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DETERMINATION OF FLUID VISCOELASTIC PROPERTIES USING RESONANT MICROCANTILEVER

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The dynamic characteristics of a microcantilever depend on the fluid in which the beam is immersed. In this paper, the analysis of the frequency response of microcantilevers immersed either in viscous or viscoelastic fluid is performed in order to determine if this vibrating device could be used as a 'microrheometer'.

INTRODUCTION

Resonant microcantilever can be used as viscometer. To achieve such a goal the change in the microcantilever resonant frequency is monitored when the oscillating cantilever is immersed in a viscous fluid. This working principle has been used in various fluids (water, ethanol, glycerol/water mixture) to determine their kinetic viscosities, in a range from 1 to 200 $mm^2.s^{-1}$ [1-4]. The limitation of this principle for higher fluid viscosity is due to the important viscous losses in the fluid. In the first part of this paper, we propose to widen this viscosity measurement range by studying the whole frequency spectrum of a microcantilever immersed in a newtonian liquid instead of its resonant frequency shift. In the second part of this paper, the extension of this study to fluids with maxwellian behavior is proposed. The aim of this theoretical study is to know if microcantilevers can be used as a microrheometer to determine the viscoelastic properties of maxwellian liquids. Then, some preliminary measurements are presented to validate if microcantilevers can be used to characterize the mechanical properties of various fluids (silicon oils and binary mixtures made of cetylpyridinium chloride and sodium salicylate, CPCl-Sal).

CANTILEVER VIBRATIONS IN A VISCOUS FLUID

The analytical expressions of the resonant frequency and its associated quality factor for a rectangular beam in a viscous fluid have been established by Sader [5]. This model is known to be in good adequation with numerous experiments. The expressions of both resonant frequency and quality factor are obtained by the resolution of the equation governing the beam dynamic deflection (1):

$$EI\frac{\partial^4 w(\omega,t)}{\partial x^4} - m_L\omega^2 w(\omega,t) = F(x) + F_{fluid} \qquad (1)$$

where E is the Young's modulus of the cantilever material, I the cantilever moment of inertia, m_L the cantilever mass per unit length, ω the angular frequency, x the spatial coordinate along the length of the beam, F the actuation force per unit length and F_{fluid} the force per unit length exerted by the surrounding medium on the microstructure.

F_{fluid} is the solution of the three-dimensional Navier-Stokes equations [6]:

$$F_{fluid} = -j\omega g_1 w(\omega,t) + \omega^2 g_2 w(\omega,t) \qquad (2)$$

with g_1 and g_2 terms due respectively to the dissipative and inertial parts of the drag force exerted by the fluid:

$$g_1 = \frac{\pi}{4}\rho_f b^2 \omega \Gamma_i \qquad (3)$$

$$g_2 = \frac{\pi}{4}\rho_f b^2 \Gamma_r \qquad (4)$$

where b is the microcantilever width, ρ_f the fluid mass density, Γ_r and Γ_i are the real and imaginary parts of the hydrodynamic function.

Using the equations (1-4) it is possible to obtain, for a given actuation, the microcantilever deflection for each frequency. In Fig. 1, the spectra (amplitude and phase) of a microcantilever deflection in the case of electromagnetic actuation have been plotted for different fluid viscosities.

Regarding Fig. 1, it can be deduced that, even if there is no resonance phenomenon (high viscosity), the measurement of the microcantilever deflection can be used to determine the fluid viscosity by measuring at a given frequency (chosen in accordance to the viscosity range) the deflection amplitude or by measuring the frequency or the phase slope at the phase inflection point.

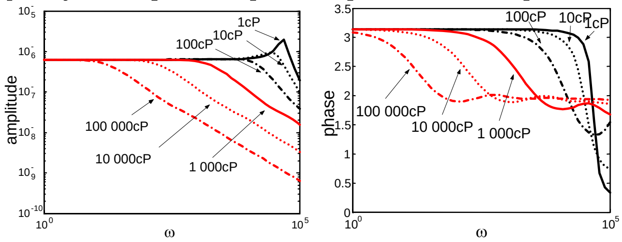

Fig. 1. Theoretical amplitude and phase of a silicon cantilever (3µm, 50µm, 400µm) oscillating in fluids with different viscosities.

CANTILEVER VIBRATIONS IN VISCOELASTIC FLUID

Could a microcantilever be used as a rheometer in order to measure the viscoelastic properties of fluids ($G'(\omega)$ and $G''(\omega)$)? To answer this question, we introduced the shear modulus $G = j\omega\eta$ in the simplified expression of the hydrodynamic function proposed by Maali et al. for the case of viscous fluid [7].

Then, the obtained expression of $\Gamma_r + j\Gamma_i$ as a function of the imaginary shear modulus is used for the study of a viscoelastic fluid (complex shear modulus) by replacing G by $G' + jG''$. As shown in [8-9] this can be done because the cantilever is larger than the mesh size of the rheological network.

By extracting the real and imaginary parts of the resulting hydrodynamic function, it comes for viscoelastic fluid:

$$g_1 = \frac{\pi}{2}\frac{b_2 G''}{\omega} + \frac{\pi}{4\sqrt{2}}\sqrt{\rho_f}\, b \left[\begin{array}{l} (b_1 - a_2)\sqrt{\sqrt{G'^2 + G''^2} + G'} \\ +(a_2 + b_1)\sqrt{\sqrt{G'^2 + G''^2} - G'} \end{array} \right] \quad (5)$$

$$g_2 = \frac{\pi}{4} a_1 \rho_f b^2 + \frac{\pi}{2} b_2 \frac{G'}{\omega^2} + \frac{\pi}{4\sqrt{2}} \frac{\sqrt{\rho_f}\, b}{\omega} \left[\begin{array}{l} (a_2 + b_1)\sqrt{\sqrt{G'^2 + G''^2} + G'} \\ +(a_2 - b_1)\sqrt{\sqrt{G'^2 + G''^2} - G'} \end{array} \right] \quad (6)$$

where a_1=1.0553, a_2=3.7997, b_1=3.8018 and b_2=2.7364 are Maali's parameters [7].

These analytical expressions (5-6) have been used in (1-2) to simulate the case of maxwellian viscoelastic fluids for which:

$$G' = \frac{G_0 \tau^2 \omega^2}{1 + \tau^2 \omega^2} \quad \text{and} \quad G'' = \frac{G_0 \tau \omega}{1 + \tau^2 \omega^2} \quad (7)$$

with G_0 the elastic plateau modulus and τ the terminal relaxation time of the maxwellian fluid.

Simulations have been made for maxwellian viscoelactic fluid (high viscosity and no elastic effect for frequencies below the relaxation frequency, and no viscosity and elastic effect for frequencies larger than the relaxation frequency). An example of simulation is plotted in Fig. 2 where it can be seen that due to low viscosity for angular frequency above $1/\tau$, resonant phenomenon occurs even if the fluid is a 'compact gel'. The characteristics of the resonance (frequency and phase slope) depend on G_0. Moreover for angular frequency near $1/\tau$, the microcantilever deflection depends on τ.

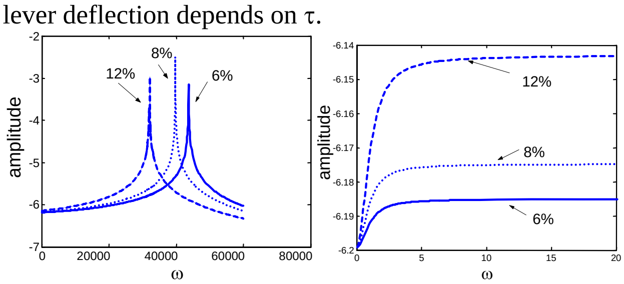

Fig. 2. Theoretical amplitude of deflection for a silicon cantilever (3µm, 50µm, 400µm) oscillating in a maxwellian fluid.

EXPERIMENTS

All experiments were carried out with a commercial AFM lever with piezoelectric actuation (Nanodevices AFM MPA-11100). The amplitude and phase of the deflection have been measured by a optical system. In Fig. 3 some measurements with silicon oils (20cP to 30000cP) are plotted. Even if there is no resonance frequency for high viscosity, the term $f_0 \left[d\Phi/df \right]_{f_0}$, with Φ the deflection phase and f_0 the frequency corresponding to $\Phi = 0$, could be used as information on the fluid viscosity. Measurements with viscoelastic fluids CPCl-Sal with different concentrations are plotted in Fig.4. It can be noted that the plateau modulus G_0 increases with the concentration, and consequently the term $f_0 \left[d\Phi/df \right]_{f_0}$ decreases. As predicted with simulations, in spite of the gelatinous aspect of the CPCl-Sal solution, a sharp resonance occurs (high value of $f_0 \left[d\Phi/df \right]_{f_0}$).

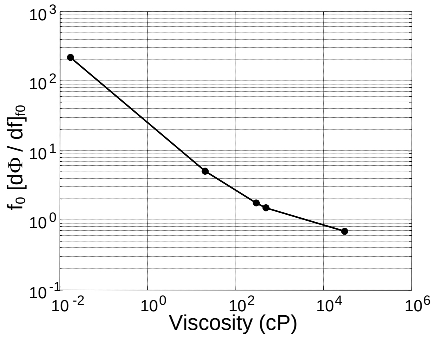

Fig. 3. Measurements in air and in silicon oil (20cP to 30000cP). $f_0 \left[d\Phi/df \right]_{f_0}$ versus viscosity.

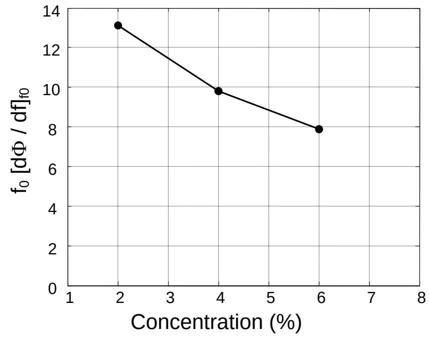

Fig. 4. Measurements in CpCl-Sa (2% to 6%). $f_0 \left[d\Phi/df \right]_{f_0}$ versus concentration.

CONCLUSIONS

Our first encouraging results indicate that the proposed new cantilever sensor might be a useful tool to determine the rheological properties of a wide range of fluids and could thus be used as a powerful microrheometer.

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