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Simple model of an electrolytic capacitor taking into account the temperature and aging time

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Abstract With their large capacity and low price, electrolytic capacitors are used in many fields of power electronics, mainly for filtering and energy storage functions. Their characteristics change strongly according to the frequency, temperature, and aging time. These last two parameters are generally not taken into account in electrical simulation software. To realize these precise simulations, we propose to determine parameters of an electrical model of electrolytic capacitor by using the genetic algorithm method. This identification was carried out for a large electrolytic capacitor with rated value of 4,700 $\mu\text{F}/500\text{ V}$. The comparison between measurements and model for different temperatures and aging times gives good results. The model shown in this paper has a precision that is much better than a standard model for temperatures lower than 0°C . But this model does not comprise too many elements, which allows it to be integrated easily in simulation software.

1 Introduction

With their large capacity and low price, electrolytic capacitors are used in many fields of power electronics, mainly for filtering and energy storage functions. Their characteristics change strongly according to frequency, temperature, and aging time. These parameters are generally not taken into account in electrical simulation software.

In many applications such as transport, aerospace engineering, or in the military field, the variations in temperature and degradations of characteristics with aging time are significant. A model taking into account characteristics variations versus temperature and aging time is essential.

In this study, we propose a simple model taking into account these characteristics variations of the electrolytic capacitors according to temperature, aging time, and frequency. One of our objectives is to obtain a good compromise between simplicity and precision.

The majority of the models described in the literature use the physical characteristics of the component [1, 2, 3, 4, 5, 6]. Unfortunately these models are very seldom available. This paper led us to use the component's impedance function of the frequency and DC leakage current.

The parameters of this electrical model capacitor are calculated by using a genetic algorithm method [7]. The model used could be integrated in any type of simulation software. This identification was carried out for a large electrolytic capacitor with a rated value of 4,700 μ F/500V.

2 Presentation of the capacitors

The capacitors taken as an example for this study are aluminum electrolytic capacitors rated 4,700 μ F, 500 V, 85°C. They are mainly used for filtering and energy storage.

The capacitors running can be represented by its impedance according to frequency. These impedance variations (magnitude and phase) are represented in Figs. 1 and 2.

Fig. 1 Magnitude variation of $Z(f)$ versus temperature

Fig. 2 Phase variation of $Z(f)$ versus temperature

We notice that the temperature is a significant factor upon the impedance variations, mainly for temperatures lower than 0°C.

The wearout of an aluminum electrolytic capacitor is due to vaporization of electrolyte which leads to a drift of the main electrical parameters of the capacitor [8, 9, 10]. To observe this evolution, we applied an accelerated thermal aging to the capacitors, under their nominal voltage and maximum temperature (500 V and 85°C, respectively). In Figs. 3 and 4, for this aging test, we represent the impedance Z (magnitude and phase) versus frequency, measured at 25°C for the aging times of $t=0$ h, 2,200 h, 4,700 h, and 5,700 h.

Fig. 3 Magnitude variation of $Z(f)$ versus aging time measured at 25°C

Fig. 4 Phase variation of $Z(f)$ versus aging time measured at 25°C

3 Parameters identification versus frequency

The parameters identification is carried out with a genetic algorithm. It identifies directly the elements of an equivalent circuit.

It is usual to find the parameters of the model starting from measurements of impedance and phase of the capacitor according to the frequency.

Optimization is defined by a function minimizing the error between measurement and calculation of the algorithm. This function ($invf$) is the reverse of the fitness function:

$$invf = \sum_1^i (P_r - P_{rc})^2 + (P_i - P_{ic})^2 \quad (1)$$

where:

- i =Number of experimental frequency points
- P_r =Real part of the measured impedance
- P_{rc} =Real part of the calculated impedance
- P_i =Imaginary part of the measured impedance
- P_{ic} =Imaginary part of the calculated impedance

The data required for the algorithm are as follows:

- A range of the parameters to be determined (order of magnitude)
- The iteration number
- Precision of calculation
- Methods of selection, crossover, and mutation
- A file containing the points of measurement according to frequency

4 Models used

Different electrical models can be used to represent the electrolytic capacitor. Two principal constraints must be considered for this study:

- It must be easily integrated into any type of simulation software
- It must be used in the frequency range of the identified components (DC to several MHz)

All electrical models can be written in the Laplace system in the following form:

$$Z(s) = \frac{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s^1 + b_0}{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s^1 + a_0} \quad (2)$$

where:

- Z =Transfer function of the model
- s =Laplace operator
- a_i, b_i =Coefficients

To have a physical representation of the model and satisfy the previous conditions, it is necessary to have $m \geq n$.

4.1 Model_1

Most of the series or parallel simple models are not appropriate. The simplest model usable is represented in Fig. 5.

Fig. 5 Simple electrical model

The Laplace formulation corresponding to this model is given by:

$$Z_0(s) = \frac{(R_{a0}L_0C_0 + R_{b0}L_0C_0)s^2 + (R_{a0}C_0R_{b0})s + R_{a0} + R_{b0}}{L_0C_0s^2 + C_0R_{b0}s + 1} \quad (3)$$

We prefer the following model, represented Fig.6, which is a better representation of the technology of an electrolytic capacitor:

$$Z_1(s) = \frac{(R_a + R_b)R_cL_1C_1s^2 + (R_aR_bR_cC_1 + L_1R_c + L_1R_b + L_1R_a)s + (R_a + R_c)R_c}{R_cL_1C_1s^2 + (R_bR_cC_1 + L_1)s + R_b} \quad (4)$$

The capacitance C_1 represents total capacitance between the anode and the cathode. Resistance R_a includes different terms: terminals resistance, tabs resistance, foils resistance, resistance of the impregnated electrolyte paper, dielectric resistance, and tunnel-electrolyte resistance. R_c represents the leakage current, which depends on the quality of the dielectric material. Inductance L_1 is especially dominated by the loop area from the terminals and tabs outside of the active winding. Resistance R_b is essential to have in a physical representation of the model (see Eq. 2); however, this one does not have much of an influence in the range of measurement.

Fig. 6 Electrical model (model_1)

The identification of parameters is carried out for a sound capacitor at different temperatures and at 25°C for different aging times until 5,700 hours.

The resistance R_c represents the leakage current (DC bias), which is given by the DC measurement according to aging time. In the case of the studied components, it is considered constant according to aging and its expression is a function of the bias voltage:

$$R_c(U) = \frac{U}{\frac{U}{R} + I_0 \exp((U - U_n)k) - I_0 \exp(-U_n k)} \quad (5)$$

where:

- U =Bias voltage (V)
- U_n =Nominal voltage of the capacitor (V)
- R =Dielectric resistance, considered constant for the weak voltages (lower than $U_n/2$) (Ω)
- I_0 =Leakage current for the voltage U_n (A)
- k =Coefficient dependant on the component

The identification with the genetic algorithm is used to find the other parameters of the model.

Figure7 shows the average error between measurement and simulation, according to temperature, for the sound capacitor. We can see that the model used is incorrect for temperatures lower than 0°C.

Fig. 7 Results of identification for different temperatures

Figure 8 shows the results of the parameters identification at 25°C for model_1 and for aging time $t=0$ h. We show only the amplitude because the phase is in agreement with this one.

Fig. 8 Results of identification at $t=0$ h and 25°C

The quality of the identification can be represented by the average error between measurement and simulation, according to the aging time. Figure 9 shows the average error in the frequency range versus aging time. We can see that the model used is correct for the positive temperature and for all aging times.

Fig. 9 Average error between measurement and calculation at 25°C

The results of identification at 25°C according to the aging time can be represented by the following set of equations:

$$\begin{aligned}
 R_a &= a_1 + a_2 t_v \exp(a_3 t_v) \\
 C_1 &= b_1 - b_2 t_v \\
 L_1 &= L_1 \\
 R_c &= R_c(U) \\
 R_b &\gg 75.4 \times 10^{-3} \Omega
 \end{aligned} \tag{6}$$

where:

- t_v =Aging time at 85°C (h)
- a_i, b_i =Coefficients which are a function of the component

The resistance R_b can be considered as constant because, in this frequency range, and for the capacitor used in this study, it does not have any influence. The variation of L_1 is negligible, so we do not consider it.

4.2 Model_2

In order to improve the results throughout the temperature range, and particularly at the negative temperatures, we propose another model, model_2, which is represented in Fig. 10. It constitutes the elements of the model_1 (C_1, L_1, R_a, R_b, R_c) identified at the maximum temperature of the component for which the impedance is weakest (85°C). At low temperature, the shape of the curve of impedance is due to the modification of the state of the electrolyte (thickening) [4]. Some elements (C_2, R_{2a}, R_{2c}) are added in series to this model in order to take into account the influence of the reduction of temperature and the shape of the curve Z versus frequency.

Fig. 10 Electrical model (model_2)

The corresponding Laplace formulation is given by:

$$Z_2(s) = Z_1(s) + Z_{02}(s) \quad (7)$$

where:

$$Z_{02}(s) = \frac{C_2 R_{2a} R_{2c} s + R_{2c}}{C_2 (R_{2a} + R_{2c}) s + 1} \quad (8)$$

and $Z_1(s)$ is the transfer function of model_1.

4.2.1 Values of the parameters of model_1 at 85°C

Model_1 here is fixed according to temperature. The values of its parameters are given at 85°C with the genetic algorithm. The results obtained are as follows:

$$\begin{aligned}
 R_a &= 10.7 \times 10^{-3} \Omega \\
 C_1 &= 4.44 \times 10^3 \text{ F} \\
 L_1 &= 18.9 \times 10^{-9} \text{ H} \\
 R_c &= 377 \times 10^3 \Omega \\
 R_b &\gg 75.4 \times 10^{-3} \Omega
 \end{aligned} \tag{9}$$

R_c resistance changes with temperature, thus:

$$R_c(U, T) = \frac{U}{\frac{U}{R(T)} + I_0(T) \exp((U - U_n)k) - I_0(T) \exp(-U_n k)} \tag{10}$$

where, for studied component:

$$\begin{aligned}
 R(T) &= 2.49 \times 10^6 \exp(18.9 \times 10^{-3} T) \\
 I_0(T) &= 452 \times 10^{-6} \exp(24.9 \times 10^{-3} T) \\
 k &= 50 \times 10^{-3} \text{ V}^{-1} \\
 T &= \text{temperature of the component (}^\circ\text{C)}
 \end{aligned}$$

4.2.2 Determination of the added elements

Initially, we identify the capacitance C_2 analytically. Then, we identify the resistances R_{2a} and R_{2c} with help of the genetic algorithm.

4.2.2.1 Determination of the C_2 capacitance

The C_2 capacitance is deducted from the frequency resonance f_c (see Fig. 11). This frequency is identifiable when the magnitude of Z is at a minimum. At the frequencies higher than f_c , and included in our range of measurement, model_1 at 85°C can be simplified into a $R_a L_1$ series circuit. By neglecting the influence of R_{2c} , we can determine the value of C_2 by the following relation:

$$C_2 = \frac{1}{L_1 (2\pi f_c)^2} \tag{11}$$

4.2.2.2 Determination of R_{2a} and R_{2c}

The method used is the same as in model_1. Here, we show only the results of the identification in Figs. 11, 12, 13, and 14 at temperatures ranging from -30°C to 40°C . Figure 15 represents the

error between measurement and simulation, according to temperature. We see that this model is much more precise than model_1, with the average error at -30°C being less than 20%.

Fig. 11 Results of identification at -30°C

Fig. 12 Results of identification at -20°C

Fig. 13 Results of identification at 0°C

Fig. 14 Results of identification at 40°C

Fig. 15 Average error between measurement and calculation

4.2.3 Variations of the added elements versus temperature

The variation of R_{2c} for temperatures lower than 85°C can be represented by Eq. 12, which is of exponential form. For temperatures higher than 0°C, the influence of R_{2c} becomes negligible compared to the equivalent circuit composed of R_{2a} and C_2 . R_{2a} and C_2 have an influence only for temperatures lower than 0°C, and their variations are expressed by Eq. (13):

$$R_{2c}(T) = (85 - T)a_{c1} \exp(-a_{c2}T) \quad (12)$$

$$\begin{aligned} C_2(T) &= b_{c1} \exp(b_{c2}T) \\ R_{2a} &= R_{2a} \end{aligned} \quad (13)$$

where:

- a_{ij}, b_{ij} =Coefficients which are a function of the component
- T =Temperature of the component (°C)

5 Coupling of the models according to temperature and aging time

According the previous results, we can describe a complete model (model_2) considering aging time and temperature of use of the component.

The resulting elements are written as follows:

$$\begin{aligned} R_a &= a_{10} + a_{20}t \exp(a_{30}t) \\ C_1 &= b_{10} - b_{20}t \\ L_1 &= L_1 \\ R_c &= R_c(U, T) \\ R_b &\gg 75.4 \times 10^{-3} \Omega \end{aligned} \quad (14)$$

where:

- a_{ij}, b_{ij} =Coefficients for the sound component at 85°C
- t =Aging time at temperature T (°C)

The time of aging t at the temperature T (°C) can be extrapolated starting from an accelerated aging by the Arrhenius law [11, 12]:

$$t = t_v \exp \left[\frac{E_a}{k} \frac{T_v - T}{(T_v + 273)(T + 273)} \right] \quad (15)$$

where:

- t, T =Time and real temperature of aging
- t_v, T_v =Time and temperature of accelerated aging
- E_a =Activation energy (E_a is estimated to be 0.405 eV)
- k =Boltzmann's constant

The elements complementary to model_2 are described by Eqs. 12 and 13.

6 Application to other sizes of capacitor

The model described is based on the technology of the electrolytic capacitor. It can also be applied to other capacitors with different rated values.

We have measured the impedance according to the temperature for different components of following rated values:

- 3,300 μF , 450 V
- 11,000 μF , 360 V
- 15,000 μF , 350 V
- 33,000 μF , 63 V

In Fig. 16, we represent the impedances measured at -30°C . The shape of the curves is near to which was previously studied. So, it is possible to validate the method with other components.

Fig. 16 Impedance measurement for different capacitors at -30°C

7 Conclusion

The parameters of the electrolytic capacitors, which depend strongly with temperature and aging time, are generally not taken into account in electrical simulation software.

We have shown in this paper that a simple electrical model of the electrolytic capacitor could be used for represented running at various temperatures and different aging times.

A genetic algorithm integrated into Matlab software determines the parameters of this model. The identification of the parameters is carried out with the frequencies measurement. Optimization is defined by a function minimizing the error between measurements and the calculation of the algorithm. The identification gives good results for a large temperature range. This simple model can be integrated as a transfer function or a schematic representation in order to be used in various types of simulation software. This method can also be applied to other types and models of capacitors.

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