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Abstract

This paper is concerned with the characterization of elastic targets immersed in a fluid and submitted to an acoustic impulse. Time-frequency methods have already been used in the case of scatterers of simple geometric shape. We have chosen the wavelet transform for its particular properties, such as linearity and local analysis at $\Delta f/f = c^{ste}$. We have developed an algorithm based on the behavior of the phase of the transform, which enables us to extract modulation laws (related to the dispersion law of the phase velocity), even for close echoes. In the case of spherical elastic shells, we have applied this method to both experimental and simulated signals. We point out the good agreement between theoretical and experimental results.

1- Introduction

We are interested in the information about targets of simple geometric shape, obtained from the analysis of their acoustic response. This problem requires, on one hand, the knowledge of the mechanisms generation of surface waves, and, on the other hand, the choice of relevant parameters for the identification [1-12].

In this paper, we limit ourselves to the study of signals backscattered from spherical elastic shells immersed in water. We suppose that the emitted signal is of short duration and consequently broad-band.

When one studies experimentally a problem of scattering by targets, there are two classical ways of proceeding:

-The first method uses long wave trains (about a hundred periods) and the study takes place in a monochromatic regime. The parameters obtained from backscattered echos are basically the position and width (Γ) of the acoustic resonances. The MIIR (Methode d'Isolation et d'Identification des Résonances) developed by the group in the laboratory of the CNRS-Le Havre [3] gives a simple experimental access to these parameters. They are classifying parameters that can be used in identification procedures.

-On the other hand, the second method uses very short (wide band) signals, and we measure in some sense the acoustic response of the target. In this case, the parameters that can be estimated are essentially arrival times of energy packets. These packets correspond to surface waves that turn several times around the target while losing energy during the travel.

Work done at the Laboratory of Mechanical and Acoustic (L.M.A) and the I.C.P.I (Institut de Chimie et Physique Industrielles) with the help of Wigner-Ville techniques[10] has shown that one can measure precisely the dispersion law of group velocity of the various waves involved.

In this work we want to show that, under some not very restrictive assumptions concerning the relative behavior of the analyzing wavelet and the signal, one can determine with precision the modulation laws of each surface wave (dispersion law of phase and group velocity); this allows a characterization of the target to be analyzed through an acoustic signature.

From a general point of view, the back-scattered pressure field can be decomposed into a sum of contributions: specular echo which is observed in first, corresponding to the "geometric contribution", and surface waves which can be observed during very long time in the surrounding medium (surface waves and its successive echos around the target). It corresponds to waves that reradiate energy in the liquid medium.

If the kind of surface waves are the same for targets of the same shape and thickness, the dispersion law of their phase velocity varies with respect to the characteristics of the target (density, celerity). Complete characterization of surface waves requires time-frequency methods. We will focus on the estimation of frequency modulation laws of the different wave-packets (surface waves) by the use of the wavelet transform. The use of this method is justified by its linearity and localization properties.

2- Description of the physical problem.

This problem has been mainly approached by the works of Junger, Überall, Flax, Derem [1-2,13-14].
In particular, Überall has proved the relation between the ringing of the surface waves around the scatterer and the resonance phenomenon [13]. For instance, a resonance for the mode n appears when there is n+1/2 wave length in the circonference for a spherical shell, and n wave -length for a cylinder one.

The surface waves can be splitted in two classes: the waves which have a support in the fluid media (Franz waves called creeping waves, Stoneley waves [16]), and the ones where the support is on the elastic scatterer (Rayleigh and whispering waves or gallery waves). Franz and whispering waves are related to the geometry of the target.

For an incident plane wave, the pressure scattered by a spherical elastic shell is given by [15]:

$$(1) \qquad P_d(r,t)= P_0\int e^{i\omega t}\int \eta\, e^{-i\pi/2(\eta-1/2)}(J_{\eta-1/2}\frac{N_\eta(k_1r)}{D_\eta(k_1r)} + h_{\eta-1/2}^{(1)}(k_1r)\, P_{\eta-1/2}(\cos\theta))d\eta\, d\omega,$$

where:

k_1 represents the wave number in the fluid media (water), r is the distance between the center of shell and the observation point, θ the angle between the emssion and the reception transducer. t is the time variable.

P_0 represents the amplitude of the incident wave;

N_η and D_η are functions depending of the target geometry. They are written as (6x6) determinant taking into account the boundary conditions [5,7,15,16]. The resonances coming from the interfaces (shell) correspond to the singularities of these functions (complex zeros of the denominator D_η) [7,13];

$h_\eta^{(1)}$ represents the spherical Hankel functions of the first kind;

P_η are the Legendre functions.

η: complex variable

The integral of the expression (1) on a path Γ can be decomposed into a sum of integrals due to the different singularities (branch-points and poles). Each integration around these points corresponds to a surface wave. These waves have their own time and frequency behavior.

The extraction of the resonances position of the target is done classically, in the case of a monochomatic source, by spectral analysis of the different terms of the modulus of the farfield form function [3] expressed by the discret sum (n integer variable):

$$|F_\infty(k_1r,\theta)| = \frac{1}{k_1r}\left|\frac{P_{diff}(r)}{P_{inc}(r)}\right| = \frac{2}{k_1r}\left|\sum_{n=0}^{\infty}(-i)^n(2n+1)\frac{N_n(k_1r)}{D_n(k_1r)}P_n(\cos\theta)\right|$$

By this method, only the resonances corresponding to a small imaginary part can be observed.

3- Continuous wavelet transform.

3-1: recalls and definitions

The continuous wavelet transform of an arbitrary signal s is given by the scalar product between this signal and elementary functions, called wavelets [17,18]. These functions are translated and dilated copies of a basic function g, named analyzing wavelet. The wavelet transform is given by :

$$S(b,a) = \langle g_{b,a} \mid s\rangle = \int \bar{g}_{b,a}\, s(t)\, dt = \frac{1}{a}\int \bar{g}\left(\frac{t-b}{a}\right)s(t)dt \quad , \quad b \in R, \qquad a>0$$

$\bar{g}$ denotes the complex conjugate of g, and g verifies the admissibility condition. Moreover we assume the wavelet to be progressive.

We use here the L_1 normalization for pratical reasons given in [21].

3-2: Extraction of amplitude and frequency modulation laws [11,12, 19-23].

The information obtained by the continuous wavelet transform is redundant and all the points in the half-plane have not the same importance. It appears natural to look for the most significative ones. The intuitive way is to define some trajectories representative of frequency modulation laws of the signal, along which the energy of the transform is concentrated.

From a theoretical point of view, the phase of the continuous wavelet transform with a progressive wavelet posesses many good properties. The adopted criteria for the trajectory definition is the coherency between the instantaneous frequency (phase derivative with respect to the translation parameter) of the transform and the wavelet's one. For example (fig. 1,2,3), we shall define a "ridge", as a set of points b(a) taking place like notes on a musical partition.

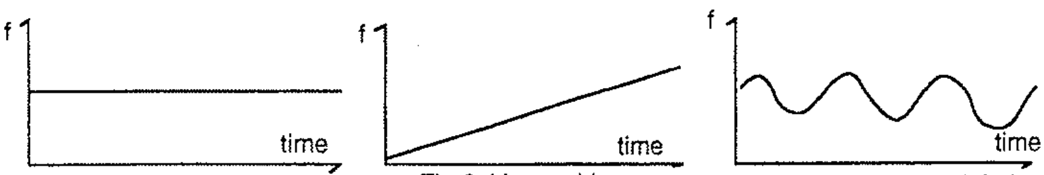

Fig 1: Sine wave — Fig 2: Linear chirp — Fig 3: Frequency modulation

Intuitively, this criteria can be seen as a generalization of the one adopted for the estimation of spectral lines [21]. The phase derivative of the wavelet transform of a monochromatic signal gives exactly the frequency of the signal. We shall now study the wavelet transform of a linear chirp with a Morlet wavelet.

Define the signal s(t) as: $s(t)=\exp\left(i(\omega_1 t+\alpha t^2)\right)$

and the wavelet $g(t)=\exp\left(i\omega_o t-\frac{t^2}{2}\right)$

If we call $\phi_T(b,a)$ the phase of the wavelet transform, one can easily show that:

$$\frac{\partial\phi_T(b,a)}{\partial b}=\frac{\omega_o}{a}+\frac{1}{1+4\alpha^2a^4}\left(\omega_1-\frac{\omega_o}{a}+2\alpha b\right)$$

The ridge definition by $\frac{\partial\phi_T(b,a)}{\partial b}=\frac{\omega_o}{a}$ implies that: $\omega_1+2\alpha b=\frac{\omega_o}{a}$

Then, along this trajectory, we have: $\frac{\partial\phi_T(b,a)}{\partial b}=\omega_1+2\alpha b$, and the phase derivative of the wavelet transform gives us exactly the frequency modulation law of the signal (Fig 4).

For the purpose of this paper, one may focus on waves packets with linear frequency modulation law and gaussian amplitude defined by: $s(t)=\exp\left(i(\omega_1 t+\alpha t^2)\right)\exp\left(\frac{-t^2}{2\sigma^2}\right)$.

Along the ridge, one can show that: $\frac{\partial\phi_T(b,a)}{\partial b}=\omega_1+\left(\frac{2\alpha}{1+\frac{a^2}{\sigma^2}}\right)b$

In this case, the estimated ridge doesn't give the true frequency modulation law. Nevertheless the error is small and decreases when σ increases and the estimation of the ridge is again valid, even for wave packets containing few oscillations (Fig 5).

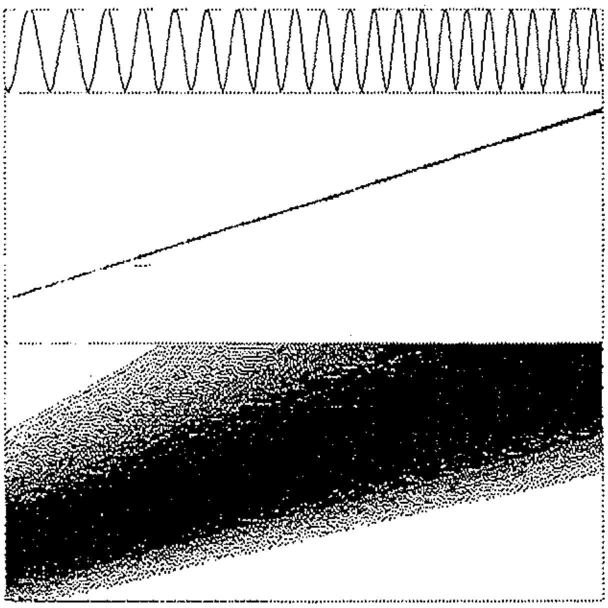

Fig: 4

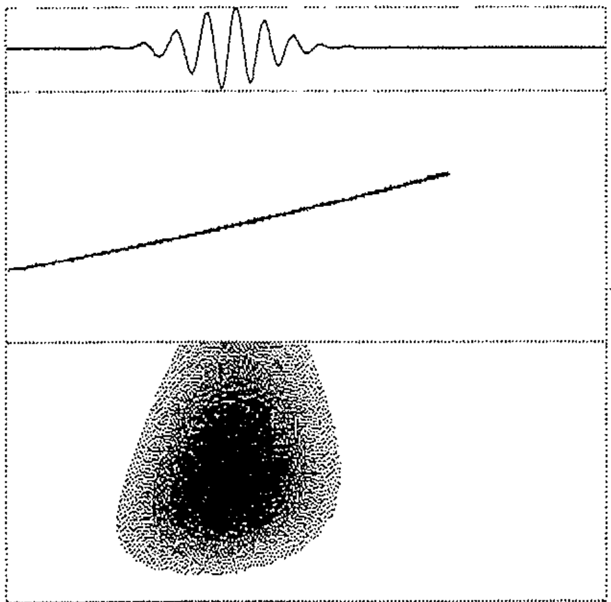

Fig: 5

Fig 4 and 5: Influence of the amplitude modulation law on the ridge estimation

In fact, at the time we write this paper -more than a year after the conference- we know that this criteria is valid for a large class of signals, asymptotic with respect to the wavelet [21-23]. This method compared to the one used in [11,12] can be applied to waves packets containing few oscillations. The stationary phase approximation is a general framework for the study of this kind of signals. Here, we have just described the intuitive idea based on the generalization of spectral lines and supported by some explicit examples.

4- Time-scale analysis of echoes scattered by a spherical elastic shell.

We consider spherical scatterers immersed in water. The thickness of the shells is small with ratio of radii r_i/r_e of 0.9 (Fig. 6,7), and 0.96 (Fig. 8), (r_i, r_e : internal, external radius r_e=3cm). They are made of duralumin (ie: an alloy of 95% aluminium and 3,5% copper), except for the last one which is made in nickel-molybdene. To cover a wide band of analysis two transducers have been used. The first has a central frequency of 500kHz with a band-width of 400kHz at -3dB (Fig 6), the second has a central frequency of 250kHz with a band-width of 150kHz (Fig 7,8) .

We have applied a wavelet analysis both to backscattered echoes (experimental and numerical signals). We present here experimental results. For the different cases studied (Fig. 6-8), we have displayed the impulse response of the target (a) and two complementary informations of the wavelet transform: the phase (b) and the modulus (c). We have take out the specular return (Fig. 6 and 8) in order to study only the surface waves and to extract their modulation laws.

The modulus of the transform is shown in linear scale with a density of points. The modulus is maximum (minimum) when it is black (white). The phase representation is between (0, 2π). The ridge is represented by black dots over the phase diagram. The abscissa represents the translation parameter (time). The ordinate represents the dilation parameter in hyperbolic scale (linear in frequency), decomposed in 150 voices. The analyzing wavelet (standard Morlet wavelet) has been defined with the following caracteristics: ω_0=5.336, and g(t)=0, for a=1, when $|g(t)| \leq 10^{-5}$.

The sampling rate of the time signal is 10^7Hz. The analysis is respectively performed with a frequency range of [208.33kHz to 833.33kHZ], [156kHz to 625kHz] and [78kHz to 500kHz].

Roughly speaking, the dispersion law of the surface waves can be seen in observing the inclination of the waves packets in the time-frequency space. We note also an attenuation corresponding to a loss of energy during the time (analysis of the modulus). The dispersion law of the phase velocity can be estimated from the phase of the wavelet transform by extraction of the ridge.

The analysis of two successive waves packets of the same type allows us to extract the dispersion law of the group velocity.

We can see on the figures c in accordance with the theoretical study of the scattering, the specular echo or geometric contribution (1st waves packet) and the surface contributions.

Figure 6: The shell has a ratio of radii 0.9. The second structure observed is the whispering gallery wave (fig.6.c). The fourth structure corresponds to the same wave after it has turned around the shell. The 3rd wave-packet which is not dispersive has not been studied for the present time. For each echo, the ridge gives an estimation of the dispersion law of the phase velocity (fig 6.b) for the gallery wave. The range of the ridge frequency is here 366kHz-669kHz. The horizontal ridges are characteristics of the non-dispersive echo. This ridge is located on the central frequency of the transducer.

Figure 7: displays the analysis of the response of the same shell than figure 6, but with a different transducer. The specular echo is not analysed.The modulation law varies with respect to the frequency of the excitation. The frequency of the ridge is here, over 223kHz to 398kHz. The behavior of waves depends of the central frequency of the excitation.

Figure 8: represents the wavelet transform of a signal scattered by a nickel-molybdene shell with a ratio of radii r_i/r_e=0.96. The external radius r_e is the same that the duralumin shell (fig 6). The echos structure is similar to fig 6. The analysis of this figure shows the relation between the modulation law and the characteristic (celerity and density) of the shell. Ridge frequency is 136kHz to 401kHz.

5- Conclusion

The dispersive behavior of the surface waves has led us to choose a time and scale method. By an another way the separation of close echoes requires a linear method.

The ridge associated to the continuous wavelet transform seems to be a promising tool for the systematic study of this type of waves. We may consider the response of the target as an acoustic signature with respect to the analyzing wavelet.

Acknowledgements

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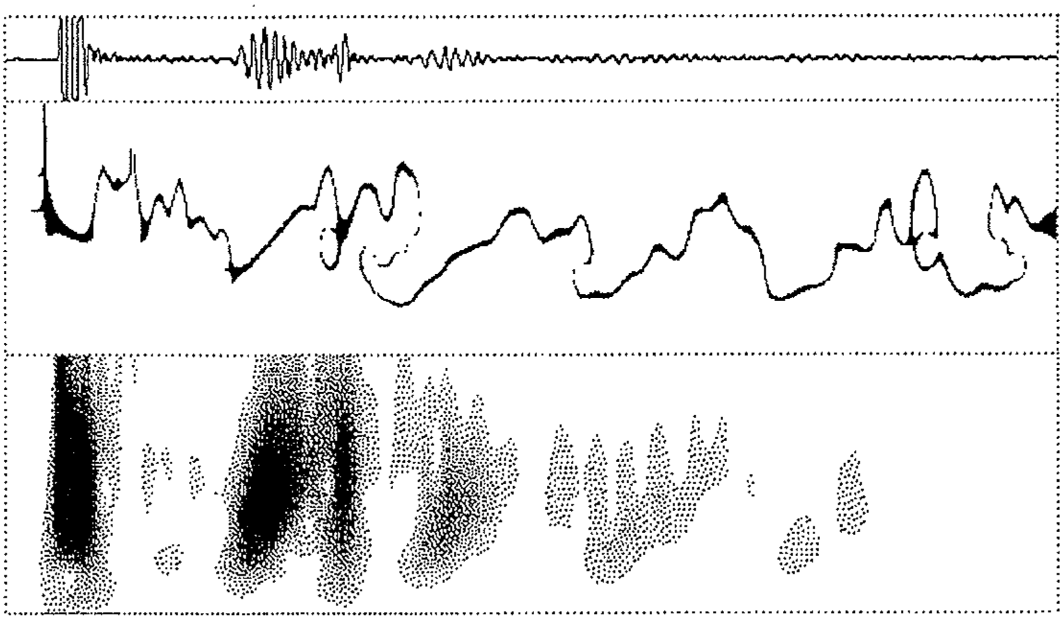

Fig. 6: Impulse response of a thin spherical shell of duralumin with an external radius $r_e = 0.03m$. $r_i/r_e = 0.9$ (r_i is the internal radius). (whispering-gallery waves 2 and its successive echo after to have turn round the target 4, 5, 6). Central frequency=500kHz Ridge frequency is 366kHz to 669kHz

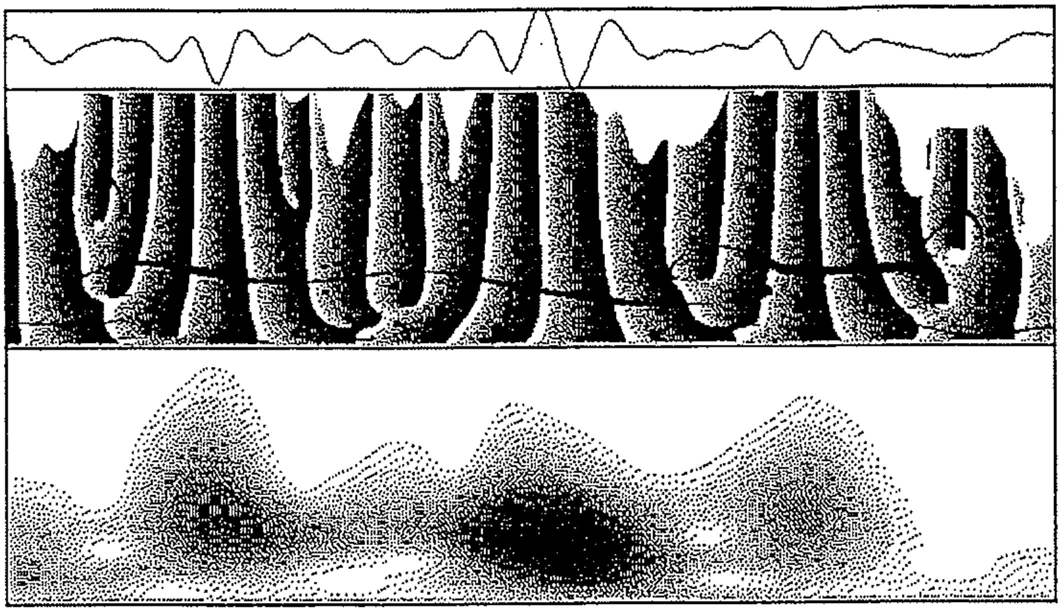

Fig. 7: Impulse response of a thin spherical shell of duralumin with an external radius r_e= 0.03m. r_i/r_e= 0.9 (r_i is the internal radius). Central frequency=250kHz Ridge frequency is 223kHz to 398kHz

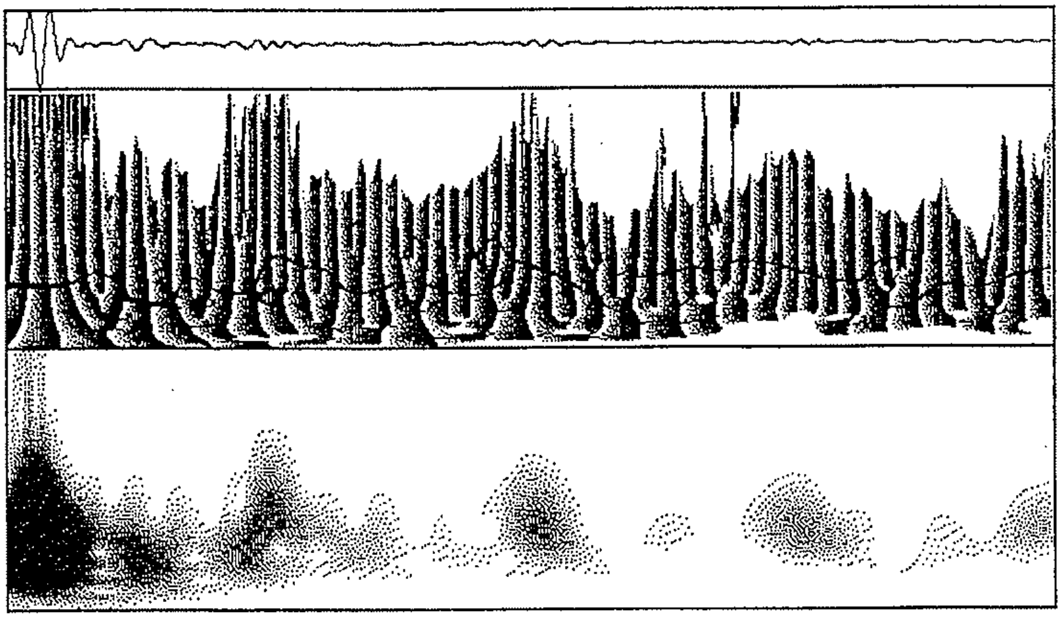

Fig. 8: Impulse response of a thin spherical shell of nickel-molybdene with an external radius r_e= 0.03m. r_i/r_e=0.96 (r_i is the internal radius).Central frequency=250kHz Ridge frequency is 136kHz to 401kHz specular echo with whispering waves (surface waves)

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