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Abstract

This article presents the different ways used to characterize A/D converters through the "CanTest System" test bench and its functionalities. The different ways used to characterize A/D converters dynamically could be organized into three parts: time domain, statistical and spectral analysis. The most commonly used algorithms in time domain analysis are presented. Due to instrumentation error, some may give "inaccurate" results. We explain why and propose a new method based on a direct estimation of conversion noise to avoid this problem. The different methods and applications linked to statistical analysis are also described. We explain the three different methods used to measure the differential non-linearity (DNL) and the integral non-linearity (INL). A comparison between these three algorithms in simulation and through an experimental case shows that their behaviours differ when the histogram presents edging effects. The final study presented in this paper concerns the A/D converter performance evaluation by analysis in frequency domain. From the output power spectrum, relationships are given for the computation of all spectral parameters. We present a correction to include in the computation of spectral parameters when the analog input does not span the entire full-scale range of the A/D converter under test. When the input generator provides a sine wave with harmonics, we propose the "Dual-Tone" method in order to separate the distortion associated with the component and the one associated with the generator. In the last part of this paper, we describe the test bench giving its specifications (acquisition system, frequency range, ...). Some typical measurements and comments about the interest of analysis in different modes are given also.

Key Words

A/D converter, statistical analysis, spectral analysis, time domain analysis.

I. Introduction

The A/D converter is the major component linking the analog world to the numerical one. This conversion reduces the quality of the analog signal which is converted in a numerical form. It is therefore important to measure the systematic error and defects introduced by the analog to digital conversion. In this paper, we present a theoretical and experimental approach for the dynamic characterization of the A/D converter.

In the first part of this paper is presented a theoretical background of the three major methods for the A/D converters characterization. First, the analysis of the ADC output signal is considered in time domain. The obtained parameters, which allow an ADC performance evaluation, are related to two standards : the JEDEC standard giving parameter definitions [1] and the IEEE-1057 Standard setting the measurement methods [2]. One of these parameters is the number of effective bits (n_{eff}). Indeed, an r-bit A/D converter is associated with a systematic error, the quantization noise, whose theoretical power B_{rms} depends on r. When an A/D converter with the same resolution is tested, the measured quantization noise has a power B_q greater than B_{rms}. So, an estimation of a number of effective bits associated with the measured quantization noise could be given by comparison with the theoretical one. In the time domain analysis, the B_{rms} is deduced from the error signal, defined as the difference between the input and output signals of the ADC. In this case, in order to estimate the input signal, the output numerical signal is computed and fitted using a least-square method to obtain an estimation of its amplitude, offset, frequency and phase [2]. This method implies the resolution of a nonlinear equation system by iterative method, requiring initial values. These values generally correspond to the test setup with a perfect input signal generator. If generators were really perfect, any classical resolution algorithm (Newton, Gradient, Simplex, ...) could converge to a correct result. But, in the case where the input frequency value is not precise, these algorithms would not converge. In order to eliminate this influence, we propose a method giving a direct estimation of the ADC conversion noise. It is based on the eigendecomposition value of the correlation matrix data [3]. This method will be presented in the third section of this paper.

Another analysis concerns statistics. Basically, a histogram is built, giving the number of occurence codes $H[i]$, depending on the code value (i). To be normalized, $H[i]$ is then divided by the total number of samples N. The ADC transfer characteristic parameters, like differential nonlinearity DNL and integral nonlinearity INL, are directly deduced from the histogram. In this paper, three different characterization methods related to the statistical analysis are presented. *Method 1* is the comparison between the effective histogram and the theoretical histogram obtained when considering a perfect ADC [4]. *Method 2* and *Method 3* are based on the cumulative histogram and compare the cumulative frequency occurences of successive codes [5], [6].

For *Method 1* and *Method 2*, the amplitude and offset of the input signal have to be known and for *Method 3* a reference quantum q_{ref} is necessary. This assumption justifies a study of each method's robustness towards the estimation errors. Both simulations and experimental results showed a better behaviour for *Method 2* and *Method 3* [7].

Spectral analysis is then considered. From the power spectrum are deduced the signal-to-noise ratio, with or without harmonic distorsion ($SNDR$ or SNR), the harmonic distorsion rate (THD), and the number of effective bits (n_{eff}) [8], [9]. For the ADC input signal, all algorithms consider a full-scale and perfect sine wave.

In the first step, we show how parameters are deduced from the theoretical equations, when the hypothesis quoted earlier is fulfilled. Then we present eventual modifications by adding correction terms in the evaluation of $SNDR$, SNR, THD and n_{eff}, when, for instance, the input signal is not exactly a full-scale one.

A major concern in spectral analysis is the measurement of the harmonic distorsion rate. Indeed, in the case of an input signal presenting harmonics, it becomes impossible to separate the ADC distorsion from the signal generator one. To solve this problem, we developed a method called "DUAL-TONE", using as input signal a sum of two sine waves. A theoretical study shows that the resulting intermodulation lines in the power spectrum are due to the single ADC nonlinearity. Experimental results confirm this assertion [10].

The "CanTest System" test bench, described in the last part of this article, proposes an automatic system dedicated to the dynamic characterization of A/D converters developed in our laboratory. The different elements included in this system are presented with their characteristics (sample frequency, bandwidth of the acquisition system, size of acquisition, ...) [11]. In this section, several components characterized in the three analysis modes are also presented and the adequate analysis as a function of the A/D converter applications are discussed.

II. Theoretical background

Before describing the different types of analyses, the process of the analog-to-digital conversion is shown by the transfer function of the ADC. Fig.1 shows the different elements of transfer diagram for an ideal linear ADC. The analog input full-scale range of the A/D converter is devided into 2^r equal quanta and each of these quanta is associated with an integer code value.

[Fig. 1 about here.]

The quality of the conversion depends on the variation of the width of each quantum, the linearity of the transfer function and so on. The goal of the dynamic characterization of A/D converter is to point out defects of the A/D converter under test. Generally, we use as stimulus a sine wave with a high spectral quality (well defined mathematically) and the experimental response of the A/D converter is compared with the theoretical one.

A. Analysis in time domain

As quoted earlier, the mainly used input signal for temporal analysis is a sine wave defined by :

$$V_{in}(t) = A\sin(2\pi f_{in}t + \varphi) + O \tag{1}$$

where A, O, f_{in}, φ are respectively the amplitude, offset, frequency and phase. In order to achieve an uniform distribution of samples, [2] gives a relation between the input and clock frequencies, f_{in} and f_s, the number of samples N and an odd integer k_{np}:

$$N f_{in} = k_{np} f_s \tag{2}$$

If this relation is fulfilled, the acquisition test is made in coherent frequencies mode. It implies also that the number of periods contained into the acqusition is integer and all the samples are differentes themselves. An output vector of N samples, called $d[i]$ with $i \in [0, N-1]$, is collected. The quantification process induces an error represented by the difference between $d[i]$ and the input signal. The purpose of this kind of analysis is to determine the signal error, to compute the quantization noise power and after the number of effective bits. We recall that the noise is given by :

$$r[i] = d[i] - \hat{A}\sin\left(2\pi i\frac{\hat{f}_{in}}{f_s} + \hat{\varphi}\right) - \hat{O} \tag{3}$$

$$B_{rms} = \sqrt{\frac{1}{N}\sum_{i=0}^{N-1} r[i]^2} \tag{4}$$

and leads to the n_{eff} value using:

$$n_{eff} = r - \log_2 \frac{B_{rms}}{B_q} \tag{5}$$

where B_q is defined as the theoretical quantization noise which a rms value closed to $\frac{q}{\sqrt{12}}$. As it is difficult to know the different parameters of the input signal, we estimate them by a least square method. This method tends to minimize the mean square error defined by:

$$J_r = \sum_{i=0}^{N-1} \left\{ d[i] - A \sin\left(2\pi i \frac{f_{in}}{f_s} + \varphi\right) - O \right\}^2 \tag{6}$$

To solve this problem, we use iterative methods such Newton, Gradient or Levenberg-Marquardt, all based on the resolution of a system of nonlinear equations. If the input frequency is precisely known, it goes to a linear system [2]. So, the choice of the algorithm is made depending on the good knowledge of the input frequency.

B. Statistical analysis principle

The test setup for a statistical analysis consists in applying a sine wave at the input of the ADC in coherent frequencies mode. Then a histogram, $H[j]$, is built from the N acquisition samples. It gives the number of occurence of each code value[1] normalized by the total number of sample acquired during the test procedure. Here, we present different methods used to analyze this histogram.

The first one, called *Method 1*, is based on the comparison between $H[j]$ and the theoretical histogram value $H_{th}[j]$, defined as :

$$\begin{aligned} H_{th}[j] &= \int_{T[j]}^{T[j+1]} \frac{dV}{\pi\sqrt{\hat{A}^2 - \left(V - \hat{O}\right)^2}} \\ &= \frac{1}{\pi}\left\{ \arcsin\left(\frac{T[j+1]-\hat{O}}{\hat{A}}\right) - \arcsin\left(\frac{T[j]-\hat{O}}{\hat{A}}\right) \right\} \end{aligned} \tag{7}$$

[1] For a non-signed A/D converter the code value range is defined as $j \in [0, 2^n - 1]$ and all methodologies described in this paper concern this kind of converter. But it will be easy to extend the presented work to a signed A/D converter with $j \in \left[-2^{n-1}, 2^{n-1} - 1\right]$

where $T[j]$ is the decision level of the code j (i.e. the transition voltage between the code $j-1$ and the code j). $\hat{A}$ and $\hat{0}$ are estimations of the amplitude and offset of the input signal, given by :

$$\begin{cases} \hat{A} = \frac{j_{max}+1-j_{min}}{2} \\ \hat{O} = \frac{1}{j_{max}+1-j_{min}} \sum\limits_{j=j_{min}}^{j=j_{max}} j \end{cases} \tag{8}$$

j_{min} and j_{max} are respectively the left and right codes having the maximum values of $H[j]$. In accordance with the different standards [1], [2], the differential and integral nonlinearities are respectively given by :

$$DNL(j) = \frac{T[j+1]-T[j]}{q_{ref}} - 1 = \frac{H[j]-H_{th}[j]}{H_{th}[j]} \tag{9}$$

and

$$INL(j) = \sum_{l=j_{min}}^{j} DNL(l) \tag{10}$$

Note that q_{ref} is the theoretical quantum width of the A/D converter transfer function and the vector INL should be corrected by a best straight line fit (linear mean square method [12]).

Method 2 and *Method 3* are based on the cumulative histogram, $CH[j] = \sum\limits_{k=0}^{j} H[k]$ from which we can find the experimental decision levels :

$$\begin{cases} T[j] = \hat{O} - \hat{A}\cos(\pi CH[j-1]) & \textit{for the Method } 2 \\ T[j] = -\cos(\pi CH[j-1]) & \textit{for the Method } 3 \end{cases} \tag{11}$$

Thus, DNL is calculated from the $T[j]$ values by :

$$DNL[j] = \frac{T[j+1]-T[j]}{q_{ref}} - 1 \tag{12}$$

where $q_{ref} = 1$-lsb (the unit lsb expresses the least significant bit) in the case of the *method 2* and $q_{ref} = \frac{T[j_{max}]-T[j_{min+1}]}{j_{max}-(j_{min}-1)}$ in the other case corresponding to the mean value of the quantum width associated with the normalized transfer function.

C. Spectral Analysis Principle

This method is based on the use of the Discrete Fourier Transform (DFT) applied to the output sinusoidal data ($d[i]$). In order to use the FFT algorithm, the number of samples, N, must fulfill this condition: $log_2 N$ is an integer. Then, the FFT gives a discrete complex spectrum, which components are given by :

$$D[n] = \sum_{i=0}^{N-1} d[i]\, e^{-j2\pi i \frac{n}{N}} \qquad n = 0 \ldots N-1 \tag{13}$$

Usually, the spectral parameters of the converter under test are extracted from the power spectrum $\left(|D[n]|^2\right)$. When computing the power spectrum of a signal, digitally sampled (N samples) at frequency f_s, we obtain discrete lines separated by a frequency interval Δf_{sp}, with $\Delta f_{sp} = f_s/N$. Only half of the spectrum has to be considered (Shannon theorem), corresponding to lines at frequencies $n \times \Delta f_{sp}$ with $n \in [1, N/2]$. Fig. 2 shows a power spectrum acquisition made with the CanTest System. The component under test is the 8-bit TDC1048 from TRW. By a qualitative approach, a description of the TDC1048 behaviour shows nonlinearities with the appearance of the second and third harmonics of the fundamental frequency.

[Fig. 2 about here.]

The purpose of this section is to give a quantitative performance evaluation of the ADC under test. It is explained how to compute the power spectrum data to obtain the spectral parameters. For a good estimation of the dynamic parameters, the coherent sampling mode will be used in order to avoid leakage [13]. Therefore, the input and clock frequencies, f_{in} and f_s, must verify the relation (2). It means that there are k_{np} entire periods of the input signal into the acquisition and that the fundamental line is at the position k_{np}.

Different parameters may be related to the dynamic spectrum. The first parameter is the noise floor, NO_M, defined as the noise mean square in the spectrum. In the ideal case, it is related to the ADC resolution, r, and the number of samples N, contained in the acquisition by:

$$(NO_M)_{dB} \equiv -6.02 \times r - 10log_{10}N + 1.25 \tag{14}$$

It is also possible to compute this parameter from the power spectrum without the offset line ($n = 0$) and the fundamental one ($n = k_{np}$) by the following relation:

$$\widehat{NO}_M = \sqrt{\frac{\sum_{n=1,\, n\neq k_{np}}^{N/2} |D[i]|^2}{\frac{N}{2} - 1}} \tag{15}$$

By a comparison between (14) and (15), the effective ADC resolution can be found. But in the case of harmonic distortion, the noise floor estimation is strongly biased. So, the usual algorithm computing the number of effective bit n_{eff} is based on the comparison between the theoretical and experimental signal-to-noise ratios.

The Total Harmonic Distortion , THD, which traduces the nonlinearities of the ADC transfer function is one of the most useful spectral parameters. This value is given by the ratio between the quadratic sum of the harmonic power and the fundamental power:

$$THD_{dB} = 10\log_{10} \frac{\sum_{h\geq 2} \left(|D[h \times k_{np}]|^2 - (\widehat{NO}_M)^2 \right)}{|D[k_{np}]|^2} \tag{16}$$

where k_{np} and $h \times k_{np}$ correspond respectively to the input ($h = 1$) and to the h^{th} harmonic frequencies. Before concluding this section, we have to give two different definitions of the signal-to-noise ratio: it can be considered without harmonic distortion (SNR_{dB} without harmonics line at $h \geq 2$ for the noise) or with distorsion ($SNDR_{dB}$). It is then calculated by:

$$\widehat{SNR}_{dB} = 10\log_{10} \frac{|D[k_{np}]|^2 - (\widehat{NO}_M)^2}{\sum_{i\neq k_{np},\, h\times k_{np}} |D[i]|^2 + h \times (\widehat{NO}_M)^2} \tag{17}$$

and

$$\widehat{SNDR}_{dB} = 10\log_{10}\frac{|D[k_{np}]|^2 - (\widehat{NO}_M)^2}{\sum\limits_{i\neq k_{np}} |D[i]|^2 + (\widehat{NO}_M)^2} \tag{18}$$

Considering a input sine wave with a full-scale amplitude range, the theoretical signal-to-noise ratio for a perfect A/D converter is given by:

$$SNR_{dB} = SNDR_{dB} = 6.02 \times r + 1.76 \tag{19}$$

After computing the experimental signal-to-noise ratio (18) and comparing with the theoretical one (19), we can express the equivalent resolution by the number of effective bits defined as :

$$n_{eff} = \frac{\widehat{SNDR}_{dB} - 1.76}{6.02} \tag{20}$$

In this section, we have given the different classical algorithms used in the dynamic characterization of the A/D converters. In the case of perfect test condition, each of them could be used without problem. But, as the resolution or/and the sampling frequency increases, the test setup decreases in quality. It implies some drawbacks for each of these methods and the next sections try to give some solutions.

III. Direct quantization noise B_{rms} estimation

In the section dedicated to the time domain analysis, we have described two algorithms in order to compute the error signal. They are based on the knowledge input frequency: either its value is well know with high precision and the equation system is linear, either this condition is not fulfilled and the system is non linear. To have a comparison between the algorithms efficiency, we have studied the evolution of the estimation noise versus the difference between the real signal frequency f_{in} and the initial frequency estimation $f_{in_{ie}}$ [2]

[2] This parameter is one of the initial values necessary to the resolution of the non-linear equation by an iterative method such Newton, Gradient, ...

defined as:

$$\Delta f_{in} = \frac{f_{in_{ie}} - f_{in}}{f_{in}} \tag{21}$$

We have shown that no algorithm provides a good estimation of the quantization noise when $\Delta f_{in} > 0.1\%$ [12]. For a good convergence of any algorithms, the input frequency must be well known in the case of linear system, and for the non-linear system, the initial estimated value should be close to the true one. Due to instrumentation inacurracy, it is essential to solve that problem.

This is why we proposed an original method, derivated from a parametric spectral estimation method [14], to evaluate B_{rms} directly, without calculation of the input signal parameters [3]. The model considered is a sinusoidal signal with additional noise:

$$g(i) = a\sin(2\pi f_0 i + \varphi) + b(i) \tag{22}$$

where f_0 represents the signal frequency normalized by the clock frequency, ($f_0 \in [0, 0.5]$). The phase φ is defined by a uniform distribution within the domain $[0, 2\pi]$. The noise component, $b(i)$ corresponds to a sequence created by a white noise process with gaussian distribution, zero mean and variance σ_n^2. The notation used is :

- $\Sigma_A = diag(\lambda_i)_{0 \le i \le L}$ is a diagonal matrix which main diagonal is the vector $(\lambda_1, \lambda_2, \ldots)$.
- $G_L(i)$ is a observed signal vector of size L contained in $g(i)$
- S_L is a Vandermonde matrix which terms are:

$$S_L = \begin{bmatrix} 1 & 1 \\ exp(j2\pi f_0) & exp(-j2\pi f_0) \\ \vdots & \vdots \\ exp(j2\pi f_0(L-1)) & exp(-j2\pi f_0(L-1)) \end{bmatrix} \tag{23}$$

- P : Power of the component signal $(a^2/4)$
- I_L : Identity matrix of size $(L \times L)$.

The correlation matrix of the signal $g(i)$, $R_g = E\left\{G_L(i)G_L^H(i)\right\} = S_L P S_L^H + \sigma_n^2 I_L$ could be expressed in orthogonal base of subspace obtained by the Eigenvalue decomposition (EVD):

$$R_g = UDU^H = \begin{bmatrix} U_S & U_B \end{bmatrix} \begin{bmatrix} \Sigma_S & \\ & \Sigma_B \end{bmatrix} \begin{bmatrix} U_S \\ U_B \end{bmatrix}^H \tag{24}$$

where U is a unitary matrix verifying $U^H U = I_L$. The eigenvectors associated with the eigenvalues contained in Σ_B were called noise subspace vectors. This subspace was characterized by the eigenvalues verifying :

$$\lambda_3 = \lambda_4 = \ldots = \lambda_L = \sigma_n^2 \tag{25}$$

In the same way, we defined the signal subspace as the space spanned by the eigenvectors related to Σ_S. In fact, due to the limited set of data, we consider an estimated correlation matrix and the eigenvalues of the noise subspace are related by :

$$\hat{\lambda}_3 > \hat{\lambda}_4 > \ldots > \hat{\lambda}_L \tag{26}$$

We can introduce a new method based on the calculation of the noise variance :

$$\hat{\sigma}_n^2 = \frac{1}{L-2} \sum_{j=3}^{L} \hat{\lambda}_j \tag{27}$$

where L is the dimension of R_g choosen in order to have N/L equal to an integer. The B_{rms} noise can be estimated by :

$$B_{rms} = \sqrt{\frac{1}{N/L} \sum_{k=1}^{N/L} (\hat{\sigma}_n^2)_k} \tag{28}$$

The good results obtained from simulations with this method are corroborated by experimental results. An experimental study is made using a 8-bit successive-approximation from Analog Devices, the AD7574. The number of effective bits was calculated by this method and by the FFT method. The acquisition was made in characteristic mode with f_{in} in the range $[10Hz, 1kHz]$, $f_s = 5kHz$ and $N = 2048$. Figure 3 shows there is only a slight difference between the results.

[Fig. 3 about here.]

The interest of this method are multiple. The first is that the quantization noise could be estimated without the knowledge of all the necessary parameters used in the time domain analysis for solve the equation system. The second interest concerns its advantage from the FFT. Indeed, computing the number of effective bits or the quantization noise necessits some conditions such the coherent frequency mode and a number of samples equal to $\approx 4 \times 2^r$ for the acquisition. In the contrary case, the windowing approach should be used in order to avoid leakage but the power estimation of the different components contained into the signal is difficult [15]. So, with our method it will be easy to compute B_{rms} or n_{eff} without these drawbacks.

IV. Robustness of the three different statistical analysis algorithms

The second section gives the theoretical background of the statistical analysis principle, assuming a perfect input sine wave or with a higher spectral purity than the ADC under test. In the real case, it will be very difficult to found a generator which fulfills this assumption. When the amplitude of the input signal doesn't sweep completely the ADC full-scale, or/and an additive noise appears during the acquisition, an edge effect occurs. It means that the measured histogram doesn't show abrupt transitions to zero outside the codes i_{max}, i_{min}. Figure 4 shows an experimental histogram measured with the CanTest System. The ADC under test is a 16-bit one fed by a synthetizer HP3326 providing the clock signal and by a Bruel and Kjaer 1049 generator for the input signal.

[Fig. 4 about here.]

We can see a large edge effect, due to longer acquisition time, depending on the number of samples and the clock frequency, that induces drift of amplitude in the input signal. In this case, the theoretical histogram estimated from this acquisition will be strongly biased. It is then evident the first method could be provided wrong results concerning the integral and differential nonlinearities.

In [12] and [7] has been shown that the systematic errors generated by the edge effect get worse when the treatment is made using *Method 1*. In the other hand, *Method 2* and *Method 3* have an identical behavior. However, *Method 2* and *Method 3*, based on the comparison between cumulative occurences of two successive codes, give a good result, close to the realistic behavior. It is shown in figure 5 where the first curve is obtained with *Method 1* and the second one with *Method 2* or *Method 3* (we noted that these two last methods gave identical results).

[Fig. 5 about here.]

V. Concerning the drawbacks of the spectral analysis

In the experimental case, it is difficult, even impossible, to make an acquisition with a full-scale range amplitude. So, a correction has to be introduced in the measurement of $S\widehat{ND}R_{dB}$, function of the ratio between the estimated input amplitude and the full-scale range of the converter, and function of the total harmonic distortion defined in (16).

$$S\widehat{ND}R_{dB_{mod}} = S\widehat{ND}R_{dB} - 10\log_{10}\left\{S\widehat{ND}R_{dB}^{2} \times THD^2\left(1-\left(\frac{2\hat{A}}{PE}\right)^2\right)+\left(\frac{2\hat{A}}{PE}\right)^2\right\} \tag{29}$$

where $\hat{A}$ is an estimation of the input amplitude computed from the acquisition, and with $PE = 2^n$. Figure 6 shows a typical result that you can obtain with the CanTest System in spectral analysis.

[Fig. 6 about here.]

The user can visualize the different spectral parameters in a window (the table gives here the amplitudes of the fourteen first harmonic lines). The type and the name of the ADC as well as the different spectral parameters are indicated besides. All we presented here is correct when the spectral purity of the input signal is superior to the one of the component under test. When the stimulus signal has larger harmonic distortion, it becomes necessary to use an original method called "Dual-Tone" [10]: it allows the separation of the different sources of distortion.

The principle of the "Dual-Tone" method is based on the modification of the input signal: it is constituted of the addition of two sine waves with the same pic to pic amplitude $2 \times A$ equal to the half ADC full-scale ($A = FS/4$). Their frequencies f_{in1} and f_{in2} are closed, and a new relationship of the acquisition in coherent frequency mode is defined:

$$\frac{N}{f_s} = \frac{k_{np1}}{f_{in1}} = \frac{k_{np2}}{f_{in2}} \tag{30}$$

where k_{np1} and N, k_{np2} and N are respectively prime of each other. [10] gives the different relations linking the spectral parameters in the "Dual-Tone" to the "Single-Tone" mode. The first concerns the total harmonic distortion slightly equal to the InterModulation Distortion (IMD):

$$THD_{ST} \cong IMD \tag{31}$$

where IMD is defined by :

$$IMD_{dB} = 10 \log_{10} \left\{ \frac{\sum_{i \in N^*} \sum_{j \in Z^*} \left(|D[i \times k_{np1} + j \times k_{np2}]|^2 \right)}{2A^2} \right\} \tag{32}$$

It means that the distortion generated by the input signal doesn't appear in the intermodulation lines. The IMD takes therefore in account only the ADC nonlinearities corresponding to the THD measured with a perfect generator in "Single-Tone" mode.

Concerning the effective number of bits determination, a new theoretical relation between the signal-to-noise ratio and the resolution r in "Dual-Tone" mode is given:

$$SNR_{DT_{dB}} = 6.02 \times res - 1.25 \tag{33}$$

In the same way, the signal-to-noise plus distortion ratio is now given by:

$$S\widehat{ND}R_{DT_{dB}} = 10\log_{10}\left\{\frac{\left|D[k_{np1}]\right|^2 + \left|D[k_{np2}]\right|^2 - 2(\widehat{NO}_M)^2}{\sum\limits_{i\neq k_{np1},k_{np2}} \left|D[i]\right|^2 + 2(\widehat{NO}_M)^2}\right\} \tag{34}$$

Fig.7 shows simulations based on a 14 bits ADC spectral acquisition with a transfer function modelled by $V_{out} = V_{in} + 10^{-7}V_{in}^2$. Fig.7.a displays a power spectrum with a perfect input signal, the evaluation parameters depend only from the ADC transfer characteristic errors. Fig.7.b corresponds to a non-ideal input signal (with harmonic distorsion), which induces a degradation of the spectral parameters.

[Fig. 7 about here.]

Fig.8 shows the spectral behavior in the "Dual-Tone" mode of the same simulated converter, fed by the non-ideal input signal quoted earlier. Spectral parameters are closed to the ones of the reference acquisition (Fig.7.a). Experimental measurements [10] also corroborated this validation of the "Dual-Tone" method.

[Fig. 8 about here.]

VI. ADC dynamic evaluation system and exprimental results

The Cantest system was developed at the IXL laboratory, dedicated to the dynamic characterization of A/D converters. The whole bench is organized as shown in Fig.9. It allows data acquisition and parameter evaluation for a wide range of ADC, by different analytic methods. The main characteristics of CanTest are its adaptability to many external configurations, a very well developed user-interface, and the availability of a large number of data computation algorithms extracting specific characterization parameters.

[Fig. 9 about here.]

CanTest runs on Apple computers (Quadra and Power PC series), and a specific acquisition board plugged into the computer has been developed to optimize data acquisition. The whole software, driving the acquisition and computing the results, has been written in C language.

Two different modes are available for data acquisition: memorization of until 1Meg-samples, or real-time data processing. Numerical samples, outputs of the ADC under test, are issued from a customized evaluation board. The input and clock signal generators, as well as the power supplies, are external elements, possibly driven by the CanTest system software through HPIB interface. The sampling frequency can go up to 55MHz, the system handles over-sampling mode (for sigma-delta ADC for example), and converters resolution will be up to 24 bits.

The software gives to the user the opportunity to set all the test variables, for the programming of the input and sampling signals, or even for specific parameters of the analysis algorithms. Automatic test sequences can be programmed, and characteristic paramaters can be studied when varing with input or sampling frequency and also with amplitude of the input signal (for the variation study of SNR versus the input amplitude for a Sigma-Delta ADC for instance). In memorization mode, sets of data or of results can be saved or loaded for further treatement, averaging or simple comparison. The real-time acquisition mode is useful for ADC characterization when modifying external constraints, such as temperature, and can display for example the evolution of the numerized signal frequency spectrum.

The menus, windows and dialog boxes based presentation of the CanTest software makes it a convenient and useful tool, already used in various industries. Meanwhile, research-oriented original algorithms have been developped for precise applications or emerging concerns, such as instrumentation precision, as shown earlier.

In this section, we give some results obtained by the CanTest system in different kind of analysis. The A/D converters used in this study have the same resolution (12-bits) and have been tested in the same condition (input and clock frequencies, number of sample required). Our choice has been made in order to test different structures. Whole these parameters are resumed in Table I. Note that we have also included in this table an approximation-successive 12-bit with a very high static performance (DNL in the range $[-0.1, 0.1]$-lsb).

[Table 1 about here.]

First, we show what kind of results are obtained from the "CanTest system" in time domain analysis. Excluding the sample signal window, which presents no interest, two wave forms are presented here. The first concerns an acquisition reconstructed into one of the input period [12]. This window helps any user when testing the test bench configuration (with or without saturation, coherent frequencies, ...) and moreover could be used for the bit error rate measurement setup [16]. The bite error rate (BER), defined as the number of erroneous code value per unit time, is an important parameter for the specifications of High-Speed A/D converter. Fig.10 shows the BER measurement of a 8-bit ADC from Maxim (MAX100).

[Fig. 10 about here.]

The input and clock frequencies are respectively at 10-MHz and 250-MHz. Note that this test setup needs a high-speed data acquisition system. The HP16500 with HP16517 probe from Hewlett Packardat could be included in our system using HPIB commands. Naturally, the setup condition for this measurement fulfills all the request quoted in [16] and described by the following figure.

[Fig. 11 about here.]

Another important window in time domain analysis is the error signal. Fig.12 shows the error signal of the CS5012 component from CRYSTAL.

[Fig. 12 about here.]

This component is a successive approximation converter with a good number of effective bits comparing to its resolution. But this treatment is not adapted for this kind of converter. Indeed it is a low frequency and middle resolution converter dedicated to the measurement application and it will be shown that a statistical analysis is more adequate for its evaluation (Fig.13).

[Fig. 13 about here.]

Moreover, the principle of the statistical analysis eliminates all the white noise influence and tends to a static analysis. In fact, just the transfer function is extracted through its DNL. So, this analysis provides informations about the variation quantum width which are primordial for measurement application. Fig.14 shows another results of four 12-bit different products. From the DNL measurements, the better choice will be the two components presented a lower DNL (AD9220 and ADS801).

[Fig. 14 about here.]

But, we show in this part that for an application required given spectrum specification, the four components could be used. Indeed, if the application requires an A/D converter with a model spectrum specification as quoted in Fig.15, it is easily shown that the whole converters under test fulfil the specification. From this comment, we carry out that a knowledge of the A/D converter application is necessary before a testing procedure.

[Fig. 15 about here.]

VII. Conclusion

First, we have focused on the dynamic analysis in the time domain. We give a new approach based on eigenvalue decomposition (EVD) to determine the number of effective bits by a direct estimation of the quantization noise without knowing the error signal. Now, it would be interesting to include an algorithm to detect the number of harmonics contained in the acquisition vector in order to improve the measurement precision.

The aim of the statistical analysis part is to show that the different algorithms used do not have the same performances when experimentally applied. A comparative study of these algorithms is undertaken by simulation, giving the ADC response to a noisy signal. This study allows us to recommend the two algorithms based on the cumulative histogram for evaluating the transfer characteristic errors (DNL, INL). We also propose some results in order to confirm that characterization of A/D converters for measurement applications by statistical analysis is good choice.

The spectral analysis is certainly the most commonly used method for the dynamic characterization of analog-to-digital converters. This paper intended to give definitions of the different spectral parameters, as well as their ways of determination from the Fourier transformation of an acquisition. With increasing resolution of ADC, it becomes difficult to find a generator whose spectral quality is better than the one of the converter under test. To solve that problem, we present a method based on the exploitation of intermodulation lines, which are linked only to distortion due to the ADC.

At the end of this paper, we present a powerful system to characterize different kinds of A/D converters: "CanTest". With some results issued from this test bench, we propose some type of analysis as a function of the application where the component will be used. For instance, the statistical analysis should be recommended for instrumentation and measurement applications.

References

[1] Jedec Standard No. 99, *Addendum No. 1 : Terms, Definitions, and Letter Symbols for Analog-to-Digital and Digital-to-Analog Converters* (Electronic Industries Association Engineering Department, July 1989).

[2] IEEE Std 1057-1994, *IEEE standard for Digitizing Waveform Recorders* (The Institute of Electrical and Electronics Engineers, Inc., New York, December 1994).

[3] D. Dallet, Y. Berthoumieu, T. Moreau, P. Marchegay, New Methods for Determining the Number of Effective Bits of A/D Converters Based on Eigendecomposition, *Proc. IEEE Instrum. Meas. Technol. Conf. IMTC-95* (Boston, April 24-26, 1995) 23–26.

[4] B. E. Peetz, Dynamic Testing of Waveform Recorders, *IEEE Trans. Instrum. Meas.* **vol.32 no.1** (March 1983) 12–17.

[5] J. Doernberg, H. S. Lee, D. A. Hodges, Full-Speed Testing of A/D Converters, *IEEE Journal of Solid-State Circuits* **vol.19 no.6** (December 1986) 820–827.

[6] M. Vanden Bossche, J. Schoukens, J. Ronneboog, Dynamic Testing and Diagnostics of A/D Converters, *IEEE Trans. Circuits and Systems* **vol.33 no.8** (August 1986) 775–785.

[7] D. Dallet and G. Chiorboli, Statistical Analysis of A/D Converters : A Comparative Study, 2^{nd} *International Workshop on ADC Modelling and Testing IWADC-97* (Tempere, June 1-6, 1997) 207–212.

[8] HP Note 5180A-2, *Dynamic Performance Testing of A to D converters* (Hewlett Packard Product Note 5180A-2, 1982).

[9] D. W. Doerfler, Dynamic Testing of a Slow Sample Rate, High-Resolution Data Acquisition System, *IEEE Trans. Instrum. Meas.* **vol.35 no.4** (December 1986) 477–482.

[10] M. Benkais, S. Le Masson, P. Marchegay, A/D Converter Characterization by Spectral

Analysis in "Dual-Tone" Mode, *IEEE Trans. Instrum. Meas.* **Vol.44** no.5 (October 1995) 940–944.

[11] M. Benkais, D. Dallet, S. Le Masson, *CanTest Version 7 : User Manual* (Internal Report, pp. 80, January 1995).

[12] D. Dallet, *Contribution à la caractérisation des convertisseurs analogique-numérique : Evaluation des méthodes et mises en oeuvre de nouveaux procédés* (Thèse de Doctorat, Université Bordeaux I, 12 Janvier 1995).

[13] P. Kraniauskas, *Transforms in Signals and Systems* (Addison-Wesley Publishing Company, 1992).

[14] S. Lawrence Marple Jr, *Digital Spectral Analysis with applications* (Prentice Hall Signal Processing Series, 1987).

[15] L. Benetazzo, C. Narduzzi, C. Offeli, D. Petri A/D Converter Performance Analysis by Frequency-Domain Approach, *IEEE Trans. Instrum. Meas.* **vol.41 no.6** (December 1992) 834–839.

[16] M. Benkais, L. Dulau, P. Kadionik, G. Gestin, D. Dallet, P. Marchegay, A New Method for the Bit Error Rate Measurement of ADC, 2^{nd} *International Workshop on ADC Modelling and Testing IWADC-97* (Tempere, June 1-6, 1997) 160–164.

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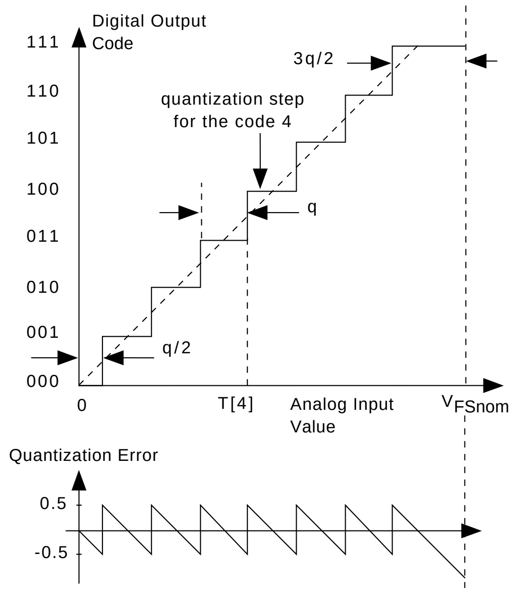

Fig. 1. Transfer function of an ideal linear ADC.

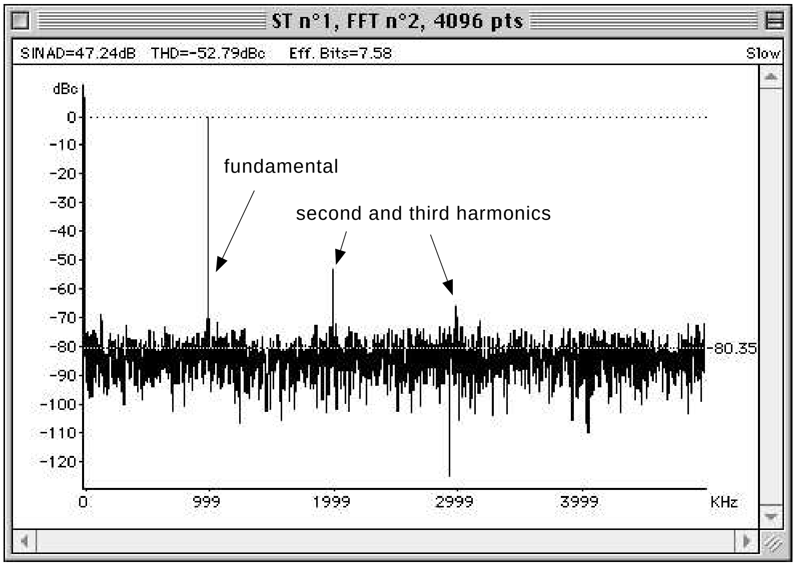

Fig. 2. Power spectrum of an acquisition made with the CanTest System, on the TDC1048 ADC.

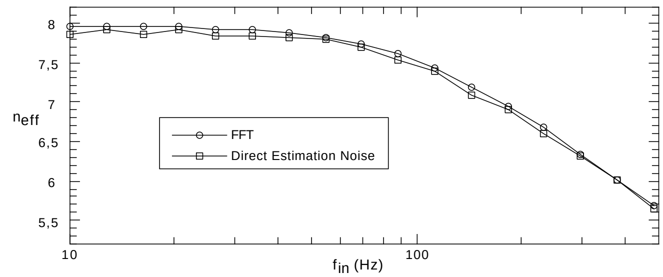

Fig. 3. n_{eff} estimation by means of FFT and direct estimation noise methods.

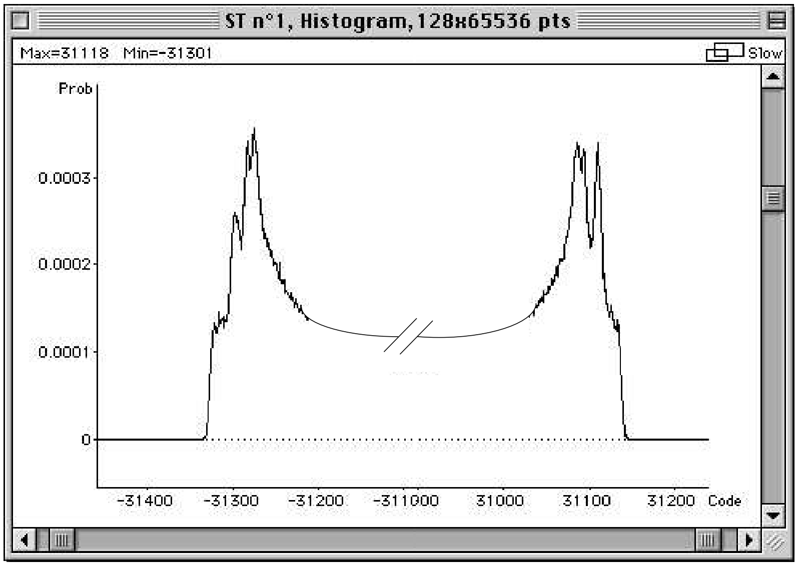

Fig. 4. Experimental histogram on a CS5317 ADC (zoom on the edge effect).

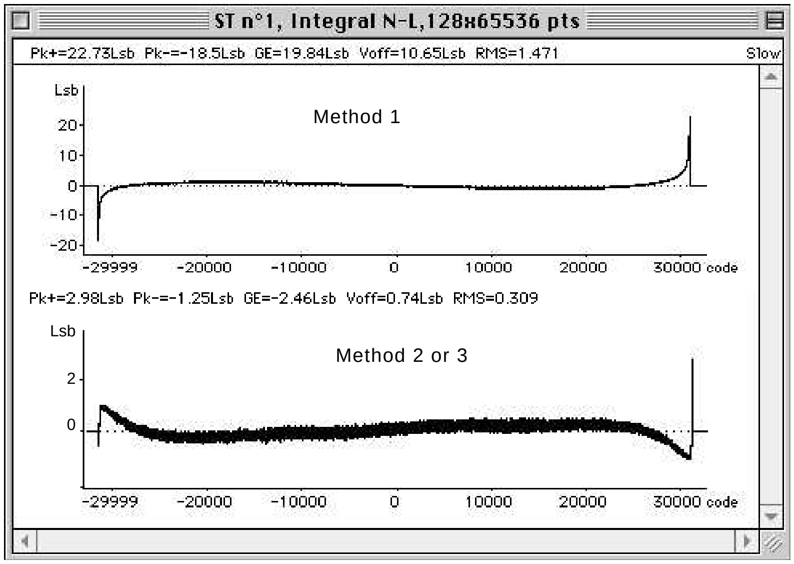

Fig. 5. Integral nonlinearities obtained with *Method 1* and *Method 2* from the CS5317 histogram.

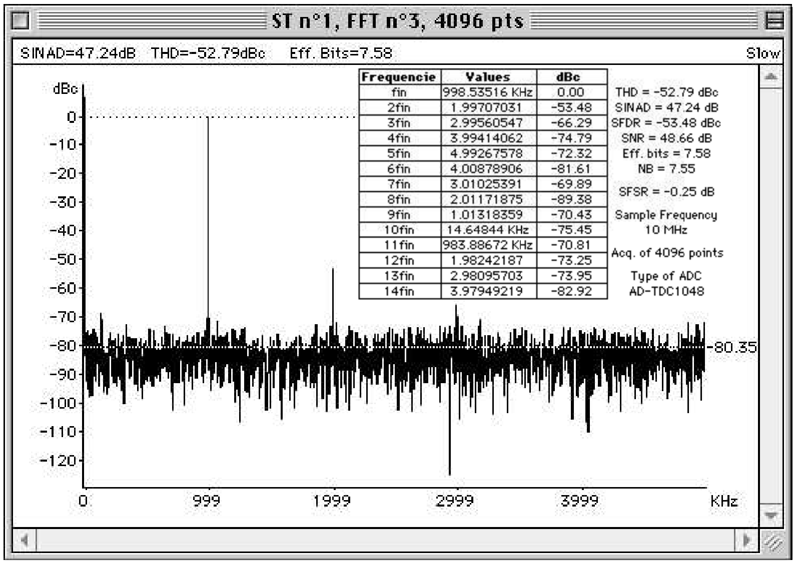

Fig. 6. Power spectrum of the TRW1048 acquisition and spectrum parameters undertaken with the CanTest System.

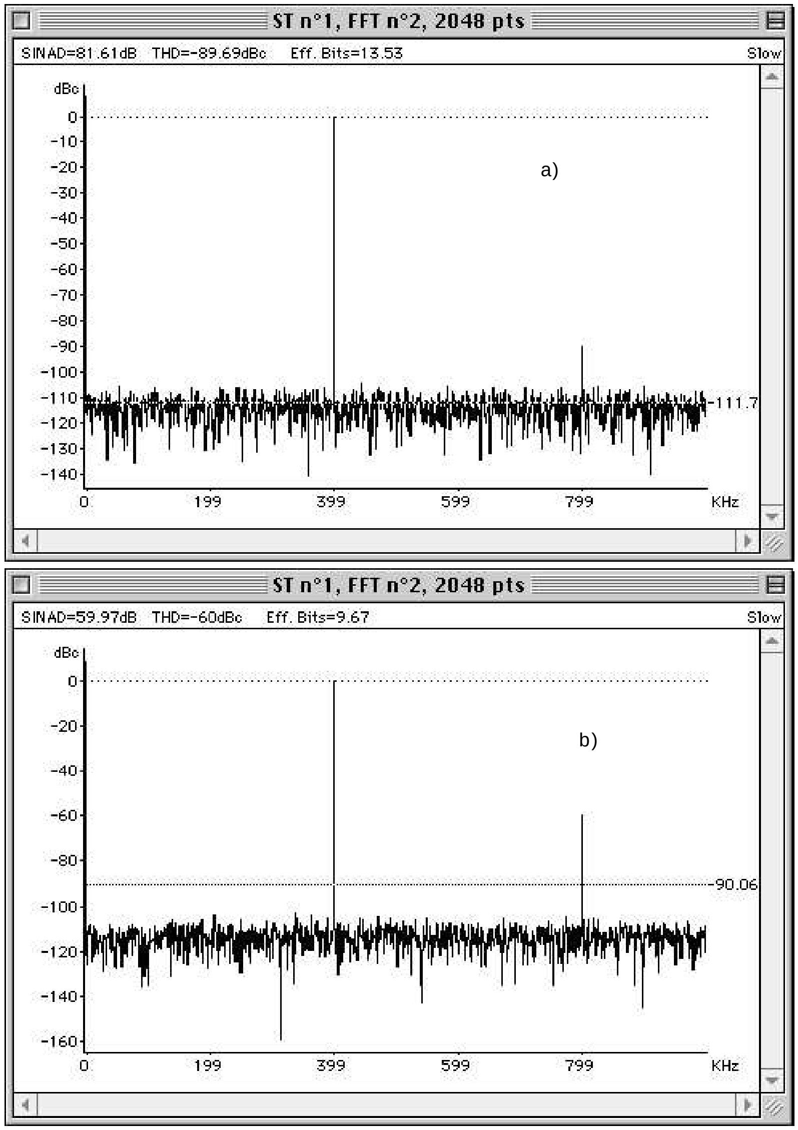

Fig. 7. Power spectrum from simulation made with a non-ideal ADC. (a) is made with a perfect input signal while (b) consider a input sine wave with harmonic distortion.

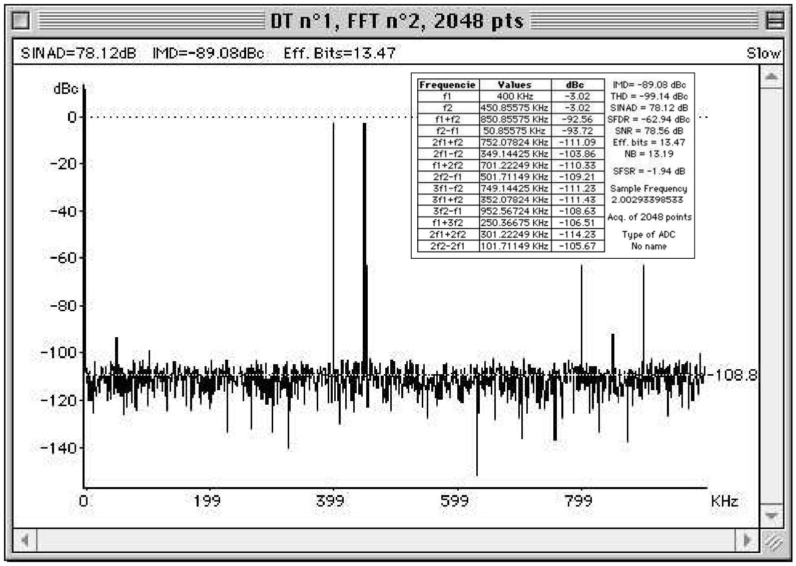

Fig. 8. Power spectrum in Dual-Tone mode.

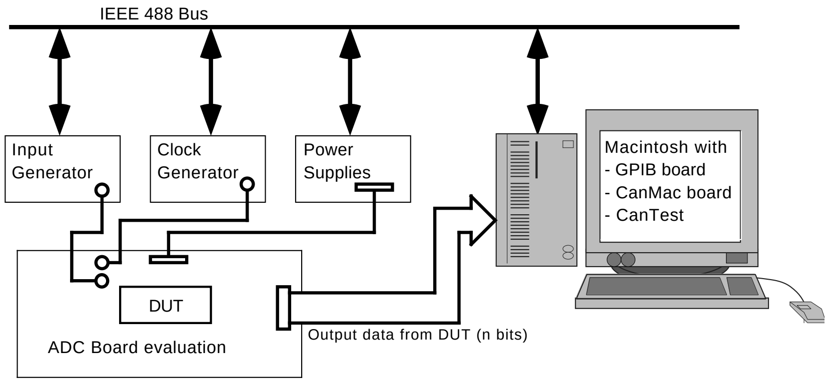

Fig. 9. Synoptic of the CanTest System.

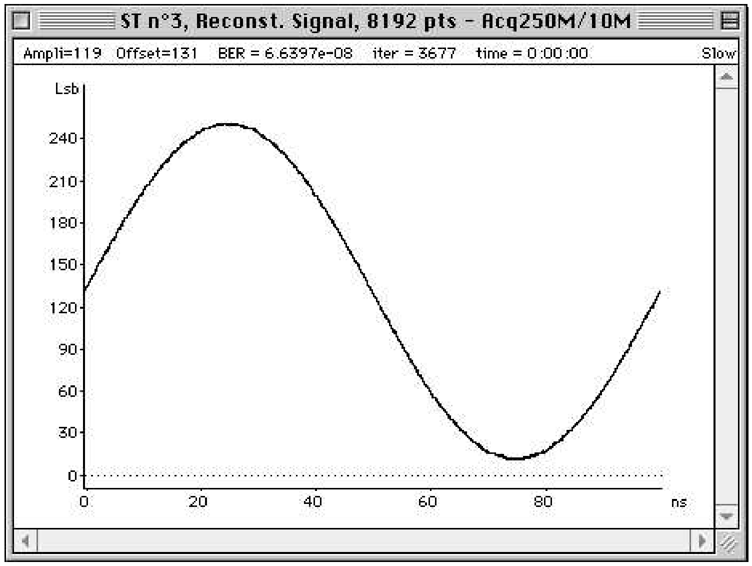

Fig. 10. Window from CanTest with the BER measurement.

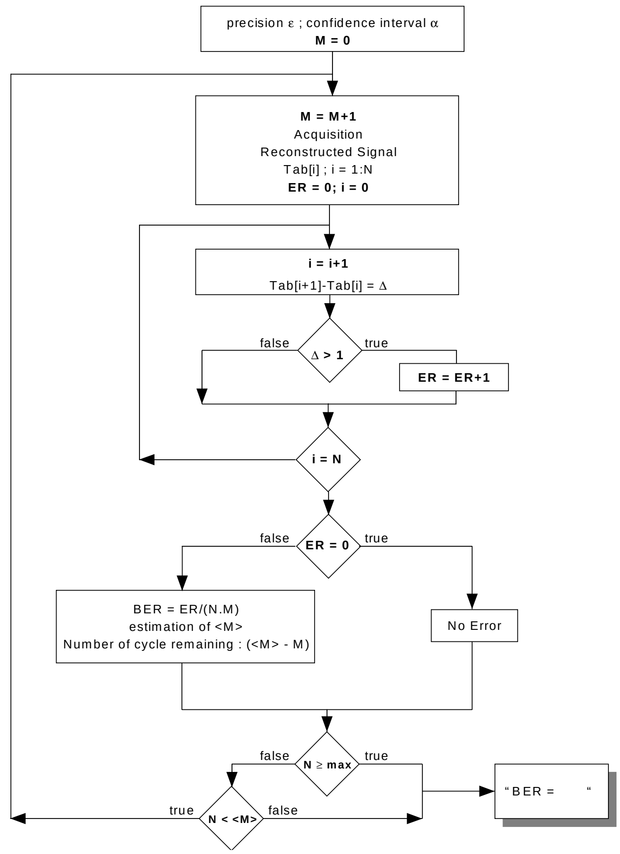

Fig. 11. Synoptic of the BER algorithm implemented in CanTest system.

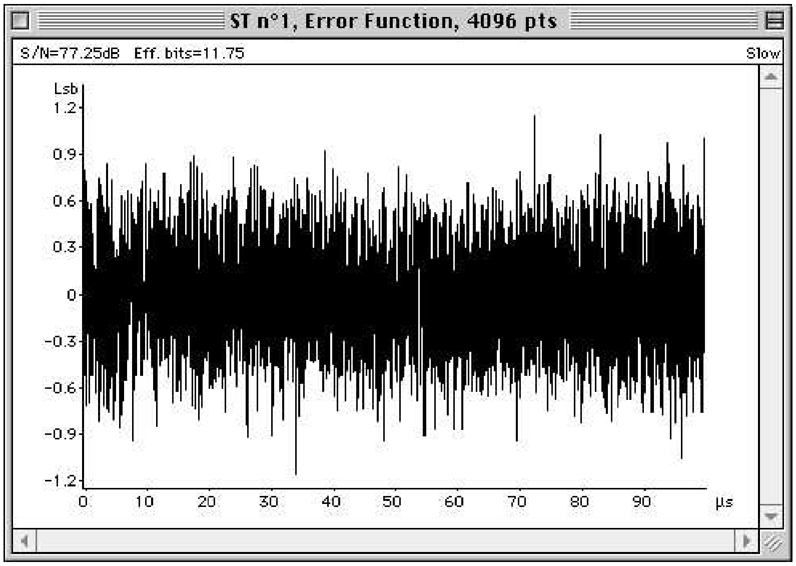

Fig. 12. Error signal with the linear algorithm.

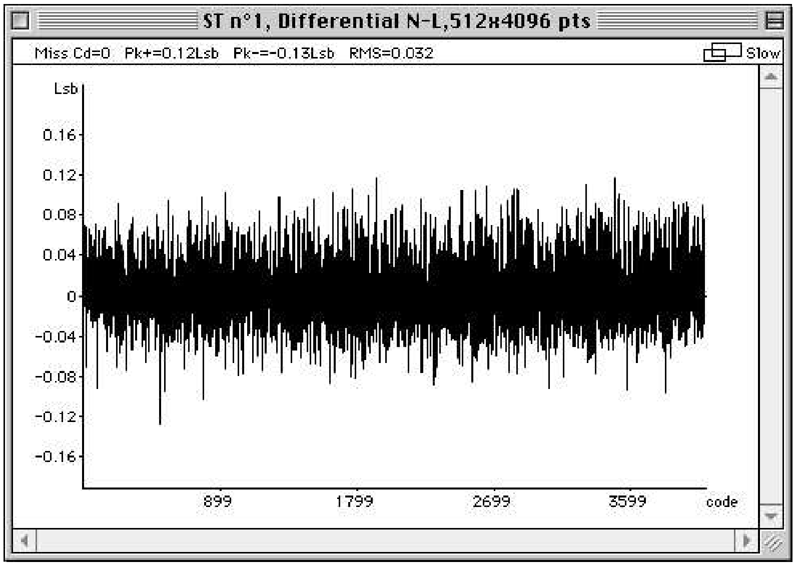

Fig. 13. Differential nonlinearity of a 12-bit at $f_{in} = 10KHz$ and $f_s = 50KHz$.

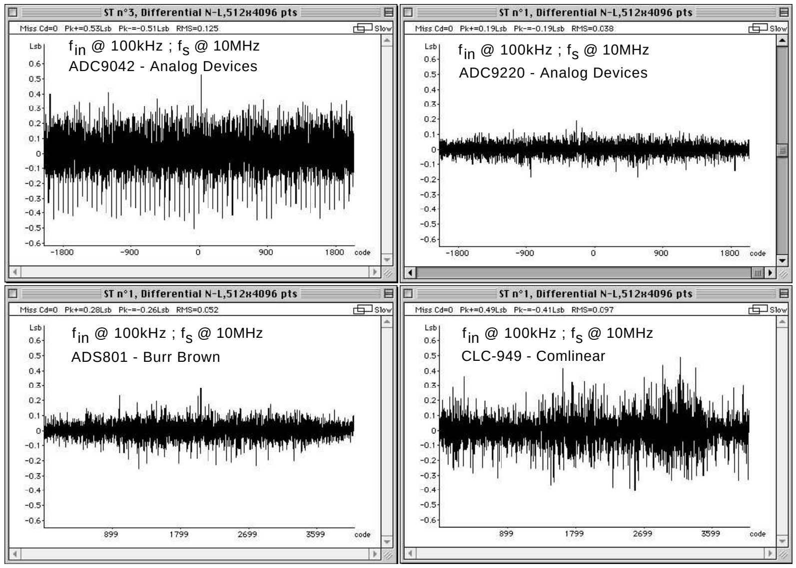

Fig. 14. Differential nonlinearity comparison for 4 ADC with the same test setup.

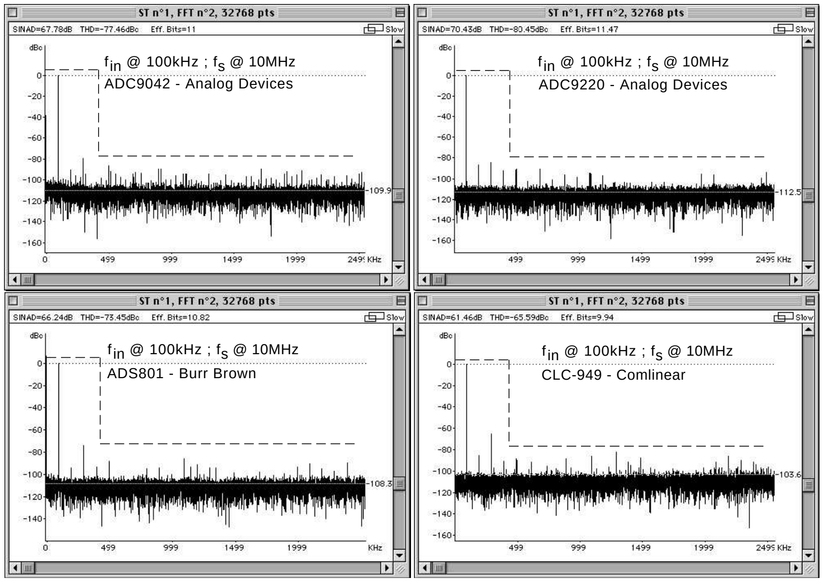

Fig. 15. Power spectrum comparison for 4 ADC with the same test setup.

List of Tables

TABLE I

Different A/D converter parameters.

A/D Converters	Architecture	Application	Compagny
ADC9042	multipass	Multichannel receivers	Analog Devices
AD9220	differential pipeline	Imaging, Communication, Data Acquisition System	Analog Devices
ADS801	differential pipeline	Digital communication, Test instrumentation, Video Digitizing	Burr-Brown
CLC949	differential pipeline	Digital communication, Test instrumentation, medical Imaging	National Semiconductor
CS5012	successive-approximation	Instrumentation	Crystal