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Experimental study on wave breaking and mixing properties in the periphery of an intense vortex

F.Y. Moulin¹, J.-B. Flór*

Laboratoire des Ecoulement Geophysiques et Industriels, LEGI-CNRS, B.P. 53, 38041 Grenoble Cedex 9, France

We experimentally investigate the effects of the breaking of inertia-gravity waves on the dynamics and transport properties of a vortex in a rotating stratified fluid. A tall cyclonic vortex with vertical vorticity axes, and Gaussian velocity profile is perturbed by planar waves, emitted by a horizontally oscillating cylinder. We show that the trapping and breaking of the waves inside the vortex lead to a deposit of negative momentum in the periphery of the vortex. Periodic waves destabilize the streamlines and chaotically mix the outer part of the vortex while the core remains coherent. In case waves are trapped, and break, mushroom-like structures form that may transport fluid in the radial direction. The two different mixing processes are discussed. Further, we consider the effects of vortex strength and wave-energy on the mixing.

Keywords: Rotating stratified flows; Wave–vortex interaction; Mixing

1. Introduction

In large-scale geophysical flows, the dominant presence of background rotation and stratification not only prompts the inverse energy cascade towards large scales leading to the

* Corresponding author. Fax: +33 4 7682 5271.

E-mail address: flór@hmg.inpg.fr (J.-B. Flór).

¹ Present address: ORI, The University of Tokyo, 1-15-1 Minamidai Nakano-ku, Tokyo 164-8639, Japan.

formation of large-scale vortices, but also support waves, e.g. generated during geostrophic adjustment. The interaction of waves with shear layers in rotating stratified fluids have been studied in detail experimentally, numerically and analytically. In studies on the interaction of waves with a jet, reflection, absorption or breaking of the waves occurred as shown experimentally (e.g. Koop, 1981), theoretically (Booker and Bretherton, 1967; Ivanov and Morozov, 1974; Olbers, 1981; Badulin et al., 1984; Badulin and Shrira, 1993) and in numerical studies by Dornbrack (1998) and Staquet and Huerre (2002). Wave–vortex interaction is a very rich and relatively unexplored subject of study. In the context of the polar vortex and the transport of ozone depleting radicals, the effect of wave breaking and the transport of material across the edge of the polar vortex, have been discussed by McIntyre (1995). Recently, Bühler and McIntyre (2003) have investigated wave–vortex interactions in the shallow-water approximation and showed that the vortex core of a potential vortex is advected by horizontally refracted surface waves. Here, we focus on the wave-induced mixing within the vortex and the related vortex dynamics. The breaking of waves inside a jet or a vortex results from an accumulation of energy of the waves trapped in the background flow. Such a trapping process is well predicted by the WKB theory (see Bretherton, 1966), which, to summarize, assumes that the phase of the wave evolves on time and space scales shorter than the scales associated with the jet or vortex. The time dependence of the wave field is given by its absolute frequency, ω_{abs} , corresponding to the wave frequency before the beginning of the interaction. During the interaction, this time dependency is conserved, and the intrinsic frequency ω_0 , defined as its frequency in a referential moving with the local mean flow velocity $\mathbf{U}$, is inferred from the Doppler shifting relationship that reads

$$\omega_{\text{abs}} = \omega_0 + \mathbf{k} \cdot \mathbf{U}, \quad (1)$$

where k is the wave vector. In the WKB theory, ω_0 satisfies the dispersion relationship for inertia-gravity waves, $\omega_0^2 = (k_h^2 N^2 + k_z^2 f^2)/(k_h^2 + k_z^2)$, where N is the buoyancy frequency of the stratified fluid, f the Coriolis parameter, and k_h and k_z the horizontal and vertical components of the wave vector, respectively.

In the case of interaction of a planar monochromatic wave with a jet, ω_0 is a function of the spatial coordinates only. Space is then divided into regions where waves can propagate or are arrested. For a barotropic jet (without vertical shear), waves approaching the layer $\omega_0 = N$ are trapped, the component of the wave number perpendicular to the layer increasing towards infinity. The accumulation of energy combined with the decrease in wavelength eventually makes the wave overturning and breaking. Here we consider interactions of inertia-gravity waves with a tall baroclinic vortex in a rotating linearly stratified fluid.

For the interaction of a planar monochromatic wave with a vortex, the equations of the WKB theory have to be solved numerically in cylindrical coordinates (see Moulin and Flór, 2004). In the case of the interaction with tall vortices, the incident wave field is partially trapped in a broad region. This is in contrast to the trapping in a unique layer predicted by the WKB theory for a barotropic jet. In addition, the wave breaking is observed only in experiments where the time scale associated with the trapping is far shorter than the time scale of the viscous damping of waves (see Moulin and Flór, 2004). For this condition to be fulfilled, we need stable vortices with a high level of shear. In experiments with isolated vortices, having net zero circulation at a finite distance, the shear values were far too low for wave breaking to occur, while highly sheared isolated vortices are unstable (see e.g. Carton

et al., 1989). Therefore, vortices were generated by siphoning off fluid through a perforated tube. These vortices are stable and confined to a small radius so that the shear is relatively high.

In the present paper, we investigate experimentally the breaking of waves and its effects on the dynamics of such a vortex and on the radial transport of material. The results presented here and in Moulin and Flór (2004) as well as related subjects on vortex–wave interactions are also presented in Moulin (2002). In Section 2, we describe the experimental methods and the modelling of the vortex, followed in Section 3 by experimental observations of the wave trapping and of a deposit of momentum inside the vortex due to the wave breaking. Section 4 reports on the transport of material driven by the wave–vortex interaction and the breaking of waves. Conclusions are drawn in Section 5.

2. Experimental procedure

Experiments were carried out in a 1 m^2 tank of 45 cm depth, mounted on a rotating table. A side view of the experimental set-up is sketched in Fig. 1. The tank was filled with a linearly salt-stratified fluid. The Coriolis parameter $f = 2\Omega$ varied between 0.4 and 0.5 rad s^{-1} , and the buoyancy frequency N of the water solution, defined as $N^2 = -(g/\rho_0)\partial\rho/\partial z$, varied between 1.15 and 1.5 s^{-1} . Experimental values of N were calculated measuring the density at different depths in the tank with a refractometer.

Fig. 1. Sketch of the experimental set-up.

To measure the horizontal velocity field, the motion of neutrally buoyant particles of 70 μm diameter, illuminated by a horizontal laser sheet, was recorded by a B&W camera from the top and processed with the CIV method developed by Fincham and Delerce (2000). Values of the vertical vorticity and the horizontal divergence were then calculated using their algorithms. To observe the mixing induced by wave–vortex interaction, we introduced fluorescein dye inside the vortex, and illuminated the flow either by a horizontal or a vertical laser sheet.

To generate an elongated vortex, a 1.5 cm diameter tube which was perforated over a vertical extent of 10 cm, was introduced vertically at the centre of the tank. A radial inflow was generated by siphoning off fluid with a pump, leading to a cyclonic vortex by conservation of angular momentum. In order to siphon off an equal amount of fluid at each depth, a 15-mm diameter tube was filled with seven small tubes of about 2 mm in diameter that ended at different levels in the tube. The circulation $\Gamma = \int U_\theta(r)dr$, with U_θ the azimuthal velocity, depends on the total volume, V , pumped through the tube. After removing the tube, this forcing results in a vortex with a Gaussian vorticity profile at mid-depth, which slowly diffuses radially outwards.

In the inviscid approximation, an inwards radial flow that is uniform over a vertical extent of $H = 10$ cm and of a total volume V , would yield a vortex with circulation $\Gamma = fV/H$. Measurements of Γ in the mid-plane just after the removal of the tube were found to be lower than this theoretical value, most likely, because we underestimated the vertical extent of the pumped fluid which is larger than the 10 cm distribution of holes.

In order to reproduce the evolution of the vortex structure, the diffusion equation for momentum was integrated numerically (see Beckers et al., 2001) for the initial condition of a tube of constant vorticity of height 10 cm and radius 1 cm. Once the value of the circulation is prescribed by the experimental measurements just after the removal of the tube, an excellent agreement was found between the experimental data and the prediction of this model. We also found a good agreement between measurements and predictions for a diffusing Lamb–Oseen vortex, which is an exact solution of the diffusion equation. Its azimuthal velocity, U_{LO} , evolves as

$$U_{LO} = \frac{\Gamma}{2\pi r} (1 - e^{-(r/R(t))^2}), \quad (2)$$

with $R(t) = 2\sqrt{\nu(t - t_0)}$, where ν is the kinematic viscosity and t_0 a reference time. The vortex radius, $R_{\max}$, defined as the radial position of the maximal velocity in the mid-plane, satisfies $R_{\max}(t) = 1.12R(t)$ for the velocity field (2). The experimental value of $R_{\max}$ just after removal of the tube was around 1.6 cm. To reproduce the experimental data, the reference time t_0 was chosen to be equal to 50 s after the tube removal.

Inertia-gravity waves were generated by vertical oscillations of a horizontal cylinder, of length $L = 30$ cm and diameter $D = 5$ cm, placed near the bottom of the tank. The amplitude of the oscillations, A_{cyl} , ranged from 0.5 to 3 cm. The vertical distance between the vortex centre and the cylinder was around 15 cm. The cylinder was placed such that the mid of the wave front propagated towards the vortex centre during the interaction. Thus, at one side, waves propagated in opposite directions to the vortex flow, while at the other side, they propagated in the same direction.

Since the cylinder was very long compared to its radius, the wave pattern was almost planar, with wave fronts parallel to the cylinder axis. The wave energy propagates along trajectories whose angle with the horizontal plane, θ , depends only on the oscillation frequency, and is perpendicular to the propagation direction of the front. The wave fronts propagate downwards while the energy propagates upwards towards the perforated tube. Only one wavelength appears in the velocity field, associated with the bimodal structure of the wave field generated by an oscillating cylinder. Though the spatial spectrum of such a wave field is large, it is observed experimentally that the dominant wavelength which appears in the velocity field, is approximately equal to twice the diameter of the cylinder.

3. Impact of the wave breaking on the dynamics of the vortex

To investigate the behaviour of waves propagating towards the vortex flow, the WKB theory is used. The planar wave generated by the oscillating cylinder is well described by a group of parallel rays originating from the cylinder, and the behaviour of these rays in the framework of the WKB theory agrees with the experimental observations.

Waves propagating along the vortex flow experience a Doppler shift (1) leading to a decrease of the intrinsic frequency ω_0 . As ω_0 decreases, the intrinsic group velocity, v_g , becomes increasingly horizontal. The wave energy is then advected by the horizontal flow of the vortex, reflects on the vortex flow and escapes without being trapped. This behaviour is similar to that of waves propagating along a barotropic jet. Wave rays associated with the wave field propagating against the vortex flow, experience a Doppler shift (1) leading to an increase of the intrinsic frequency ω_0 towards N . As ω_0 increases, the intrinsic group velocity, v_g , becomes increasingly vertical, and the wave vector increases gradually in the direction of the vortex axis because of the strong radial shear. Most of these rays are then propagating locally in a flow with strong radial shear and weak vertical shear, and exhibit a similar behaviour as rays propagating against a barotropic jet: they are trapped along a critical layer and their wave vector increases towards infinity in the framework of the WKB theory. In the experiments, trapped waves are either damped by viscosity, or eventually break (see Moulin and Flór, 2004). In the scope of the present paper, only results on experimental waves-breaking will be discussed.

Fig. 2 shows typical vertical vorticity and horizontal divergence fields of an experiment with wave breaking. Waves propagating along the flow are advected horizontally and scattered away from the field of view, and are therefore not visible in Fig. 2. Waves propagating against the flow are trapped, as shown in the divergence field Fig. 2(b). They are hardly visible in the vertical vorticity field (Fig. 2(a)) because their intrinsic frequency is close to N and their polarisation is very close to pure gravity waves. The energy of these trapped waves is transported along the flow and wrapped around the vortex centre, while the horizontal wavelength of the wave pattern decreases when approaching and turning along the vortex core. It is hard to detect wave-breaking events by PIV measurements, because of intrinsic limitations of this method, observations by other methods had to be used to ensure the occurrence of wave breaking.

To measure the impact of the wave breaking on the vortex dynamics, we compare a vortex in the absence of waves in experiment A, with a vortex in the presence of wave-breaking

Fig. 2. Vertical vorticity (a) and horizontal divergence field (b) of waves propagating into the vortex flow, obtained from PIV measurements at mid-level in the fluid. The waves are generated at the right near the bottom of the tank and propagate from below to the left, towards the vortex. Experimental parameters: $f = 0.4 \text{ s}^{-1}$, $N = 1.5 \text{ s}^{-1}$, $V = 3.2 L$, $D = 5.0 \text{ cm}$, $A_{\text{cyl}} = 2.0 \text{ cm}$ and $\omega_{\text{abs}} = 0.53 \text{ rad s}^{-1}$. The measurement is taken at $t = 45 \text{ s}$ after forcing, and $R_{\text{max}} \approx 2.5 \text{ cm}$ and $U_{\text{max}} \approx 2.5 \text{ cm s}^{-1}$, yielding $\Gamma/2\pi \approx 8.3 \text{ cm}^2 \text{ s}^{-1}$.

during a finite time interval in experiment B. By using exactly the same experimental conditions for the vortex generation, we are able to create identical vortices. By generating waves during a finite time interval, we are able to measure accurately the vortex velocity profiles before and after the interaction, and compare them quantitatively. Fig. 3 shows the results of these experiments. The azimuthal velocity profiles of the vortex just after the removal of the tube are plotted on the left of Fig. 3. The experimental data agree well with a Lamb–Oseen vortex with the same circulation Γ and the same radius R_{max} for experiments A (top) and B (bottom), showing that the vortex generation by aspiration is well reproducible. For experiment B, Fig. 3(B,2) shows the radial profiles of the azimuthal velocity at the onset of trapping; the scatter in the tail is due to the non-axisymmetric wave field. Fig. 3(B,3) shows the radial profiles of the azimuthal velocity field after wave breaking.

The continuous lines in Fig. 3 correspond to the predictions of the Lamb–Oseen vortex, and agree very well with the measurements in experiment A without waves. In contrast, a clear discrepancy is found between the Lamb–Oseen profile and the measurements of experiment B after wave breaking. In the outer part of the vortex, the azimuthal velocity has decreased, revealing a deposit of negative momentum in the vortex periphery. This is in agreement with the trapping of waves which transport linear momentum oriented against the flow. Another interesting point is that this deposit of negative momentum leads to a steepening of the velocity profile, corresponding to negative vorticity.

In order to estimate the vortex momentum and energy variations due to the wave interactions, we compare our vortex with a Lamb–Oseen vortex. In Fig. 4(a), the radial profiles of the azimuthal velocity field before the interaction with the waves Fig. 3(B,1) and after in Fig. 3(B,3), have been smoothed and fitted with the profile of a Lamb–Oseen vortex, while the

Fig. 3. Azimuthal velocity profiles, U_θ , measured at mid-depth for (A) an experiment without waves and (B) an experiment with wave breaking. Experimental parameters: $f = 0.5 \text{ s}^{-1}$, $N = 1.5 \text{ s}^{-1}$, $V = 3.5 L$, $D = 5.0 \text{ cm}$, $A_{\text{cyl}} = 2.0 \text{ cm}$ and $\omega_{\text{abs}} = 0.8 \text{ rad s}^{-1}$. The times of measurement are indicated below the figures, $t = 0 \text{ s}$ corresponding to the beginning of the PIV acquisition, 15 s after removal of the perforated tube. Waves are generated between $t = 12$ and 78 s , and break around $t = 120 \text{ s}$ in experiment B. The lines represent the Lamb–Oseen vortex, given by (2).

deviation in angular momentum of the two profiles from the Lamb–Oseen vortex is shown in Fig. 4(b). The difference in angular momentum between the initial and final profile is estimated by the surface of the grey region in Fig. 4(b) and, for $5 < r < 14 \text{ cm}$, amounts $M_{\text{dep}} = -92 \text{ cm}^3 \text{ s}^{-1}$. This is equal to 12% of the total angular momentum of the vortex calculated for $r < 15 \text{ cm}$. With the values equally derived for the kinetic energy, we deduced a decrease of $255 \text{ cm}^4 \text{ s}^{-1}$ due to the wave breaking from the discrepancy between the evolution of energy of the Lamb–Oseen vortex and of the experimental vortex. This decrease is around 20% of the amount of kinetic energy the vortex at the end of the experiment.

Let us compare these values with the energy density of the incident wave field, e_{wave} . This energy was measured from the amplitude of the vertical displacement of dye in a similar experiment with the same experimental conditions and amounted, for an amplitude of 0.5 cm , $e_{\text{wave}} \approx 0.82 \text{ cm}^2 \text{ s}^{-2}$. If this energy would be completely converted into a barotropic horizontal flow, it would produce a velocity $\tilde{u} = \sqrt{2e_{\text{wave}}} = 1.28 \text{ cm s}^{-1}$. The waves are generated during $\Delta t = 66 \text{ s}$ and propagate away from the cylinder at the group velocity $v_g = 1.57 \text{ cm s}^{-1}$ along a beam of width roughly equal to $\Lambda = 2D$ and of half a cylinder length, $L/2$, since only waves propagating against the flow are trapped and may provide energy and momentum to the vortex field. We therefore estimate the kinetic energy and momentum of the incident wave field by $LDv_g(\Delta t)e_{\text{wave}} = 12745 \text{ cm}^5 \text{ s}^{-2}$ and

Fig. 4. (a) Smoothed profiles of the azimuthal velocity U_θ deduced from experimental data of Fig. 3(B,1) and (B,3), respectively, before wave generation, and after wave breaking. The dashed lines represent predictions of the Lamb–Oseen vortex, whose azimuthal velocity U_{LO} is given by (2). (b) Difference in negative angular momentum, rv_θ , between the smoothed experimental profiles and the predictions from the Lamb–Oseen vortex plotted in (a). The difference in azimuthal velocity, v_θ , is defined as $v_\theta(r) = U_{LO}(r) - U_\theta(r)$. (···) Experiment of Fig. 3(B,1), before the generation of waves. (—) Experiment of Fig. 3(B,3), after the wave breaking. The grey-shaded area indicates the amount of momentum transferred by wave breaking from the incident wave field towards the vortex field.

$LDv_g(\Delta t)\tilde{u} = 19895 \text{ cm}^4 \text{ s}^{-1}$, respectively. For a uniform transfer in the vertical direction, along a vortex of height $H = 10 \text{ cm}$, the kinetic energy and momentum at each level, would roughly read $Ec_{av} = 1275 \text{ cm}^4 \text{ s}^{-1}$ and $M_{av} = 1990 \text{ cm}^3 \text{ s}^{-1}$, respectively.

Apparently, only a very small part of the wave energy and momentum are converted to the horizontal flow. As can be inferred from Figs. 5, 6 and 11, this loss in wave energy and momentum is most likely due to the (vertical) diapycnal mixing, horizontal mixing and viscous dissipation on small scales. Such a weak deposit of momentum in the background field was also observed numerically for waves interacting with a uniform shear flow by Staquet and Huerre (2002).

4. Mixing induced by the wave interaction and breaking

4.1. Method and observations

To study the mixing induced by the interaction of waves with the vortex and their breaking inside the vortex, we used fluorescein dye illuminated by either a horizontal or a vertical laser sheet. To generate a dye sheet, we employed the method of Hopfinger et al. (1991) and introduced a 1 mm thin cotton wire, soaked in concentrated fluorescein, vertically in the fluid just before vortex generation. The dye sheet was made by translating the vertical wire horizontally. During the vortex generation, the radial inflow and the increasing azimuthal flow wraps the vertical plane of dye around the vortex centre. Top view of the dye illuminated by a horizontal plane takes the form of a spiral, like in Fig. 5(a), while a typical side view is given by Fig. 6(a). If the radial inflow had been uniform in the vertical direction, we would

Fig. 5. Top-view dye-visualisation of the trapping and breaking of the wave in the vortex for $t =$ (a) 15 s, (b) 65 s, (c) 103 s and (d) 124 s after removal of the tube. Experimental parameters: $f = 0.5 \text{ s}^{-1}$, $N = 1.3 \text{ s}^{-1}$, $V = 3.0 \text{ L}$, $D = 5.0 \text{ cm}$, $A_{\text{cyl}} = 3.0 \text{ cm}$, $\omega_{\text{abs}} = 0.85 \text{ s}^{-1}$. Waves are generated at $t = 34 \text{ s}$ near the bottom of the tank and propagate from right to left.

have expected vertical lines of dye in the side view Fig. 6(a). However, the displacement of iso-density surfaces, required to balance the vertical shear near the top and bottom of the vortex, leads to the formation of curved lines. Because of the mixing induced by the removal of the perforated tube, the dye distribution is almost uniform near the vertical axis of the vortex.

Figs. 5 and 6 show typical time sequences of, respectively, top and side views of the dye distribution in an experiment where the waves penetrate and break in the vortex flow. Initially, when waves start to be trapped, zigzag patterns appear in both views (see Figs. 5(b) and 6(b)). As time goes on, the wave amplitude gradually increases and smaller patterns appear, until eventually wave breaking occurs, driving intense mixing of the dye shown in Figs. 5(d) and 6(d). These successive stages of the mixing are discussed below.

Fig. 6. Side-view dye-visualisation of the trapping and breaking of the wave for (a) 18 s, (b) 59 s, (c) 70 s and (d) 120 s after the removal of the tube. Waves are generated at $t = 24$ s from the right near the bottom of the tank and propagate upwards to the left, towards the vortex. Experimental parameters: $f = 0.5 \text{ s}^{-1}$, $N = 1.21 \text{ s}^{-1}$, $V = 5.0L$, $D = 5.0 \text{ cm}$, $A_{\text{cyl}} = 2.0 \text{ cm}$, $\omega_{\text{abs}} = 0.9 \text{ s}^{-1}$.

4.2. First stage: chaotic mixing

The zigzag patterns that initially appear in the top and side views of the dye distribution shown, respectively, in Figs. 5(b) and 6(b), are horizontal and vertical cross-sections of an inclined folding of the dye sheet transported by the mean flow. To understand how the propagation without breaking of waves inside the vortex could produce such patterns, we performed numerical simulations of the transport of passive tracers in a time-dependent flow. For simplicity, we modeled the vortex by a time-independent azimuthal velocity field u_θ deduced from the expression (2). Let R_f be the radius of the vortex at time t , $R_f = R(t)$. We freeze the velocity field and scale the radial coordinate $r \rightarrow R_f r$, the velocity $U_\theta \rightarrow (\Gamma/2\pi R_f) U_\theta$. The non-dimensional azimuthal velocity of the vortex is then given

by the expression (2) with $R(t) = 1$, $\Gamma = 2\pi$ and $U_\theta \approx 1/r$ away from the core. The time coordinate t , is then scaled as $t \rightarrow (2\pi R^2/\Gamma)t$. To simulate numerically the perturbation by the waves, we superposed a non-divergent time-dependent velocity field on the vortex whose non-dimensional form reads

$$\begin{aligned} \frac{u_\theta}{r} &= \epsilon \left[-\frac{2}{m} e^{-r^2} \sin(\Phi) - \frac{k_r}{mr} (1 - e^{-r^2}) \cos(\Phi) \right] f(t) \\ u_r &= \epsilon \frac{1}{r} (1 - e^{-r^2}) \cos(\Phi) f(t), \end{aligned} \quad (3)$$

where $\Phi = m\theta + k_r r - \omega t$ is the wave phase, m the azimuthal wave number, k_r the radial wave number, ϵ the non-dimensional wave amplitude, and $f(t)$ is a time function aimed to describe a gradual formation of the wave field from zero. To generate a wave field on a time roughly equal to its period we chose $f(t) = 1 - \exp(-\omega t)$. The description of the wave field by (3) reproduces the basic relevant features of the wave field: it is time periodic and the constant-phase surfaces for the trapped waves form a spiral-like pattern.

Fig. 7 shows results of numerical simulations of the transport of passive tracers for $m = 2$. In accordance with the experiment, we generated a spiral of passive tracers by advecting a straight segment of tracers with the velocity field of a Lamb–Oseen vortex (2). We added the wave velocity field (3), once the spiral pattern was well developed. Zigzag patterns clearly appear around a trajectory that satisfies $n = T_t/T_w = 2$, where T_t is the time of revolution of the considered trajectory, $T_t = 2\pi r/u_\theta$, and T_w is the period of the perturbation, $T_w = 2\pi/\omega$. From the Lagrangian point of view (see Wiggins, 1992), trajectories with n rational can be destabilized by the superposition of a periodic disturbance; the destabilized trajectories split in a succession of elliptic and hyperbolic points. In the neighbourhood of elliptic points, the distribution of tracers rotates gradually at every period while near the hyperbolic points, chaotic regions form which drive a rapid separation of initially close tracers. To reveal these elliptic and hyperbolic points, results of the same numerical simulation with tracers distributed initially along the destabilized trajectory $n = 2$ are plotted in Fig. 7(b). The elliptic points are located at the centre of the zigzag patterns, while the hyperbolic points are found in the centre of regions with a low tracer density. For other values of m zigzag patterns formed near trajectories satisfying $n = m$.

In the experiments, the wave field is more complicated than the description of Eq. (3). However, the time dependence is well defined, so that we may expect a destabilization of trajectories corresponding to integer values of n . Careful analysis of the dye distribution in the mid-plane shows that the zigzag patterns appear on trajectories satisfying $n = 2$ and $n = 3$, as illustrated in Fig. 8 for $n = 2$. Some zigzag patterns appear also near the trajectory $n = 1$ but are more difficult to detect since they are close to the well-mixed region of the vortex core. The correspondence with the experimental observations strongly suggests that the fold and stretch patterns, associated to the chaotic mixing, are due to the wave-induced perturbations of the vortex flow.

To explain the appearance of zigzag patterns in both vertical and horizontal cross-sections, we should take into account of the vertical dependence of the wave field. Therefore, we consider a wave with phase $\Phi = m\theta + k_r r - \omega t + k_z z + \Phi_0$ in Eq. (3), with k_z the vertical component of the wave vector and Φ_0 a phase reference. Then we may expect that the elliptic points will be distributed along constant phase lines at a fixed time, and that

Fig. 7. Numerical simulation of the transport of passive tracers in a periodically perturbed vortex with (a) results for an initial distribution of passive tracers along a vertical line and (b) results for an initial distribution along the resonant trajectory that satisfies $n = T_t/T_w = 2$ (circle near $r = 2$). The background flow is given by expression (2) with fixed $R(r) = 1$ and $\Gamma = 2\pi$. The periodic perturbation is given by expression (3) with $m = 2$, $\omega = 0.5$, $k = 2.0\pi$ and $\epsilon = 0.01$. The period of the perturbation reads $T_w = 2\pi/\omega$ and the period along a trajectory, T_t , is given by $T_t = 2\pi r/U_\theta$.

their angular position, θ_e , satisfies $\theta_e = \theta_0 + k_z z/m$. Since the zigzag patterns form around elliptic points, they are located along the same trajectory and generate the inclined folding of the dye sheet observed in the experiments.

4.3. Second stage: mixing by wave breaking

Waves that are trapped in the periphery of the vortex generate periodic displacements of the dye sheet that are displayed in, respectively, top and side views in Fig. 5(b) and Fig. 6(b). To filter this oscillatory displacement, a sequence of images with a time-step equal to the period of the wave field $T_w = 2\pi/\omega_{\text{abs}}$ is shown in Fig. 9 and reveals growing

Fig. 8. Patterns of folded dye lines near the trajectory $n = 2$, for the experiment of Fig. 5. Around the trajectory that satisfies $T_t = 2T_w$, where T_w is the wave period ($T_w = 2\pi/\omega_{\text{abs}}$), parts of the dye spiral shown in (a) and (b) at $t = 0$, return to their initial position, $2T_w$, later.

Fig. 9. Time sequence of the wave trapping, with wave frequency, $T_w = 7$ s for the experiment shown in Fig. 6. The time sequence begins at $t = 30$ s.

small-scale structures (see rectangular selections in Fig. 9(d–g)). These structures have a vertical extent which does not correspond to the vertical wave length of the trapped waves, and are therefore likely to be associated to the non-linear destabilization of incident waves by parametric resonance (see Benielli and Sommeria, 1998). Eventually, mushroom-like patterns begin to appear in the dye field (see circular selections in Fig. 9(h) and (i)), indicating the occurrence of the Rayleigh–Taylor instability, associated with a local inversion of the density field due to the high amplitude wave motions. This instability is studied in the case of an unstable linear density-stratification in a thin gap by Voropayev et al. (1993), revealing the formation of two-dimensional mushroom-shaped structures, that move vertically by gravity. The transport by the mean flow of the present convective structures across the laser sheet explains the brief appearance (see Fig. 10), and reveals their almost spherical shape, not dissimilar to the wave breaking of orographic gravity-wave structures observed by Gheusi et al. (2000) and Eiff and Bonneton (2000). They show that instabilities take the form of toroidal or ring-like vortex structures, inclined with respect to the horizontal. The preferential horizontal orientation of these rings may provoke a transport of fluid through the dynamical barrier at the edge of the vortex.

4.4. Radial transport of fluid

The chaotic mixing (see Fig. 8) is confined to a narrow ring and is, therefore, not very efficient to transport fluid radially. The non-linear generation of small-scale patterns by parametric resonance does not affect the transport of fluid particles in the neighbourhood of the vortex core; only the mushroom-like patterns associated with the overturning instability are able to transport fluid efficiently in both vertical and radial directions.

Fig. 10. Evolution of a mushroom-shaped structure generated during the wave breaking crossing the vertical laser plane at $t = 96, 96.6, 97.3$ and 98 s after the removal of the tube. The spatial scale is given by the thick vertical line that corresponds to a length of 2.3 cm. Experimental parameters: see Fig. 6.

The location and strength of these convective patterns should be controlled by the properties of the trapped incident waves, described to some extent by the WKB theory. In an attempt to check this, we performed two experiments with different vortex strengths, and different amounts of energy of the incident wave field prescribed by the amplitude of the cylinder oscillations. For comparison, the dye distributions are shown in Fig. 11 after an equal number of wave periods.

Fig. 11. Side view dye-visualization of the mixing induced by the wave breaking for experiment A with moderate waves breaking into an intense vortex, and for experiment B with strong waves breaking into a relatively moderate vortex. The experimental parameters are (A) $f = 0.5 \text{ s}^{-1}$, $N = 1.21 \text{ s}^{-1}$, $V = 5.0 L$, $D = 5.0 \text{ cm}$, $A_{\text{cyl}} = 2.0 \text{ cm}$, $\omega_{\text{abs}} = 0.8 \text{ s}^{-1}$ and (B) $f = 0.5 \text{ s}^{-1}$, $N = 1.16 \text{ s}^{-1}$, $V = 4.0 L$, $D = 5.0 \text{ cm}$, $A_{\text{cyl}} = 3.0 \text{ cm}$, $\omega_{\text{abs}} = 0.9 \text{ s}^{-1}$. The waves were generated at $t = 24$ s in experiment A and $t = 60$ s in experiment B after the removal of the tube, and the dye distribution observed at, respectively, $t = 124$ s and $t = 160$ s, 100 s after the onset of wave generation. The thick crosses of height and width 4 cm indicate the spatial scale.

For the interaction of an intense vortex ($V = 5L$) with a wave with moderate energy ($A_{\text{cyl}} = 2\text{ cm}$), experiment A, the mixing occurs in the region $r \in [5, 8]$, far away from the vortex core for which the radius R_{max} is predicted to be equal to 2.7 cm by the Lamb–Oseen vortex model. For the interaction of a relatively weak vortex ($V = 4L$) with more energetic incident waves ($A_{\text{cyl}} = 3\text{ cm}$), experiment B, we observe a displacement of the mixed region to the region $r \in [2.5, 7]$, closer to the vortex core. Here, the vortex radius R_{max} is predicted to be equal to 3.0 cm. The broadening of the mixed region is due to the larger amplitude of the incident wave field, which provides a larger amount of energy to the mushroom-like structures associated with the wave breaking (the energy of the trapped waves in experiment B amounts about twice of that in experiment A). Besides, since the dynamical barrier of the vortex in experiment B is weaker than in experiment A (the vortex has a smaller circulation), the convective structures penetrate more deeply into the vortex structure.

5. Conclusions

We have investigated the impact of breaking waves on the dynamics and transport properties of a tall vortex. A planar wave propagating against the flow deforms and wraps around the vortex core, while the energy accumulates in the vortex periphery until its amplitude is high enough to trigger overturning and breaking. We have shown that this wave breaking may lead to a deposit of negative momentum in the periphery of the vortex. This deposit of momentum modifies the velocity profile of the vortex from a Lamb–Oseen vortex to an isolated vortex with a positive core embedded in a negative vorticity ring, and experimentally demonstrates an energy transfer from the wave field to the vortex field.

We characterize two distinct mixing processes. The periodic wave field destabilizes resonant trajectories of the vortex, leading to local chaotic mixing; the radial transport of dye is limited to a narrow region of the vortex. The breaking of waves leads to convective mushroom-like patterns; the radial penetration of these structures into the vortex core depends on the amount of energy provided by the trapped waves.

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