Band Structure Analysis of Crystals with Discontinuous Metallic Wires

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***Abstract*— The band structure for normal propagation of crystals with finite straight metallic wires is studied for different wire diameters and lengths. The crystal is considered as a set of parallel grids. Dispersion characteristics are obtained by using a transmission line model where the parameters are calculated from the reflection and transmission coefficients of the grids. These coefficients are computed rigourously with a Finite Difference Time Domain (FDTD) code. Simulated and experimental results for two structures with the dual behavior, pass-band and stop-band, are presented. This study have potential applications in electrically controlled microwave components.**

***Index Terms*— Metallic crystals, band structure, periodic structures**

I. INTRODUCTION

RECENTLY, the propagation of electromagnetic waves in periodic structures has received important interest [1]. Potential applications have been suggested in microwave and antenna domains, such as suppressing surface waves [2], designing directive antennas [3], or creating controllable beams [4,5].

The propagation of waves in periodic structures is described by means of a band theory. Different methods have been proposed for computing the band structure of a periodic structure, *e.g.*, the average field method [6], the order-*N* method [7], and a hybrid plane-wave-integral-equation method [8]. A particular interest has been given to the dispersion characteristics of periodic structure formed by infinitely long metallic wires [1-10]. To our knowledge, the band structure of periodic materials with finite straight wires have not been studied enough. These materials are interesting for designing reconfigurable microwave components. In [5], the authors have proposed to commute between continuous and discontinuous wire structures for reconfiguring the radiation patterns of an antenna. Indeed, these two structures behave differently at low frequencies, by presenting a pass-band and a stop-band for discontinuous and continuous wires cases, respectively. To obtain the commutation, the authors have placed periodically diodes in the broken wires. When the diodes are switched on, the structure behaves as a continuous wire medium, if the impedance of the diodes are neglected, whereas when the diodes are switched off, the structure behaves as a discontinuous wire medium. However, the band structure of the discontinuous wire medium for different wire diameters and lengths have not been studied enough.

In this contribution, numerical results are presented for the pass-bands and stop-bands of these 3-D periodic structures, at normal incidence. To compute the propagation constant, a transmission line model is used, where a 2-D periodic structure (grid) of discontinuous wires is modelled by a T-circuit. The T-circuit parameters are written in terms of the S-parameters of the grid, computed rigourously using the FDTD method. Experimental results for two structures with the dual behavior of their bands are also presented.

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II. TRANSMISSION LINE MODEL AND FDTD METHOD

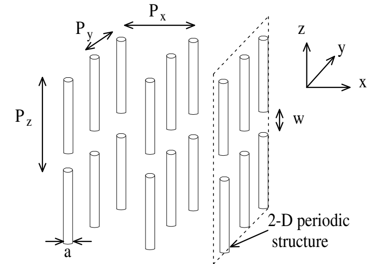

Fig. 1. Infinite 3-D periodic structure of discontinuous metallic wires in air.

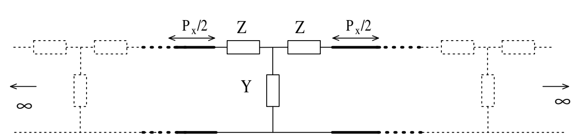

Fig. 2. Transmission line model of the periodic structure.

The infinite 3-D periodic structure of perfect metallic wires shown in Fig. 1 is considered. Its parameters are the periods P_x, P_y and P_z, the wire diameter a and the width w. The propagation of the transverse Electric field in x-direction is considered. To compute the propagation constant β_x, the transmission line model shown in Fig. 2 is used, where a 2-D periodic structure in y-direction (see Fig. 1) is modelled by a T-circuit. The propagation constant β_x of this transmission line is obtained from its well known dispersion equation, which is written [11]

$$\cos(\beta_x P_x) = (1+ZY)\cos(kP_x) + j\left(Z + \frac{Y}{2}\left(1+Z^2\right)\right)\sin(kP_x) \tag{1}$$

where k is the free space wave number. The expressions for Y and Z are derived in terms of the complex reflection and transmission coefficients r and t of the 2-D periodic structure, at normal incidence. By converting the chain matrix of the T-circuit to the S matrix, r and t can be expressed in terms of Z and Y. After inverting these relations, Z and Y are written as functions of r and t

$$Y = \frac{(r-t-1)(r+t-1)}{2t} \quad (2)$$

$$Z = \frac{-r+t-1}{r-t-1} \quad (3)$$

In this work, r and t coefficients are computed rigourously with the FDTD method, where Floquet boundaries conditions and a thin mesh ($\triangle = Period/80$) are used. It should be noted that the T-circuit model gives more precise results than the simple circuit using only the admittance Y. Indeed, the impedance Z is negligible for thin wire, but it is not for thick wire. Only the fundamental mode is considered, then the limitations $P_y \leq P_x$, $P_x \leq \lambda$ and $P_z \leq \lambda$ are used.

III. Results

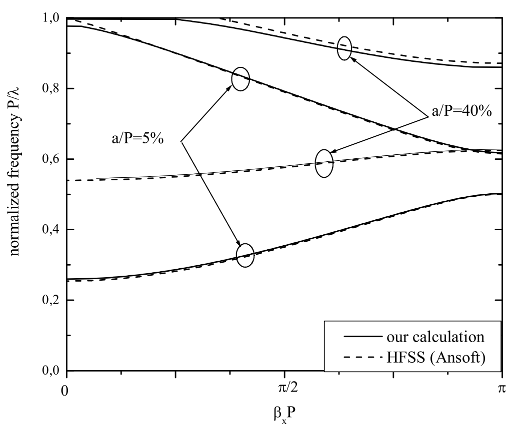

Fig. 3. Comparison of the dispersion diagrams calculated with the present method and obtained with a Finite Element method (HFSS) for $P_x = P_y = P$, $a/P = 5\%$ and $a/P = 40\%$.

To validate the method, Fig. 3 presents the computed propagation constants for a continuous wire medium ($w = 0$) compared with the results of a Finite Element method (HFSS-Ansoft), which considers all modes. A good agreement is observed between the results for both thin and thick wires. Fig. 4 shows the dispersion diagrams of structures with continuous ($P_x = P_y = P$, $a/P = 5\%$) and discontinuous ($P_z = P$, $w/P_z = 5\%$) wires. The dual behavior in the pass-band and stop-band of these structures is nearly obtained in the two first bands. The limits of the two first bands of these structures are now studied for different wire lengths and diameters. We consider $P_x = P_y = P$. We consider three values of the period in z-direction, $P_z = P$, $P_z = 2P$ and $P_z = P/2$.

Fig. 5 presents the limits of the two first bands for continuous and discontinuous cases versus the fill factor a/P, for different widths w. To obtain a dual behavior in the first band, the lower limits of stop-band for discontinuous wires and pass-band for continuous wires must match. In addition, to obtain the dual behavior in the second band, the upper end of stop-band for discontinuous wires must match with the upper end of pass-band for continuous wires. To satisfy these conditions, for small diameter, the width must be near 5% of the period. When the diameter increases, the width should be increased.

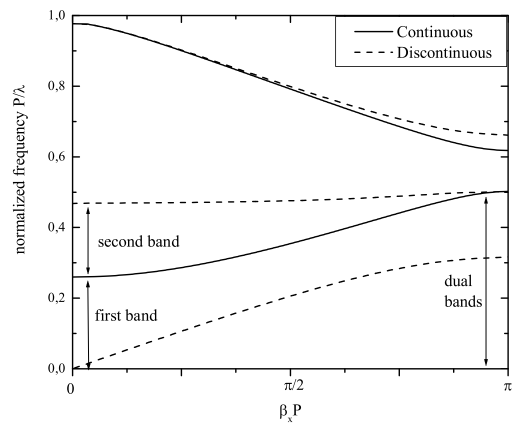

Fig. 4. Dispersion diagrams for structures with continuous wires ($a/P = 5\%$) and discontinuous wires ($P_z = P$, $w/P = 5\%$).

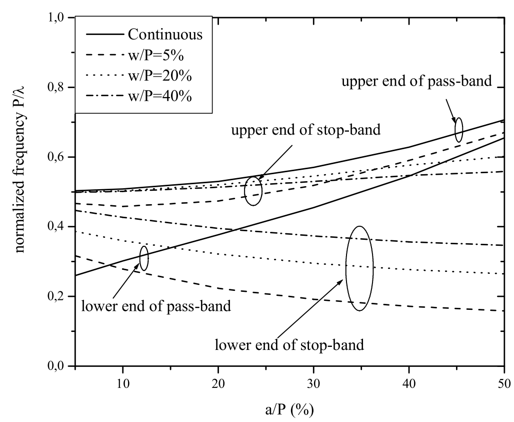

Fig. 5. Two first bands limits for structures with continuous and discontinuous wires ($P_z = P$) versus fill factor a/P, for different widths w/P.

Fig. 6 presents the same diagrams as that of Fig. 5 for the case $P_z = 2P$ (within the range $P/\lambda \leq 0.5$). From these curves, it can be seen that is difficult to obtain the dual behavior in both the first and second bands. To match the first bands, the width w must be very large ($w/P > 40\%$).

Fig. 7 presents the same diagrams as in Fig. 5 but for $P_z = P/2$. For this case, for thin wires, to match the first band limits, the width w must be very small ($w/P < 2.5\%$), whereas the second limits can be matched more easily.

The results presented in Figs. 5 to 7 are useful for the design of two structures with the dual behavior in their bands.

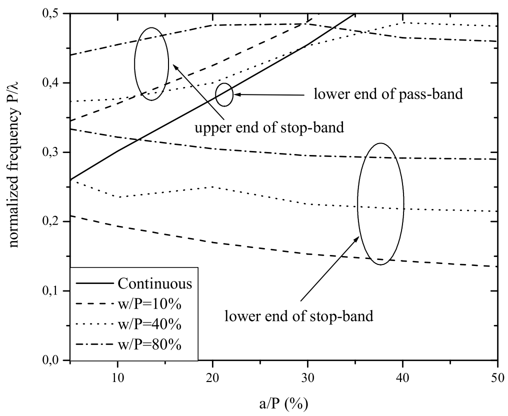

Fig. 6. Two first bands limits for structures with continuous and discontinuous wires ($P_z = 2P$) versus fill factor a/P, for different widths w/P.

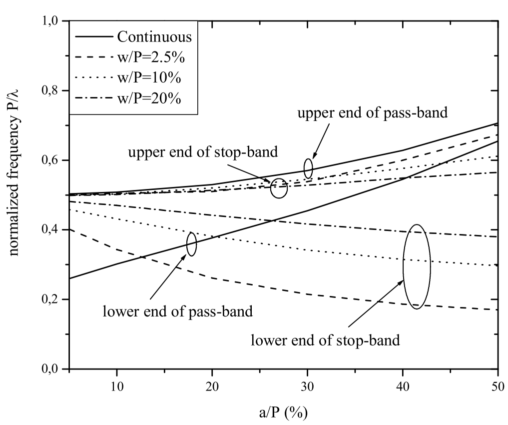

Fig. 7. Two first bands limits for structures with continuous and discontinuous wires ($P_z = P/2$) versus fill factor a/P, for different widths w/P.

To validate the proposed model, two structures with five rows in x-direction were fabricated (with wood supports) and their transmission coefficients were measured in a anechoic chamber using two horn antennas and a network analyzer. The fabricated structures have the finite dimensions $L_z = 280mm$ and $L_y = 600mm$ and the following parameters: $P_z = P = 40mm, a = w = 2mm$. The frequency range is $1GHz$-$4.5GHz$. The structures were at the distances $40m$ and $2mm$ from the transmitter and the receiver, respectively (at $2mm$ the evanescent waves are negligible, this distance has been chosen to avoid edge effects). A measurement without the structures was carried out for the normalization. For comparison, numerical simulations (FDTD) for the transmission coefficients were also carried out. Fig. 8 presents the simulated and measured transmission coefficients of the continuous and discontinuous wires structures, illustrating the dual bands. The simulated and experimental results are slightly different because of the finite dimensions of the fabricated structures.

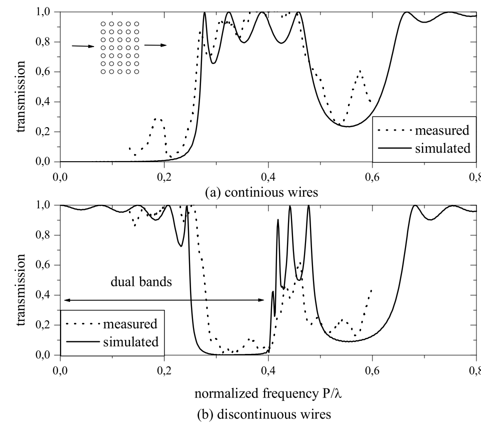

Fig. 8. Simulated (FDTD) and measured transmission coefficients for two structures with five rows of : (a) continuous wires ($a/P = 5\%$) and (b) discontinuous wires ($P_z = P$, $w/P = 5\%$).

IV. CONCLUSION

The band structure for normal propagation of crystal formed by discontinuous metallic wires has been analyzed for different wire diameters and lengths. To compute the propagation constant, the transmission line model and the T-circuit model have been used. The proposed approach has been validated with experimental results. The obtained results are useful for the design of two structures with the dual behavior of their band characteristics that can be used for controllable antennas applications.

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