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Abstract- First, we calculate the total field for an outgoing or an incoming TM cylindrical incident wave illuminating a cylindrical periodic structure of metallic wires. Then, we give analytical formulas to extract the characteristics (reflection and transmission) of the cylindrical periodic structure. To finish, we extend the study to infinite radius periodic structure by given an approximation of the dispersion diagram.

I- Introduction

Cylindrical periodic structures are not often used in antenna devices. Before associating this type of structures with an antenna it is important to understand their characteristics.

In part II, we calculate the total field due a cylindrical periodic structure of metallic wires illuminated by an outgoing cylindrical wave or an incoming cylindrical wave, outside and inside the cavity. Unlike in the plane case, in the cylindrical structure we have not access directly to the characteristics of the cylindrical surface because of the multiples reflections between the center and the surface. In part III, we show how to extract the reflection and transmission characteristics of a single cylindrical surface by the knowledge of the preceding results. In part IV, the multiple layer periodic structure characteristics are deduced from one surface characteristics.

II- Calculation of the total field

The cylindrical periodic structure of infinite long metallic wires (figures 1) have the following parameters : \( C \) is the radius of the cylinder, \( a \) is the diameter of the wires, \( P_\theta \) is the angular period, \( P_t = P_\theta * C \) and \( N = 360/P_\theta \) is the number of wires.

In figure 1a the structure is illuminated by a TM outgoing cylindrical wave and in figure 1b we consider a TM incoming cylindrical incident wave. For these two cases (case 1a and case 1b) we will calculate the total Electric field inside and outside the cylindrical cavity.

Let us call \( E(\rho) \) the tangential composing of the total electric field in function of the distance \( \rho \) to the center The total field is the sum of the incident field and the diffracted field [1].

For case 1a the expression of \( E \) is :

\[
E(\rho) = E_{inc}(\rho) + \sum_{n=1}^{N} K_n E_{dn}(\rho) \quad (1a)
\]

For case 1b the expression of \( E \) is :

\[
E(\rho) = E_{inc}(\rho)(1 + r_a \exp(-2j\eta_0 k\rho)) + \sum_{n=1}^{N} K_n E_{dn}(\rho) \quad (1b)
\]

Where \( E_{inc}(\rho) = H_0^2(k\rho) \) for the outgoing cylindrical incident wave or \( E_{inc}(\rho) = H_0^2(k\rho) \) for the incoming cylindrical incident wave, \( \eta_0(x) = \arctan \left( \frac{J_0(x)}{N_0(x)} \right) \), \( J_0(x) \) and \( N_0(x) \) are the Bessel functions of order 0, and \( k \) is the free space wave number. \( r_a \) is the reflection coefficient in the center which is equal to one when no object is placed in the center. \( E_{dn} \) is the electric field diffracted by wire number \( n \) \((n=1,...,N)\). \( K_n \) represent the unknown factors which contain the coupling between wires. Because of the symmetry theses factors don’t depend on the wire considered :

\[
\sum_{n=1}^{N} K_n E_{dn}(\rho) = K \sum_{n=1}^{N} E_{dn}(\rho) \quad (2)
\]

\( K \) is determined by imposing zero to \( E \) at a surface of a metallic wire:

\[
\sum_{n=1}^{N} K_n E_{dn}(\rho) = 0
\]
\[ K = - \frac{E_{inc}(\rho)}{\sum_{n=1}^{N} E_{dn}(\rho)} \quad , \text{for case } 1a, \]

or \[ K = - \frac{E_{inc}(\rho)(1 + r_{1} \exp(-2j\eta_{0}(kp)))}{\sum_{n=1}^{N} E_{dn}(\rho)} \quad , \text{for case } 1b \]

(3)

In fact, for all wires but one this condition will be taken in the center of a wire and for this wire the condition is taken at a distance \( a/2 \). For small diameter of wire \( a \) comparing to the wavelength, \( E_{dn} \) are approximated by the Hankel function of first order. The formulation of \( E_{dn} \) are given below:

\[ E_{dn}(\rho) = H_{0}^{(2)}(k_{0} \sqrt{\rho^{2} - 2C \cos \left( \frac{(i-1)\pi}{N} \right)}) , \quad i=1,\ldots,N. \]

(4)

The total field \( E(\rho) \) is then given by (1a) or (1b) for the two cases 1a and 1b respectively.

Let us call \( T \) the total transmission coefficient outside the cylindrical cavity and \( R \) the total reflection coefficient inside the cavity at a distance \( D \) (<C) from the center:

\[ T = \frac{E(\rho)}{E_{inc}(\rho)} \quad , \quad \rho > C + P_{t} \]

(5)

\[ R = \frac{E(\rho)}{E_{inc}(\rho)} \quad , \quad \rho = D < C - P_{t} \]

(6)

We call \( T_{o} \) the value of \( T \) when the incident wave \( E_{inc} \) is an outgoing wave (case 1a) and \( T_{t} \) when it is an incoming wave (case 1b). Similar definitions are given for \( R_{o} \) and \( R_{t} \).

### III- Characterization of a cylindrical periodic surface

\((r, t)\) are the complex reflection and transmission coefficient of the cylindrical periodic structure of metallic wire for an outgoing cylindrical wave (figures 1a). \((r', t')\) are the complex reflection and transmission coefficient of the structure for an incoming cylindrical incident wave (figures 1b). The purpose of this part is to determine \((r, t)\) and \((r', t')\) by the knowledge of \( T_{o}, T_{t}, R_{o}, \) and \( R_{t} \). Let us insist that the method that we will describe can be used even if the total field is calculated by another method that the method of part II. In [2,3] the problem of a source inside a cylindrical periodic surface have been also treated but the characteristics of the surface have not been given.

\( T_{o} \) can be expressed in terms of \((r, t)\) and \( r_{o} \) (see figure 2a):

\[ T_{o} = t \sum_{n=0}^{\infty} r_{o}^{n} \exp(-jn\eta_{0}(kC)) = \frac{t}{1 - r_{o} \exp(-j2\eta_{0}(kC))} \]

where \( \eta_{0}(x) = \arctan \left( \frac{N_{0}(x)}{\eta_{0}(x)} \right) \)

(7)

\( R_{o} \) is the wave at a distance \( D \) from the line source inside the cavity, normalized by the incident wave (figure 2b). \( R_{o} \) can be expressed in terms of \((r, t)\) and \( r_{o} \):

\[ R_{o} = \frac{1 + r_{o} \exp(-j2\eta_{0}(kC)) + r_{o} \exp(-j2\eta_{0}(kD))}{1 - r_{o} \exp(-j2\eta_{0}(kC))} \]

(8)

With expressions (7) and (8), we can calculate the reflection and transmission coefficients \((r, t)\) if we know \( R_{o} \) and \( T_{o} \):

\[ r = \frac{R_{o} - 1}{[r_{o}R_{o} + B_{0}B_{C}]}, \quad t = T_{o} (1 - r_{o}B_{C}) \]

(9)

where \( B_{C} = \exp(-2j\eta_{0}(kC)) \), \( B_{D} = \exp(2j\eta_{0}(kD)) \)

(10)

\[ T_{o} \]

\[ R_{o} \]

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Figures 2 : (a) : Partial terms of the "total" transmission coefficient \( T_{o} \) (b) : Partial terms of the "total" reflection coefficient \( R_{o} \) inside the cavity
Figures 3 : (a) : Partial terms of the "total" transmission coefficient $T_i$ (b) : Partial terms of the "total" reflection coefficient $R_i$ inside the cavity

In figure 3a we can see that $T_i$ can be expressed in terms of $t$, $r$, $t'$ and $r_0$:

$$T_i = 1 + r' \exp(-j2\eta_0(kE)) = \frac{t'\tau_0 \exp(-j2\eta_0(kE))}{1 - \tau_0 \exp(-j2\eta_0(kC))} \tag{11}$$

$R_i$ can be expressed in terms of $t'$, $r$ and $r_0$ (figure 3a):

$$R_i = \frac{t'(1 + \tau_0 \exp(-j2\eta_0(kD)))}{1 - \tau_0 \exp(-j2\eta_0(kC))} \tag{12}$$

With expressions (11) and (12), we can calculate the reflection and transmission coefficients $(r', t')$ if we know $R_i$ and $T_i$ and $(r, t)$:

$$t' = \frac{(1 - B_E)R_i}{1 + \tau_0 B_D} , \quad r' = \frac{1 + \tau_0 B_E}{B_E B_C} \left( T_i - 1 - t't_0 B_B \right) \tag{13}$$

where $B_C = \exp(-2j\eta_0(kC))$, $B_E = \exp(-2j\eta_0(kE))$, $B_D = \exp(-2j\eta_0(kD))$, $B_B = \exp(2j\eta_0(kC))$.

IV- Radius periodic structures

In Figure 4a and Figure 4b, we plot the magnitudes of $(r, t)$ and $(r', t')$ and the phases of $r$ and $r'$ obtained with $R_0$, $T_0$, $R_i$ and $T_i$ (obtained in part II) for two examples : $C=40\text{mm}$ and $C=120\text{mm}$ respectively , with $P_t$ constant. We see that $(r', t')$ tend to $(r, t)$ as $C$ increase. In Figure 5 we can see that the resonances of $T_0$ are well given by the intersections of the phase of $r$ and $2k\eta_0(kC)$. The condition of continuity $r+1=t$ have been verified also.

The value of $|T_{0i}|$ superior to one (Figure 5) correspond to a matching of the ideal source ($r_0=1$) and must not be took as a gain enhancement. We can note that an enhancement of the bandwidth of the resonance of $|T_{0i}|$ can be obtain if $\varphi_r$ increase with frequency. This can be obtained with a negative index material as it is observed in [5].

IV- Radius periodic structures

In Figure 6, we consider, now, two surfaces 1 and 2 (Figure 6), and if we consider only the reflections between these two surfaces (considering a matched source, i.e. $r_0=0$), then the transmission coefficient must be equal to:

$$T_2 = \frac{t_1t_2}{1 - \tau_1r_2 \exp(j2\eta_0(kC)) \exp(-j2\eta_0(kil\cdot P_t))} \tag{15}$$

This is the cylindrical equivalent of the Fabry-Perot cavity in the plane case.
Cylindrical surfaces

Figure 6: Two layer cylindrical periodic structure

![Figure 6](image)

Figure 7: Two layer cylindrical structure transmission coefficient $|T_2| \ (a=1\text{mm } C_1=160\text{mm}, \ Pr=40\text{mm } P_\theta=15^\circ$ $P_t=12^\circ, \ P_\theta, \ \text{constant})$

Figure 8: Multiple layer cylindrical periodic structure

![Figure 8](image)

Figure 9: Six layer cylindrical periodic structure reflection coefficient, $\beta P_t$ and $|M|$ \ $(a=1\text{mm}, \ Pr=C_1=40\text{mm } P_\theta=60^\circ/i \ (i=1,2,...), \ P_t=\text{constant})$

V- Conclusion

We have seen how to obtain the main characteristics of a cylindrical periodic surface of metallic wires and of multiple layer radius periodic cylindrical structure. These methods will be used to design new types of antennas.

References


