat transfer equation is either $u(y) = \sigma y$ or $u(y, t) = \sigma y(1 + a \sin \omega^* t)$ depending on whether a steady or unsteady problem is studied. The velocity is linear since the thermal boundary layer is thin, exactly as in the classical Lvque's problem. The shear parameter is given by $\sigma = (\text{Pr}(l_{\mu}/l_v))^2 = \text{PrRe}_c^2$.

The heat transfer law generally used for wall shear stress measurements (the Nusselt number $Nu \propto \sigma^{1/3}$) is based on the simplest solution which assumes $\partial\theta/\partial t = \partial^2\theta/\partial x^2 = 0$ in the fluid (quasi-steadiness and no longitudinal diffusion) and $k_S = 0$. More complete solutions have been considered in the past, for instance with $\partial\theta/\partial t \neq 0$ [13–15] or with $k_S \neq 0$ (in the 1D case by Bellhouse and Schultz [9] and the 2D case by Brison et al. [10]). The full simulation introduces the additional parameter of the diffusivity ratio α_F/α_S , which points to the importance of *how fast* the heat is transferred to the fluid after conduction through the solid in addition to *how much* heat is transferred this way as compared to the direct transfer on the film.

The stretching method used in the fluid and substrate following the wall normal direction is of the form $y = \eta^2$. Thus for the model shown in Fig. 1(a), the coordinate transformations are

$$S1 \text{ (solid 1): } -H_{S1} \leq y \leq 0, \quad -c_{S1}H_{S1} \leq \eta \leq 0;$$

$$y(\eta) = -\eta^2/(c_{S1}^2 H_{S1})$$

$$S2 \text{ (solid 2): } -(H_{S1} + H_{S2}) \leq y \leq -H_{S1},$$

$$-(c_{S1}H_{S1} + -c_{S2}H_{S2}) \leq \eta \leq -c_{S1}H_{S1};$$

$$y(\eta) = -(\eta + c_{S1}^2 H_{S1})^2 / (c_{S2}^2 H_{S2}) - H_{S1}$$

The coefficients c_F , c_{S1} , and c_{S2} can be modified to adapt the stretching to different zones. The maximum resolution occurs at the interfaces where the derivative $dy/d\eta=0$. The upstream weighting difference and ADI problems are formulated in the (ξ, η) domain and the resulting system was resolved by the classical Thomas algorithm.

Fig. 2 Local heat flux over the film compared with the boundary layer approximation (Lvque solution)

parameter. The effect of the trailing edge singularity increases with decreasing Pclet number. The computed global heat flux is compared with Ling [4] and Springer [6] in Fig. 3. There is an excellent agreement with the asymptotic analysis of Springer [6], and the difference between the computations and his theoretical approach is less than 1%.

3 Results

3.1 Two-Layer Configuration; Homogeneous Substrate.

The global heat transfer rates per unit span Q for the coupled glass/water (conductivity ratio $k_S/k_F=1.27$) and glass/air ($k_S/k_F=25.3$) cases are shown in Tables 1(a) and 1(b) as a function of the Pclet number $\sigma = \text{Pr} f_f^{+2}$. The notations are illustrated in Fig. 4. In the steady regime, the heat transfer from the film to the substrate Q_{fS} must be balanced by heat loss from the semi-infinite substrate to the fluid. Thus, $Q_{fS} = Q_{SF}^- + Q_{SF}^+$. This balance is, by the way, a good check for computational accuracy. The fluid is heated by the upstream heat transfer rate Q_{SF}^- before it reaches the film. This decreases both the direct transfer rate from the film Q_{fF} and the downstream transfer rate Q_{SF}^+ . The latter extends the heating of the fluid further downstream and smoothes the effect of the axial diffusion at the trailing edge singularity. The conductivity ratio k_S/k_F is the only relevant parameter describing the substrate's direct and indirect influences. The direct transfer is reduced because of the wake effect due to Q_{SF}^- and the indirect transfer

Fig. 3 Global heat flux with the longitudinal diffusion term and comparison with past studies

Table 1 Global heat transfer rates versus the shear parameter. For the notations see Fig. 4. (a) Water/glass. (b) Air/glass.

σ	Q	Q_{fF}	Q_{fS}	Q_{SF}^-	Q_{SF}^+	Q_{Lvque}
3	9,74	0,63	9,11	5,37	3,74	1,16
15	12,79	1,05	11,74	7,37	4,37	1,99
23	13,70	1,23	12,47	7,89	4,57	2,30
30	14,00	1,34	12,66	8,05	4,60	2,51
38	14,26	1,43	12,82	8,18	4,65	2,71
45	14,39	1,50	12,89	8,23	4,66	2,87

together with the ratio Q_{fS}/Q_{fF} increase with k_S/k_F .

The heat flux to the substrate Q_{fS} is smaller than the direct transfer Q_{fF} for small conductivity ratios as expected (Fig. 5(a)). On the contrary, the ratio Q_{fS}/Q_{fF} is as large as 10 for the combination air-glass with $k_S/k_F=25.3$ as seen in Fig. 5(b). For both cases the ratio Q_{fS}/Q_{fF} decreases with increasing Pclet number, because the transfer to the substrate Q_{fS} depends only slightly upon σ , while the transfer from the film to the fluid Q_{fF} increases much more rapidly with the Pclet number by forced convection.

For the combination water/glass, the direct transfer from the film to the fluid is approximately equal to the Lvque solution,

Fig. 4 Sketch of heat transfer definitions over the hot film gage

Fig. 5 Direct transfer to the fluid and transfer to the substrate compared with the Lvque solution. (a) Water/glass. (b) Air/glass. Legend on (a).

Fig. 6 Effective length of the hot film gage versus the conductivity ratio for $\sigma=30$

i.e., $Q_{fF} \approx Q_{\text{Lévéque}} = 0.81\sigma^{1/3}$ (Fig. 5(a)). For large σ values, Q_{fF} is smaller than $Q_{\text{Lévéque}}$, but the difference never exceeds 10%.

3.2 Effect of the Conductivity Ratio. The effect of heat transfer via the substrate may be characterized by an “equivalent length” l_f^e defined by $Q = Q_{fF} + Q_{fS} = 0.81\sigma^{1/3}(l_f^e)^{2/3}$. Hence, l_f^e is the dimensionless length of a hypothetical film on a perfectly nonconducting substrate, obeying the Lévéque law giving the same total heat transfer as the real film. The impact of the ratio k_S/k_F on the heat transfer process is further analyzed for a fixed Péclet number $\sigma=30$. Figure 6 shows that the effective length and the global transfer Q increase considerably with the conductivity ratio. A regression analysis shows that

$$Q = 2.44 \left(\frac{k_S}{k_F} \right)^{0.54} \quad \text{and} \quad l_f^e = 0.96 \left(\frac{k_S}{k_F} \right)^{0.81}$$

in the range $5 < k_S/k_F < 100$. Hence, the effective length is 1.5 times more sensitive to the conductivity ratio than Q . The global flux depends strongly upon the conductivity ratio and varies approximately as the square root of k_S/k_F . These results show that the hot-film gauge can be applied to the mixtures such as helium-air, because the variations of heat transfer rates depend on the conductivity of the fluid, which is a function of the composition of the binary mixture. The increase of the global heat transfer rate is obviously due to the importance of heat conduction to the substrate Q_{fS} at high conductivity ratios (Fig. 7(a)). The upstream heat transfer rate Q_{SF}^- is always larger than Q_{SF}^+ and the difference $Q_{SF}^- - Q_{SF}^+$ increase with k_S/k_F (Fig. 7(b)). Consequently, the direct transfer is weakened by the wake effect caused by upstream transfer Q_{SF}^- . The ideal Lévéque solution is never exactly reached even at low conductivity ratios such as $k_S/k_F \approx 1$.

3.3 Two-Layer Substrate. Computations were also carried out for a two-layer substrate configuration, in which a thin isolating mylar film of thickness $0.5l_f$ is sandwiched between the hot wall film and the substrate. Such a configuration exists commercially (DANTEC glue-on probe 55R47). The transfer depends on both k_{S1}/k_F (mylar-fluid) and k_{S2}/k_F (substrate-fluid). Two cases were studied, namely

- (i) $k_{S1}/k_F=1$, $k_{S2}/k_F=5.2$ (helium-mylar-glass),
- (ii) $k_{S1}/k_F=4.9$, and $k_{S2}/k_F=25.3$ (air-mylar-glass)

A low indirect transfer Q_{fS} is expected in the first case, since the ratio k_{S1}/k_F is relatively small. Case (i) is indeed somewhat similar to the water-glass combination ($k_S/k_F=1.3$). The equivalent film lengths are indeed approximately equal in these cases (Fig. 8). The indirect transfer is, however, 10% larger in case (i), compared to water-glass because of larger k_{S2}/k_F and this explains the

Fig. 7 Global heat transfer rates versus the conductivity ratio. (a) Transfer to the fluid and to the substrate. (b) Transfer rates from the substrate to the fluid upstream and downstream of the hot film.

slight differences observed in Fig. 8.

The case of air-mylar-glass is more interesting because it shows how the isolating thin film may improve the global performance of the hot film gauge. Thus, the effective length and the transfer to the substrate decrease by a factor of 2, and the direct transfer Q_{fF} is 30% larger compared with the mono-layer glass substrate. Moreover, the relationship $Q = f(\sigma^{1/3})$ tends rapidly to a Lévéque's type law $Q = A\sigma^{1/3} + B$ when the Péclet number is high enough, more rapidly with the isolating film than without. This is summarized in Fig. 9. The explanation of the decrease of the equivalent length with increasing velocity (or shear) and increasing values of the ratio k_F/k_S lies in the increased capacity of the fluid to carry away the heat conducted through the solid or, in other words, in the capacity of the fluid to cool the solid substrate heated above ambient temperature in the vicinity of the film.

The temperature at the wall decreases rapidly in a zone of radius l_f in the case of a double-layer substrate (Fig. 10). Figure 11

Fig. 8 The effective length of the hot film with a double-layer substrate

Fig. 9 Global heat flux for different substrate configurations and comparison with the Lévéque solution. For legend see Fig. 8.

shows that the local flux is also more important compared with the air-glass case. The heat flux at the trailing edge is two times larger when the substrate has a multi-layer structure.

3.4 Frequency Response. The frequency response of the hot film gauge is determined by imposing a sinusoidal shear parameter $\sigma = \bar{\sigma}(1 + a \sin \omega t)$. The periodical variations of the direct and indirect transfer rates as a function of the imposed frequency ω^* are determined for different configurations. The unsteady computations were started from the time-mean steady field and were carried out until the difference in temperature at any point was less than 0.01% between two successive imposed periods. The dimensionless parameters governing the problem are $\bar{\sigma}$, a , ω^* , k_S/k_F , α_S/α_F , k_{S1}/k_{S2} , and α_{S1}/α_{S2} with the last two parameters

Fig. 10 Wall temperature distribution for several fluid/substrate combinations

Fig. 11 Effect of the fluid/substrate configuration on the local flux for $\bar{\sigma}=30$

only entering into the consideration in the double-layer substrate configuration. To limit the number of cases considered, results for $\bar{\sigma}=30$ and $a=0.5$ are presented. The imposed frequency ω^* ranged from 10^{-6} to 10. Typical dimensions of commercial films are $l_f = 0.2$ mm, so that $\sigma=30$ in air corresponds to $u_\tau=0.5$ m/s, i.e., to a boundary layer with a free stream velocity of $\hat{u}_\infty=12$ m/s.

The code was tested by investigating the exact 1D time-dependent three-layer case of Bellhouse and Schultz [9] (F-S-F, with solid thickness l_f and with the same flow on both sides of the solid slab). Figure 12 shows the relative amplitude of the heat transfer versus frequency for the case of air and water flow over glass. The reference amplitude is that of the quasi-steady case. The oscillations of the total heat transfer rate are damped with increasing frequency because the part transferred through the solid is filtered. This part is proportionally larger in air than in water flow and is increased by the effect of longitudinal diffusion in the fluid and in the solid (which is, of course, inherently neglected in the 1D solution of Bellhouse and Schultz). Figure 12 shows also that the attenuation of the frequency response of the full simulation, which takes axial diffusion into account, starts at a frequency which is about ten times lower than in the 1D exact solution.

The anemometer measures the total heat transfer from the film to the fluid and to the solid $Q = Q_{fF} + Q_{fS}$. In the unsteady regime, there cannot be an instantaneous balance between Q_{fS} and $(Q_{SF}^- + Q_{SF}^+)$ as in the steady regime because the heat flux variations imposed at the solid surface by the velocity changes are filtered to variable degrees by the substrate, depending upon the distance to the film and the frequency.

The attenuation of the hot-film response is seen in Figs. 13 and 14 for the air-glass configuration. The cyclic variations of the relative transfer rates scaled with their time-mean values such as $Q_{fF}/\bar{Q}_{fF}$ (Fig. 13), $Q_{fS}/\bar{Q}_{fS}$, and the relative total flux $Q/\bar{Q}$ (Fig. 14) are shown versus time divided by the imposed period $P = 2\pi/\omega^*$ for respectively a low ($\omega^*=10^{-3}$) and high frequency ($\omega^*=3$). One has $\omega^* = \sigma\omega^+$, where ω^+ is the frequency scaled with the shear and the viscosity, i.e., with the inner variables of a near wall turbulent flow. Thus, $\omega^*=3$ is quite high and corresponds to $\omega^+=0.1$, which is the frequency where the energy spectrum of the velocity is maximum near the viscous layer. This is also approximately 1/15 of the Kolmogoroff frequency near the wall under these flow conditions. The low frequency $\omega^*=10^{-3}$ is nearly in the quasi-steady regime, wherein the transfer rates are in time equilibrium with the shear modulation. The real quasi-steady regime is difficult to reach, since the distance to which the heat diffuses during the period of oscillation is $\delta^2 \approx \alpha_S T$, i.e., $\delta^2 \approx (2\pi/\omega^*) \times (\alpha_S/\alpha_F) l_f^2$. The transfer through the substrate can only respond without time lag to velocity variations if $\delta \ll l_f$, i.e., if $\omega^* \ll 2\pi(l_f/l_f^2)^2(\alpha_S/\alpha_F)$. Since for air-glass combination $\alpha_S/\alpha_F = 0.017$ and $l_f^2 = 13l_f$, for $\sigma=30$, this requires $\omega^* \ll 6 \times 10^{-4}$, a very small frequency indeed. Clearly, the air-glass combination is unfavorable for two reasons: first, because of the larger transfer through the substrate due to the large value of the conductivity ratio k_S/k_F , and, second, because $\alpha_S/\alpha_F \ll 1$, which results in considerable filtering of the variations of upstream and downstream transfers via the substrate. Similar computations indicate that the quasi-steady regime is reached for $\omega^* \ll 4.7$ (water/glass) and $\omega^* \ll 9 \times 10^{-4}$ (He/mylar/glass or air/mylar/glass).

The frequency response of the direct transfer $Q_{fF}/\bar{Q}_{fF}$ is not significantly sensitive to the diffusivity and conductivity ratios. This can be seen in Fig. 13, which compares the cyclic modulation of the ideal Lévéque solution, the quasi-steady and high-frequency response of the hot film in air on a glass substrate. Figure 15 compares the direct transfer response of the air/glass with water/glass configurations for which it is recalled that the conductivity ratios vary by a factor 20. $Q_{fF}/\bar{Q}_{fF}$ is less modulated in air/glass but the difference is only a few percent. The modula-

Fig. 12 (a) Problem investigated by Bellhouse and Schultz (1968). (b) Comparison with the present numerical results. ■: Air with axial diffusion. □: Air without axial diffusion. ●: Water with axial diffusion. ○: Water without axial diffusion. △: Bellhouse and Schultz air. ◇: Bellhouse and Schultz water.

tion of the indirect transfer $Q_{fs}/\bar{Q}_{fs}$ is entirely frozen in the high-frequency regime as seen in Fig. 14, by comparing $Q_{fs}/\bar{Q}_{fs}$ in the quasi-steady $\omega^* = 1.7 \times 10^{-4}$ and the high-frequency $\omega^* = 3$ regimes. It can be easily shown that the amplitude of the global heat transfer A_Q is the sum $A_{Q_{fs}} + A_{Q_{fF}}$ if the phase shifts can be neglected, which is approximately the case here. Since in the high-frequency regime $A_{Q_{fs}} \approx 0$, the amplitude of the global transfer decreases drastically by some 60% in the high-frequency regime. For low conductivity ratios as for water/glass configuration, the decrease of the amplitude $A_{Q_{fs}}$ is only 12%, and the global transfer responds acceptably well in the high-frequency

regime (Fig. 16).

Some values of the frequency response of the wall hot film are plotted in Fig. 17 for $\sigma = 30$. The amplitude of the global flux A_Q is scaled with the amplitude in the quasi-steady regime $A_Q(\omega^* = 0)$ that was determined here at $\omega^* = 10^{-6}$. The limiting value of the attenuation is 1/5 when the mean flow parameter $\sigma = 30$ in the air-glass case. This value is in reasonable agreement with the reduced turbulent intensity of the wall shear stress measured by some authors in air flows [18]. It is seen that the amplitude is already reduced by a factor of 1/2 when $\omega^* = 3 \times 10^{-4}$, while the thin insulating mylar film improves somewhat the response com-

Fig. 13 Direct flux from the film to the fluid for air/glass case

Fig. 14 Total flux and indirect flux from the film to the substrate for the air/glass configuration

Fig. 15 Cyclic modulation of the direct flux for different configurations compared with the ideal Lvque solution

pared with the air/glass case, but this improvement is far from being totally satisfactory. The thermal boundary layer itself has a finite frequency response owing to the finite transit time over the film l_f/u_τ . But this frequency limit ($\omega^* \approx 1$) is much higher than the limit imposed by the transfer via the substrate.

To show how severe the filtering effect of the substrate may be in the air/glass case, consider a turbulent boundary layer with $\hat{u}_\infty = 12$ m/s and $Re_\delta = 5 \times 10^4$. The Kolmogoroff frequency is

roughly 5 kHz in this case. The dimensionless frequency $\omega^* = 0.1$ at which the attenuation is 1/5 is only 10 Hz, i.e., nearly three orders of magnitude smaller than the Kolmogoroff frequency.

The comparison the water/glass with the He/mylar/glass reveals the predominance of the conductivity ratio in the dynamic response rather than the diffusivity ratio. The ratio of conductivities are respectively $k_{Mylar}/k_{He} = 1$, $k_{Glass}/k_{He} = 5.2$, and $k_{Glass}/k_{Water} = 1.3$. The diffusivities vary considerably from a factor 4000 to 1000 with $\alpha_{Glass}/\alpha_{Water} = 4190\alpha_{Mylar}/\alpha_{He}$ and $\alpha_{Glass}/\alpha_{Water} = 1157\alpha_{Glass}/\alpha_{He}$. The distribution $A_Q(\omega^*)/A_Q(0)$ for the He-mylyar-glass is below the water/glass configuration because k_{Glass}/k_{He} is higher than k_{Glass}/k_{Water} , but the large variations of the diffusivities do clearly not play a significant role in the heat transfer process.

The frequency response for the water/glass combination is reasonably acceptable. The attenuation is only 20% at $\omega^* = 10$. The ratio $A_Q(\omega^*)/A_Q(0)$ is roughly 0.65 at the Kolmogoroff frequency $\omega_K^* = 2\pi f_K^* = 60$ in a turbulent fully developed water channel flow with a Reynolds number based on the half-channel height of 10^4 . Thus wall shear stress measurements in a fully developed turbulent water channel flow by a hot film on glass can be reliably performed, except maybe at the smallest turbulent scales. Recall that the satisfactory frequency response of the case water/glass does not mean that there is no parasitic heat conduction to the substrate. On the contrary, the ratio Q_{fs}/Q_{fF} varies between 1 and 3/5 as seen in Fig. 5(a). Yet, this ratio reaches values as large as 10 in the case air/glass (Fig. 5(b)). The heat loss ratios of roughly $Q_{fs}/Q_{fF} \approx 1$ do not affect considerably the frequency response at least up to $\omega^* = 1$.

4 Conclusion

A numerical investigation of heat transfer of wall-mounted hot film gages shows that heat transferred to the fluid via the substrate has important effects. For high values of the conductivity ratios k_S/k_F , the transfer from the film to the substrate may be considerably larger than the direct transfer to the fluid. For the air/glass combination, the transfer to the fluid through the substrate is about 10 times larger than the direct transfer from the film. Because of the hot wake, the downstream transfer from the substrate to the fluid is roughly half of the upstream value.

The importance of conduction into the substrate increases with decreasing velocity. A thin layer of material with low conductivity inserted between the film and substrate attenuates this effect.

The frequency response of the total heat transfer from the film, as measured by an anemometer set, is dominated by the filtering effect of the transient heat transfer via the substrate when $k_F \ll k_S$ and $\alpha_F \gg \alpha_S$. For the air/glass combination, the response is attenuated as soon as $\omega^* > 10^{-6}$. This limit is much lower than the cutoff frequency of the thermal layer.

There is a limit of the spanwise length l_{fz} of the hot film probes to avoid spanwise averaging of the turbulence producing structures in the near wall region. According to Blackwelder and Hariotonidis [19], this requires $l_{fz}^+ = l_{fz}(u_\tau/\nu) < 20$. The streamwise sensing length of some commercial wall hot-film gages is, $l_f = \frac{1}{8}l_{fz}$ to prevent both the spanwise conduction effects and to ensure two-dimensionality. Therefore, the previous condition gives $l_f^+ < 2.5$. The corresponding Pclet number concerning the measurements of the wall shear stress and wall shear stress intensity in water is $\sigma = Pr l_f^+ < 44$. Clearly the axial diffusion through the leading and trailing edges singularities has to be taken into account under these circumstances. Note that the estimations given above constitute an upper limit. More careful measurements require at least $l_{fz}^+ < 10$ and $\sigma < 10$. Neglecting the axial diffusion term in the energy equation when the Pclet number is small may generate serious errors in predicting the frequency response of the hot film. These points have to be taken into account in MEMS sensor designs.

Fig. 16 Modulation of the total and indirect fluxes for the water/glass configuration

Fig. 17 Frequency response of the hot-film gage in different fluid/substrate configurations. : Water-glass. : Helium-mylyar-glass. : Air-mylyar-glass. : Air-glass. : Air-glass.

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Nomenclature

A_q	= amplitude of the quantity q
a_q	= $A_q/\bar{q}$, relative amplitude of the quantity q
a	= relative amplitude of the shear parameter
$h(\hat{x})$	= local heat transfer coefficient
H	= $\int_0^{l_f} h(\hat{x}) d\hat{x}$, global heat transfer coefficient
K	= number of grid points over the hot film
k	= conductivity
l_f	= streamwise length of the hot film
l_f^e	= $(Q/Q_{L\acute{e}v\acute{e}que})^{3/2}$, dimensionless equivalent hot-film length
Nu	= H/k_F , Nusselt number
P	= $2\pi/\omega^*$, dimensionless period of the imposed velocity oscillations
Pr	= ν/α , Prandtl number
Q_{fF}	= dimensionless direct total heat flux from the film to the fluid
Q_{fS}	= dimensionless heat flux from the film to the substrate
$Q_{L\acute{e}v\acute{e}que}$	= $0.81\sigma^{1/3}$, dimensionless direct heat flux over the film of the L\acute{e}v\acute{e}que solution.
Q_{SF}^-	= dimensionless heat flux from the substrate to the fluid upstream of the hot film
Q_{SF}^+	= dimensionless heat flux from the substrate to the fluid downstream of the hot film
Q	= $Q_{fF} + Q_{fS} = \int_{Film} q_w dx$, dimensionless hot-film global heat transfer rate per unit span
$\bar{q}$	= time mean of the quantity q
q^+	= quantity q nondimensionalized with the shear velocity u_τ and viscosity ν
q_w	= $-(\partial\theta/\partial y)_{y=0}$, local heat flux
q_w^*	= $-(\partial\theta/\partial y)_{y=0}\sigma^{-1/3}$, dimensionless local heat flux scaled with the shear parameter
Re_δ	= $\hat{u}_\infty\delta/\nu$, turbulent boundary layer Reynolds number
Re_f	= $l_f u_\tau/\nu$, hot film Reynolds number
T	= temperature
$\hat{t}$	= dimensional time
t	= $(\hat{t}/l_f^2)\alpha_F$, dimensionless time
$\hat{u}$	= dimensional velocity
u	= $\hat{u}/u_\tau$, dimensionless velocity
u_τ	= $\sqrt{\tau/\rho_F}$, shear velocity
$\hat{x}$	= dimensional streamwise distance
x	= $\hat{x}/l_f$, dimensionless streamwise coordinate
$\hat{y}$	= dimensional wall normal distance
y	= $\hat{y}/l_f$, dimensionless wall normal coordinate

Greek symbols

α	= diffusivity
δ	= boundary layer thickness
η	= wall normal stretched coordinate
ν	= kinematic viscosity
ρ	= density

σ	= $\Pr(l_f u_\tau/\nu)^2$, shear parameter
$\bar{\sigma}$	= time-mean shear parameter
θ	= $(T-T_\infty)/(T_f-T_\infty)$, dimensionless temperature
τ	= wall shear stress
$\hat{\omega}$	= dimensional angular frequency of the imposed oscillations
	= dimensionless angular frequency
ω^+	= $\hat{\omega}(\nu/u_\tau^2)$, imposed frequency in wall units.
ξ	= streamwise stretched coordinate

Abbreviations

F	= fluid
S	= substrate
WHF	= wall hot film

Subscripts

F	= fluid
S	= substrate
f	= hot film
∞	= bulk

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