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REDDUCTION OF MULTISTAGE ROTOR MODELS USING CYCLIC MODESHAPES

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ABSTRACT
This study deals with the reduction of models of bladed disks assemblies through the use of mono-harmonic cyclic modeshapes. The first part of this paper describes how the classical substructuring technique in cyclic symmetry can be extended to rotors in the case of non compatible meshes to compute mono-harmonic eigenvectors. The second part of the paper then presents the method employed by the authors to seek the full modes of the rotor in a subspace generated from a set of such modeshapes. All the concepts developed here are illustrated on a representative sample.

NOMENCLATURE

I Interface
D Disk
R Intermediate ring
S Sector
[c] Observation matrix
[C] Coupling/damping matrix
[E] Harmonic recovery matrix
[I] Identity matrix
[K] Stiffness matrix
[M] Mass matrix
[Z] Dynamic stiffness matrix
[Φ] Matrix of eigenvectors
[T] Basis of a subspace

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INTRODUCTION

The current industrial practice is to model separately each bladed disk composing a rotor. In the particular case where the disk is tuned, i.e. it presents a property of cyclic symmetry, a Fourier decomposition of the disk response is performed and the response on the whole disk is decomposed into periodic components defined on one sector [1–3]. When slight deviations occur from blade to blade due to manufacturing processes or service wear, i.e. if mistuning is present, full disk models have to be considered to correctly represent the variations in the response of the mistuned disk with respect to that of the tuned disk. This leads to huge finite element matrices, as recent problems have up to ten million DOF per disk. Since such computations are impractical, many reduction techniques have been considered as recently reviewed by Castanier and Pierre [3]. They often include methods derived from the tuned system analysis coupled with either a Component Mode Synthesis (CMS) method [4–7] or a non-CMS method [8–10], assuming in every case that mistuned modes are given by projecting the individual blade mistuning onto the tuned system modes. In the case of large mistuning, the CMS technique has been improved by building a reduction basis made of tuned system normal modes and blade static or quasi-static modes [11].

The question is model the behaviour of such disks when assembled together to form a rotor. Bladh et al. [12] have studied the influence of mistuning on such multistage assemblies, underlining that one of the critical issue in such problems is the choice of the boundary conditions to take the inter-disk coupling into account. The latter was shown as mandatory, as it induces a specific form of mistuning, called rotor mistuning, that moves drastically the vibratory characteristics of the rotor away from those of the individual tuned disks. To perform such studies at a reasonable cost, Laxalde et al. [13] extended the classical cyclic symmetry approach to the case of such assemblies of tuned disks, with a strong assumption that the disks have compatible interfaces, which vanishes in practical blade disk design.

In the first section of this paper, the authors briefly recall some interesting features of the dynamics of rotors made of tuned disks and pay particular attention to the harmonic content of the modes.

In the second part of the paper, a set of fixed diameter solutions of the coupled rotor problem are built. The case studied is that of bladed disks whose individual meshes are imposed by design requirements such as the number and the geometry of the blades. This often leads to incompatible meshes between adjacent stages, and a displacement coupling as proposed by Laxalde poses a problem. The use of an intermediate rim mesh dealing with this incompatibility is detailed here. This intermediate ring gets rid of the issue of the choice of the proper boundary conditions as pointed out in [12].

In the last section, a very specific reduction strategy is employed to restore the modes of the full rotor. The approximate solution is sought in a subspace built from a set of cyclic modes completed with sector modes with fixed interfaces. This techniques is very interesting because it requires only a few vector manipulations.

An evaluation of the accuracy of the chosen methodology is conducted throughout the paper for a simplified but representative two-stage rotor model.

DYNAMICS OF MULTISTAGE ASSEMBLIES

In this section, one will consider a rotor that can be decomposed in two disks $D^1$ and $D^2$. As an illustration, the sample rotor presented in Fig. 1 will be used throughout the rest of this paper. It is composed of two disks with 12 and 15 blades meshed with brick elements connected through an intermediate ring composed of tetrahedrons. The material is titanium whose properties are $E = 120$ GPa, $v = .33$ and $\rho = 4700 \text{ kg.m}^{-3}$. The radius at the roots of the blades are 200 mm and 205 mm respectively, the radius at their tips are 280 mm and 285 mm respectively. Their thickness varies from 6 mm at the root to 2 mm at the top. Their widths are 43 mm and 46 mm at the top. The frequency bandwidth of interest for this study is $[0, 14000 \text{ Hz}]$.

Coupled problem

The proposed methodology assumes the existence of a single mesh of the whole shaft but where each disk is meshed by rotating its nominal sector. This typically leads to an incompatible mesh at the interfaces $T^1_a$ and $T^2_a$.

Rather than trying to introduce an approximate continuity constraint, one assumes that the sectors are meshed in such a way that there is a ring between the two disks that corresponds to a physical part of the shaft. One then enforces nodes of the disk interfaces, $T^1_a$ and $T^2_a$, to be part of the ring mesh so that there is no other node in the ring mesh. If the dimensions of the ring section are similar to the characteristic mesh length in the sectors, the ring mesh can be generated automatically whatever the case.
by a Delaunay triangulation performed with built-in functions in MATLAB. This procedure aims to build tetrahedrons from a given node of any of the two rims, \( J_1 \) and \( J_2 \), and its three nearest neighbours. The result is a ring mesh with coincident nodes for \( J_1 / I \) and \( J_2 / I \).

Since the ring model is constrained to have no interior nodes, the shaft model DOFs are the combination of those of the disks. In the implementation of this procedure, the ring matrices are assembled separately. One thus needs to generate a localization matrix relating the ring DOFs to those of the two disks. This can be done using a general observation equation

\[
\begin{bmatrix} q^R \\ q^a \end{bmatrix} = D \begin{bmatrix} q^1 \\ q^2 \end{bmatrix}
\]

(1)

The shaft equations are then obtained by a simple assembly of the disk matrices (which are disjoint since the disks share no DOFs) and a ring matrix which can be localized into the disk DOFs using the observation equation (1). This thus leads to dynamic equations, in the frequency domain, of the form

\[
[Z(\omega)]_{N \times N} \{q\}_{N \times 1} = \{f(\omega)\}_{N \times 1}
\]

(2)

with

\[
Z = \begin{bmatrix} Z^1 & 0 \\ 0 & Z^2 \end{bmatrix} + \begin{bmatrix} c_d^1 & 0 \\ 0 & c_d^2 \end{bmatrix}^{\top} \begin{bmatrix} Z^R \\ 0 \end{bmatrix} \begin{bmatrix} c_d^1 & 0 \\ 0 & c_d^2 \end{bmatrix}
\]

(3)

where \([Z^1]\), \([Z^R]\) and \([Z^2]\) are the dynamic stiffnesses of \( D^1 \), \( R \) and \( D^2 \) respectively.

### Analysis of the Free Response

As underlined [13] and [12], dynamics of multistage rotors are more complex than simply superimposing the individual dynamics of each bladed disk. The idea proposed in [13] is to solve Eqn. (2) for rotor shapes that have a single Fourier harmonic. This was done using a mesh compatibility assumption which no longer poses a problem when using an intermediate ring as proposed here.

The fact that the modes of a single tuned disk are associated with a single Fourier/Floquet harmonic is due to the invariance of the structure by rotations of one sector. Group theory can then be used to prove specificities of modes which have to be invariant by certain group transformations. For multi-stage assemblies, this structural invariance no longer holds and the true modes are not mono-harmonic.

The multi-harmonic nature of particular modes will be very much linked to the level of coupling between various disks and the non-coincidence of various diameter modes in each disk. To analyze this idea, one computed the 200 first modes of the full system, whose frequencies are comprised between 0 and 14000 Hz.

To have a quick overview of the participation of each stage, the author used a relative participation factor such as introduced in [12]. For each rotor mode, this factor gives the fraction of strain energy that lie in each bladed disk:

\[
R^d = \frac{\{q^d\}^{\top} [K] \{q^d\}}{\{q\}^{\top} [K] \{q\}}
\]

(4)

The resulting curve giving \( R^d \) is displayed in Fig. 2. In this figure, one can clearly see that there are only a few families of modes localized on a single disk.

To investigate further in this direction, the rotor modes are then decomposed into a Fourier Series associated with each periodic group of nodes for each direction of translation. Given a mode, each Fourier coefficient is normalized with respect to the maximum to indicate where the motion is localized. In general, one would expect that high diameter modes which have little disk coupling should be mono-harmonic and low diameter modes to be less so. The result is that it is quite difficult to find a systematic trend. Thus Fig. 3 shows a coupled mode that is nearly mono-harmonic and a localized mode that is clearly multi-harmonic. Nevertheless, modes still come in pairs with very close frequencies and modeshapes. On the contrary, modes that could be thought having \( \delta = 0 \) always come single.
a single harmonic δt valued DOF vector.

Theory

with an overestimation of eigenvalues. Modeshape estimates will be approximations of the true modes Rayleigh's theorem on constraints [14] tells us that the resulting localized modes that have a single Fourier harmonic. This was done using MONO-HARMONIC EIGENVECTORS

Figure 3. Multi-harmonic localized mode at 4250 Hz (up) and mono-harmonic coupled mode at 2987 Hz (down) and their harmonic content.

The idea proposed in [13] is to solve Eqn. (2) for rotor shapes that have a single Fourier harmonic. This was done using a mesh compatibility assumption which no longer poses a problem when using an intermediate ring as proposed here. Nevertheless, assuming a mono-harmonic response is a form of constraint. Rayleigh's theorem on constraints [14] tells us that the resulting modeshape estimates will be approximations of the true modes with an overestimation of eigenvalues.

Theory

Let us consider one of the disks 𝒟d. With a classical cyclic symmetry approach [1–3], a real DOF vector {q^d} that contains a single harmonic δ is restored on a full disk 𝒟d from a complex valued DOF vector {q_0^d} defined on the initial sector S^{d,0}. One thus has, in the local coordinate system of any sector S^{d,x},

\[ \{q^{d,x}\} = \Re \left( \{q_0^d\} e^{jx\delta} \right) \] (5)

One defines a recovery matrix \[ E^d(\delta) \] such that

\[ E^d(\delta) = \left( \begin{array}{c} 1 e^{j\delta} \ldots e^{j(N^d_s-1)\delta} \end{array} \right) \otimes [I_{N^d_{r,0}}] \] (6)

\[ [I_{N^d_{r,0}}] \] is an identity matrix whose size is the number of physical DOF of sector S^{d,0}. ⊗ is the Kronecker product. This allows to restore the eigenvector of harmonic δ on the whole disk by separating its real and imaginary parts

\[ \{q^d\} = \left[ \Re(E^d(\delta)) \; \Im(E^d(\delta)) \right] \left\{ [\Re(\hat{q}_0^d)] \; [\Im(\hat{q}_0^d)] \right\} \] (7)

The reduced dynamic stiffness matrix is projected onto the subspace generated by the mono-harmonic solution (7) as described in [15]:

\[ \{q^d\}^T [Z^d] \{q^d\} = \frac{N^d_s}{2} \left( \begin{array}{c} [\Re(\hat{q}_0^d)] \; [\Im(\hat{q}_0^d)] \end{array} \right) \left[ \begin{array}{cc} Z^{d,0} & 0 \\ 0 & Z^{d,0} \end{array} \right] \left( \begin{array}{c} [\Re(\hat{q}_0^d)] \; [\Im(\hat{q}_0^d)] \end{array} \right) \] (8)

For a mono-harmonic multi-stage solution, each disk model is reduced using (7), the rotor model (3) thus becomes

\[ \left[ \hat{Z}_8 \right] = \left( \begin{array}{c} \Re(\hat{q}_0^d) \\ \Im(\hat{q}_0^d) \\ \Re(\hat{q}_0^d) \\ \Im(\hat{q}_0^d) \end{array} \right) = \left\{ \hat{q}_8 \right\} \] (9)

with

\[ \left[ \hat{Z}_8 \right] = \frac{1}{2} \left[ \begin{array}{cccc} N^1_s Z^{1,0} & 0 & 0 & 0 \\ 0 & N^1_s Z^{1,0} & 0 & 0 \\ 0 & 0 & N^2_s Z^{2,0} & 0 \\ 0 & 0 & 0 & N^2_s Z^{2,0} \end{array} \right] + \left[ \hat{Z}_R^8 \right] \] (10)

\[ [\hat{Z}_R^8] \] is obtained by restoring \textit{a priori} the modeshapes to the DOF set of the intermediate ring:

\[ \left[ \hat{Z}_R^8 \right] = \left[ E^d_1(\delta) \; 0 \right]^T [Z^R] \left[ E^d_2(\delta) \; 0 \right] \] (11)

\[ [E^d_1(\delta)] \] and \[ [E^d_2(\delta)] \] are obtained from Eqn. (7) substituted in Eqn. (1).

The presence of the number of blades of each disk in (10) is due to the fact that because all the energy contained in the intermediate ring is counted, it is mandatory to count all the energy that lies in the disks.

Equation (9) is incomplete without the additional intersector continuity conditions that impose a phase shift between adjacent
sectors by $2\pi\delta/N_d^d$ for each disk. This leads to a constraint equation applied to the generalized DOFs of each disk [16]:

$$\begin{bmatrix} c_1 \cdots c_i \cdots c_N^d \end{bmatrix} - e^{i\delta\alpha^i} \begin{bmatrix} c_1^i \cdots c_i^i \cdots c_N^i \end{bmatrix} \begin{bmatrix} \hat{q}_8^i \end{bmatrix} = \begin{bmatrix} 0 \cdots 0 \end{bmatrix} \quad (12)$$

$[c_1^i]$ and $[c_i^i]$ are observation matrices that isolate left-interface and right interface DOF of a given sector.

The previous equations are valid in the general case but some particular cases have to be underlined:

- $\delta = 0$ is a particular case where $\{\hat{q}_8^i\}$ is real. Thus one has

$$\{q^i\}^\top [Z^i] \{q^i\} = N_d^d \{q_8^i\}^\top [Z^d,0] \{q_8^i\} \quad (13)$$

The reduced rotor model is then described by

$$\begin{bmatrix} \bar{Z}_0 \end{bmatrix} \begin{bmatrix} \hat{q}_8^i \end{bmatrix} = \{\hat{f}_0\} \quad (14)$$

with

$$\bar{Z}_0 = \begin{bmatrix} N_s^1 Z^{1,0} & 0 \\ 0 & N_s^2 Z^{2,0} \end{bmatrix} + \tilde{Z}_0^R \quad (15)$$

The a priori restitution on the intermediate ring is given by Eqn. (11) with $\delta = 0$. The inter-sector continuity constraint takes the form

$$\begin{bmatrix} c_1 \cdots c_i \cdots c_N^d \end{bmatrix} - e^{i\delta\alpha^i} \begin{bmatrix} c_1^i \cdots c_i^i \cdots c_N^i \end{bmatrix} \begin{bmatrix} \hat{q}_8^i \end{bmatrix} = \begin{bmatrix} 0 \cdots 0 \end{bmatrix} \quad (16)$$

- The other particular cases occur when $\delta = N_d^d/2$, if $N_d^d$ is even. In this case, $\{q_8^i\}$ is real for the concerned disk only. Suppose that $\delta = N_s^1/2$, the reduced rotor model is

$$\begin{bmatrix} \bar{Z}_8 \end{bmatrix} \begin{bmatrix} \hat{q}_8^i \\ \Re(\hat{q}_8^i) \\ \Im(\hat{q}_8^i) \end{bmatrix} = \{\hat{f}_8\} \quad (17)$$

where

$$\begin{bmatrix} \bar{Z}_8 \end{bmatrix} = \begin{bmatrix} N_s^1 Z^{1,0} & 0 & 0 \\ 0 & N_s^2 Z^{2,0} & 0 \end{bmatrix} + \tilde{Z}_8^R \quad (18)$$

The a priori restitution on the intermediate ring is still given by Eqn. (11) with $\delta = N_s^i/2$ where the imaginary part of $[E^1(\delta)]$ is zero. The constraint equation becomes

$$\begin{bmatrix} c_1 \cdots c_i \cdots c_N^d \end{bmatrix} - e^{i\delta\alpha^i} \begin{bmatrix} c_1^i \cdots c_i^i \cdots c_N^i \end{bmatrix} \begin{bmatrix} \hat{q}_8^i \end{bmatrix} = \begin{bmatrix} 0 \cdots 0 \end{bmatrix} \quad (19)$$

Illustration on the Sample Rotor

To illustrate this, the mono-harmonic modeshapes with $\delta$ comprised between 0 and 7 were computed in the frequency range $[0, 14000 \text{ Hz}]$. The classical frequency versus harmonic coefficient curve displayed in Fig. 4 sums up the results obtained. The rigid body modes are clearly seen in this figure.

![Illustration on the Sample Rotor](image)

The assumption in [13] is that rotor modes are mono-harmonic. While we showed earlier that is not true for all modes, many true rotor modes are nearly mono-harmonic and can thus be compared to a mono-harmonic modeshape given by one of the problem described above. For instance, Fig. 5 presents a sample result in the latter case that confirms the previous statement where two rotor modes at 4250 Hz and 4251 Hz are very well correlated with two cyclic eigenvectors with $\delta = 2$ at 4249 Hz. The comparison is made using a standard Modal Analysis Criterion, which quantifies the shape correlation between two given modes:

$$MAC(u, v) = \frac{\{u\}^\top \{v\}}{||\{u\}||^2 ||\{v\}||^2} \quad (20)$$
On the other hand, it is obvious that multi-harmonic rotor modes, such as the mode at 2987 Hz presented in Fig. 3, cannot be represented properly with a single mono-harmonic mode-shape.

REDUCED ORDER MODEL OF THE ROTOR

In order to restore the multi-harmonic modes of the rotor, a strategy to reduce the rotor model (3) by seeking its response in a subspace generated by a set of cyclic modes-shapes is presented in this section.

Methodology

In the previous section, the use of an intermediate ring to enforce disjoint DOF sets made the reduction of the mono-harmonic rotor model quite simple, since reduction due to symmetry constraints could be applied separately for each disk. Rather than reusing the mono-disk strategy proposed in [16], it is thus proposed here to introduce inter-sector interface elements. Taking the nominal sector of each disk, one divides, as shown in Fig. 6, the model in an interface set, whose elements contain at least one node of the left sector interface, and a sector set, whose elements contain no node of the left interface.

When building the full rotor model, elements in the interface set are included in the ring model, while the disk models now only contain the repetition of elements in the sector set. The resulting interface thus contains the inter-sector elements and inter-disk rings, as illustrated in Fig. 7.

The form of equations is still (3), but [Z^R] now also accounts for interface elements, and the contributions of these elements are now removed from [Z^1] and [Z^2].

The principle of the proposed rotor reduction is to reduce each sector model (that no longer contains inter-sector interface elements) in such a way that the rotor model will contain exactly a set of target fixed sector modes (to represent blade motion) and target mono-harmonic modes of the tuned rotor (introduced in the previous section). One thus starts with a set of vectors defined on the reference sector of each disk.

Figure 5. Pair of mono-harmonic rotor modes (left), corresponding cyclic eigenvectors with δ = 2 (right) and their correlation (center)

Figure 6. Sector superelement (blue) and intersector elements (red)

Figure 7. Rotor model with interface elements in red
These vectors are often not independent so that using them as a reduction basis would lead to poor numerical conditioning. Rather than traditional orthogonalization with respect to mass and stiffness, the Orthogonal Maximum Sequence scheme [17] was used here. While originally designed for sensor placement, this scheme proves to be faster and has the significant advantage to return generalized coordinates that are DOFs of the original model. It is however important to place fixed interface modes firsts since the result is dependent of the order of the initial vectors.

Application to the Sample Rotor

The sample rotor displayed in Fig. 7 is used, where disk 1 has 12 blades and disk 2 has 15 blades. The full mesh initially counts 12636 DOF, but the computations of the cyclic modes shapes only involve two sectors with 552 physical DOF each divided into a sector set with 468 DOF and an interface set with 84 DOF.

The initial vector set is composed of all sector modes with 468 DOF and an interface set with 200 reference modes only with one less family of cyclic modes. The latter results are compared to those obtained when the cyclic modes are taken in the frequency range \([0, 14000] \text{Hz}\). For each harmonic it gives 14 cyclic modes shapes, to which must be added 2 rigid body modes shapes with \(\delta = 0\) (1 rotation and 1 translation) and 4 rigid body modes shapes with \(\delta = 1\) (2 translations and 2 rotations). After orthonormalization of the vector sets \([M_d^{\text{d0}}]\) and \([M_d^{\text{d2}}]\), the superelements describing the initial sectors of disk 1 and disk 2 have 164 and 164 DOF respectively, leading to a full assembled problem that has only 4428 DOF. The projections of the matrix problem lead to the matrices whose some components are already displayed in Fig. 8. The results obtained with this technique are excellent: Fig. 9 displays the MAC error in the frequency range \([0, 7000] \text{Hz}\). For each harmonic it gives 14 cyclic modes shapes, to which must be added 2 rigid body modes shapes with \(\delta = 0\) (1 rotation and 1 translation) and 4 rigid body modes shapes with \(\delta = 1\) (2 translations and 2 rotations). After orthonormalization of the vector sets \([M_d^{\text{d0}}]\) and \([M_d^{\text{d2}}]\), the superelements describing the initial sectors of disk 1 and disk 2 have 164 and 164 DOF respectively, leading to a full assembled problem that has only 4428 DOF. The projections of the matrix problem lead to the matrices whose some components are already displayed in Fig. 8. The results obtained with this technique are excellent: Fig. 9 displays the MAC error in the frequency range \([0, 14000] \text{Hz}\), which corresponds to 200 reference modes, each one being paired with one mode computed with the reduced-order model.

The previous results are compared to those obtained when the cyclic modes are taken in the frequency range \([0, 4000] \text{Hz}\), that is with one less family of cyclic modes. The latter results

\[
\begin{align*}
[T_{d_{\text{init}}}^{d}] &= \begin{bmatrix}
\Phi_{f_{\text{d0}}}^{d} & \mathfrak{H}([\Phi_{rcyc}^{d}]) & \mathfrak{I}([\Phi_{rcyc}^{d}])
\end{bmatrix}
\end{align*}
\]  

(21)

where the generalized DOF are related to the physical DOF by

\[
\{q\}_{N\times1} = [M_{\text{d1}}^{\text{d0}}]_{N \times N_d} \{q_{d1}\}_{N_d \times 1}
\]  

(25)
The generalized DOF can be divided into groups associated with one sector of each disk so that a very graphical representation of the modes can be used for direct observations. Moreover, the harmonic content of the generalized modes can be analyzed. As an illustration, one considers the two modes that approximate the real modes displayed in Fig. 3. The representation of the amplitude of each DOF, displayed in Fig. 11, corresponds to the mono-disk multi-harmonic mode at 2987 Hz. The harmonic content is not easily distinguishable on this figure, whereas it can give very useful indications about where the motion is located. It also gives a very interesting knowledge of which vectors of the basis are involved. Notice in Fig. 11 that only a few DOF are used. These DOF are directly related to the set of sector modes with fixed interface, stating that the true mode with harmonics 4, 5 and 6 is well represented by such modes, since it seems that it has almost no disk motion, as can be seen in Fig. 3.

To quantify the harmonic content of the reduced modes-shapes, the procedure applied to the modeshapes of the full assembly is used. The harmonic content of the reduced two modes displays above is given in Fig. 10 and corresponds exactly to that already displayed in Fig. 3. These excellent results encourage the author to focus on the forced response of the bladed disk assemblies, especially by dealing with excitations whose engine order is different from a disk to another.
CONCLUSIONS
In this paper, the authors extended the cyclic symmetry concepts, first introduced in [13], to rotor models for which the nominal design of the individual bladed disks leads to incompatible meshes. From a deep study of the dynamics of such assemblies, it was shown that true rotor modes are not generally mono-harmonic, because in this case, the group theory that applies to each single tuned disk vanishes. This was used as the starting point of a new reduction technique that aims to seek the rotor modes as linear combinations of a set of mono-harmonic cyclic modeshapes computed for a model describing a sector of the rotor.

The results obtained for a very simple but representative sample are excellent and stimulate the authors to engage further investigations in computing the forced response of rotors, where each stage is exposed to a particular engine order, due to its immediately preceding stator vanes.

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REFERENCES