Optimization of an Offset Reflector Antenna Using Genetic Algorithms

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S. L. Avila, W. P. Carpes, Jr., and J. A. Vasconcelos

***Abstract*—This paper presents the application of genetic algorithms in the optimization of an offset reflector antenna. The antenna shape is designed in order to obtain a uniform radiation pattern on the Brazilian territory. Modified genetic operators are proposed with the aim to increase the efficiency of the real coded genetic algorithms used here.**

***Index Terms*—Genetic algorithms (GA), real coded GA, reflector antenna, stochastic optimization.**

I. Introduction

Electromagnetic optimization problems generally involve several parameters, which can be continuous or discrete and are often bounded. Moreover, the objective functions that arise in electromagnetic optimization problems are often nonlinear, stiff, multiextremal, and nondifferentiable. Genetic algorithms (GAs) are robust stochastic-based methods which can handle the common features of electromagnetic optimization problems that are not readily handled by other traditional optimization techniques. An overview of GAs for electromagnetic optimization can be found in [1] and [2].

In the design of reflector antennas, the required radiation pattern is generally obtained by using a set of feeds and a parabolic reflector [3], as shown in Fig. 1(a). However, this technique can be difficult to implement and the resulting structure is bigger and more expensive than that obtained using a single reflector with a single feed.

In this paper, a GA is used to design the shape of a single-feed offset reflector antenna [modeled reflector, as in Fig. 1(b)]. The main goal is to obtain an antenna whose radiation pattern covers uniformly the Brazilian territory, with maximum average directive gain.

After a simple revision about GAs, we propose modified genetic operators with the aim to increase the efficiency of the real coded GA.

II. Genetic Algorithms

GA optimizers are well-known tools in the electromagnetic community. They are particularly effective when the goal is to find an approximate global optimum (maximum or minimum) for multimodal functions, in a near-optimal manner. Among the advantages of GAs, we can quote that: they can optimize with continuous or discrete parameters and not require gradient information; the possible discontinuities present on the fitness function have little effect on the overall optimization performance; they are resistant to becoming trapped in local optima; they can deal with a large number of parameters and are well suited for parallel computers; they provide a list of semi-optimum parameters instead of a single solution; they work with numerically generated data, experimental data or analytical functions; and they can be employed in a wide variety of optimization problems [1], [2].

The use of GAs requires the choice of a set of genetic operations among many possibilities. Nowadays, an important issue is related to the techniques used to increase the efficiency of GAs. The strategy used in coding the variables, for instance, is important. It is also very important to have an efficient exploration of the search space (the space of all feasible solutions). The latter requires that the GA operators (selection, crossover, and mutation) be properly implemented.

We carried out some simulations in order to compare the most used coding schemes: binary and real. We used the Rastrigin function as a test function and we used the same procedures presented in [4]. The results obtained with real coding were practically the same obtained using binary coding. So, the coding scheme must be chosen according to the nature of the variables and the programming language used.

A. Modified Genetic Operators

Real coding is well suited to a large class of programming languages and to problems with a great number of variables. For this reason, we have developed genetic operators for a real coded

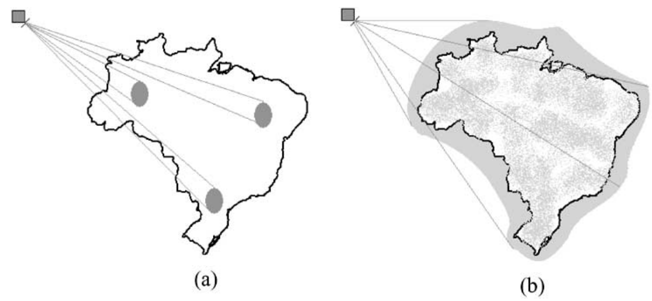

Fig. 1. Radiation pattern with (a) set of feeds and (b) modeled reflector.

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GA that allow an effective exploration of the search space. We consider a case where the population is given by

$$X^n = \begin{bmatrix} X_1^{n,1} & X_2^{n,1} & \cdots & X_{\mathrm{nvar}}^{n,1} \\ \vdots & \vdots & \vdots & \vdots \\ X_1^{n,\mathrm{nbpop}} & X_2^{n,\mathrm{nbpop}} & \cdots & X_{\mathrm{nvar}}^{n,\mathrm{nbpop}} \end{bmatrix} \quad (1)$$

where each line represents an individual (a point in the optimization space) at generation n, nbpop is the population size and nvar is the number of optimization variables. In the evolutionary process, we group the individuals in pairs and, for each pair (i, j), it is verified if crossover will take place with a probability P_c. If it is the case, crossover is performed to yield two offspring according to the following:

$$X_{\mathrm{kcross...dir}}^{n+1,i} = 0.9X_{\mathrm{kcross...dir}}^{n,i} + 0.1X_{\mathrm{kcross...dir}}^{n,j} \quad (2)$$

and

$$X_{\mathrm{kcross...dir}}^{n+1,j} = (1-\alpha)X_{\mathrm{kcross...dir}}^{n,i} + \alpha X_{\mathrm{kcross...dir}}^{n,j} \quad (3)$$

where kcross is a random integer variable with uniform distribution on $[1, \mathrm{nvar}]$ defining the crossover cut point [5]; dir is a random binary variable that indicates the direction to perform the crossover operation $\mathrm{dir} = 0$ if the direction is from the cut point to the last variable nvar or $\mathrm{dir} = 1$ in the other direction; and α is a random multiplicative coefficient with uniform distribution on $[-0.1, 1.1]$ [6]. Also, $X_{\mathrm{kcross...dir}}^{n,i}$ represents a portion of the individual i containing all variables from $X_{\mathrm{kcross}}^{n,i}$ to $X_{\mathrm{dir}}^{n,i}$ (or from dir to kcross if $\mathrm{dir} < \mathrm{kcross}$). The offspring variables not included in the interval $\mathrm{kross} \ldots \mathrm{dir}$ are directly copied from the respective progenitor. With the approach of (2), known as biased crossover, one child inherits most of its genetic material from one of the parents. In this case, in order to improve the average fitness of the population at each generation, it is necessary that

$$f(X^{n,i}) > f(X^{n,j}) \quad (4)$$

where $f(\cdot)$ represents the fitness function. In (3), the offspring is generated as a linear combination of variables of the two parents.

In a similar fashion, mutation is performed (with a probability P_m) with the addition of a perturbation vector (γ) to the portion of the individual that will suffer mutation. At the beginning of the evolutionary process, the perturbation vector is taken as

$$\gamma_{\mathrm{kmut...dir}}^{n,i} = 0.05\beta\, \mathrm{range}_{\mathrm{kmut...dir}}^{n,i} \quad (5)$$

where range is defined by the allowable limits of each variable and β is a random number with uniform distribution on the interval $[-1, 1]$. In this case, the mutation corresponds to a maximum change of $\pm 5\%$ of the range for each variable. This amount of mutation allows an efficient exploration of the search space without making the process too erratic. At the end of the evolutionary process, the perturbation vector changes to

$$\gamma_{\mathrm{kmut...dir}}^{n,i} = 0.05\beta \frac{\sum_{i=1}^{\mathrm{nbpop}} X_{\mathrm{kmut...dir}}^{n,i}}{\mathrm{nbpop}}. \quad (6)$$

In this case, γ depends on the average value of the variables that will suffer mutation. This strategy allows a better location of the maximum of the objective function roughly detected using (5).

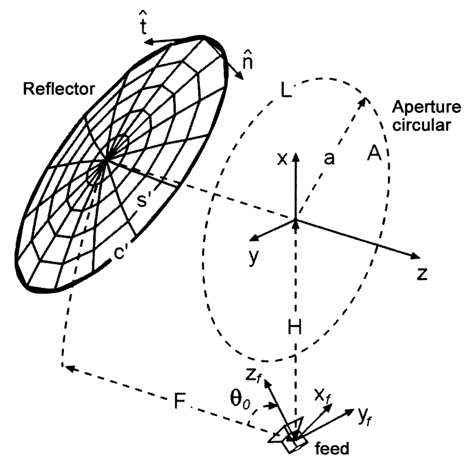


Fig. 2. Geometry of the single-feed offset reflector antenna.

TABLE I
EXPANSION COEFFICIENTS FOR THE OFFSET PARABOLOID

Coefficient	C_{00}	C_{01}	C_{10}
Value	$-\frac{1}{\sqrt{2}}F + \frac{1}{8\sqrt{2}}\frac{2H^2+a^2}{F}$	$-\frac{a^2}{8\sqrt{6}F}$	$\frac{aH}{4F}$

In order to investigate the efficiency of the genetic operators proposed here, they have been applied in the minimization of a rotated Rastrigin's function [6] with 30 variables. The allowed range for all variables corresponds to the interval $-5.12 \le X \le 5.12$. These values give an objective function with 10^{30} minima. The global minimum for the rotated Rastrigin's function is $X_{1...30} = 0$. The simulation was run 100 times and in all cases convergence has been attained in about 100 generations with a population of 200 individuals (convergence criterion: $|X_i|_2 \le \sqrt{0.02}$). The improvement techniques proposed in [4] were also used. We observed that convergence was reached with approximately 200×100 evaluations of the objective function for a search space containing 10^{30} possible solutions. This demonstrates the great efficiency of the real coded GA presented here.

III. Offset Reflector Antenna

The offset reflector antenna to be optimized is a single-feed and single-reflector structure with circular aperture (as shown in Fig. 2). Its surface is parameterized according to [7]

$$z'(t,\phi) = \sum_{n=0}^{N}\sum_{m=0}^{M}(C_{nm}\cos n\phi + D_{nm}\sin n\phi)F_m^n(t) \quad (7)$$

where t and ϕ are spherical coordinates of the paraboloid, C_{nm} and D_{nm} are expansion coefficients, and $F_m^n(t)$ is the modified Jacobi polynomial [7].

As reference shape, we take an offset paraboloid with $D = 2a = 1.524$ m; $F = 1.506$ m; $H = 1.245$ m; and $\theta_0 = 42.77°$. The corresponding expansion coefficients are given in Table I [7]. The feed is an x-polarized $\cos^n\theta$ source [8] with an edge taper of -12 dB $(n = 14.28)$.

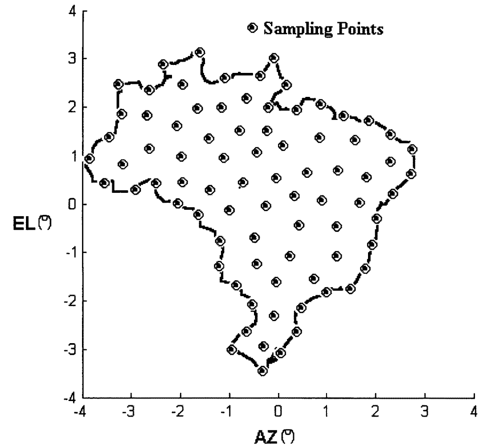


Fig. 3. Sampling points on the Brazilian territory.

A. Antenna Analysis

For a perfect electric conducting reflector surface, the physical optics (PO) approximation and the equivalency theorem [8] state that the reflector can be replaced by the equivalent current source

$$\vec{J}_{\rm PO} \approx 2\hat{n} \times \vec{H}_{\rm inc} \tag{8}$$

where $\vec{H}_{\rm inc}$ is the incident magnetic field. The radiated far field [8] calculated using PO can be written as

$$\vec{E}_{\rm PO} \approx -j\frac{k\eta}{4\pi}\frac{e^{-jkr}}{r}\int\limits_{s'}[\vec{J}_{\rm PO} - (\vec{J}_{\rm PO}\cdot\hat{r})\hat{r}]e^{jk\vec{r}'\cdot\hat{r}}\,ds' \tag{9}$$

where k and η are, respectively, the wavenumber and the intrinsic impedance of the medium, s' is the reflector surface, $\hat{r}$ is the unit position vector of the observation point, and $\vec{r}'$ is the vector of the source point.

The PO analysis is not accurate when used for predicting far-angular regions or cross-polarized fields. In order to improve the precision of (9), edge diffraction must be taken into account. The far-field contribution due to an equivalent edge current can be written as

$$\vec{E}_{\rm fr} \approx -j\frac{k\eta}{4\pi}\frac{e^{-jkr}}{r}\int_{c'}\left\{[\vec{J}_{\rm FR} - (\vec{J}_{\rm FR}\cdot\hat{r})\hat{r}] - \frac{1}{\eta}\hat{r}\times\vec{M}_{\rm FR}\right\}e^{jk\vec{r}'\cdot\hat{r}}\,dc' \tag{10}$$

where c' is the reflector rim and $\vec{J}_{\rm FR}\, e\, \vec{M}_{\rm FR}$ are the equivalent edge electric and magnetic currents [9].

The total field is then given by the direct (feed field $\vec{E}_f$) and indirect (PO field $\vec{E}_{\rm PO}$ and fringe field $\vec{E}_{\rm fr}$) components

$$\vec{E} = \vec{E}_f + \vec{E}_{\rm PO} + \vec{E}_{\rm fr}. \tag{11}$$

IV. Antenna Optimization

The optimization procedure has a goal to design a reflector surface shape that produces a uniform coverage of Brazilian territory [10] with maximum average gain. In order to get information on how good a given radiation pattern is, N sampling points are placed on the whole Brazilian territory, as shown in Fig. 3. The fitness function is defined as

$$f = G_{\rm av} = \frac{1}{N}\sum_{i=1}^{N} G_i \tag{12}$$

where G_i is the directive gain for the sampling point i.

The GA was implemented with 27 variables [28 expansion coefficients of which the first, C_{00}, was kept fixed. $N = 7$ and $M = 4$ in (7)]. In the optimization process, the value of the coefficients C_{01} and C_{10} were modified around those given in Table I; the remaining coefficients were modified around zero. The population was composed of 50 individuals. We used a real coded GA employing the modified genetic operators presented in Section II, reproduction by roulette wheel, and the improvement techniques proposed in [4].

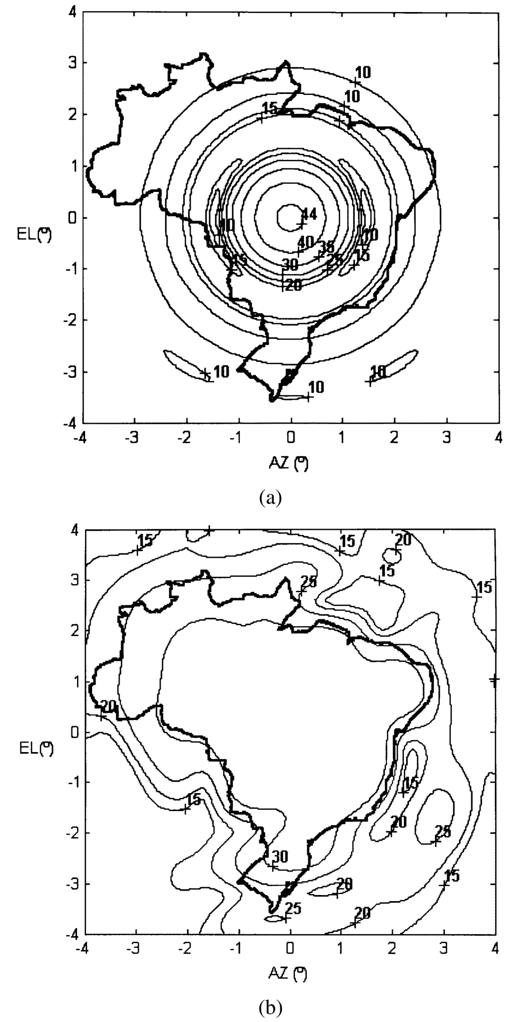


Fig. 4. Radiation pattern (in decibels): (a) reference parabolic reflector and (b) optimized shape.

TABLE II
COMPARISON BETWEEN ANTENNAS

Reflector	Max. Gain (dBi)	Average Gain G_{av} (dBi)
Paraboloid	44.64	18.37
Optimized	39.13	30.74

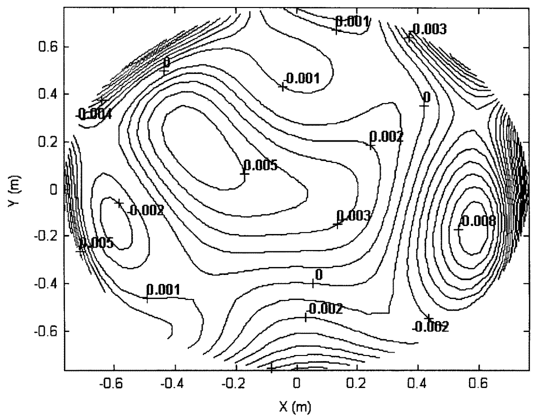
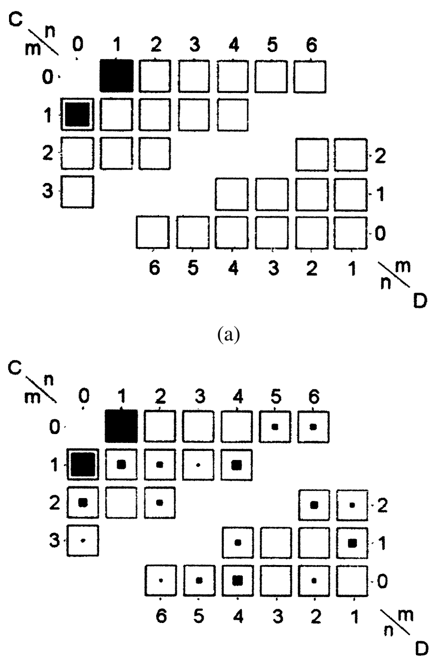


Fig.5. Difference between optimized and reference parabolic shapes.



(a)

(b)

Fig. 6. Graphical representation of the 27 expansion coefficients for (a) reference paraboloid and (b) optimized shape. Diagonal of each dark square is proportional to $\log_{10}|C_{\rm nm}|$ (or $\log_{10}|D_{\rm nm}|$) normalized to the range [10^{-3}, 100].

Table II presents the maximum and average gains of the optimized antenna together with the corresponding values of the parabolic antenna (used as reference). Fig. 4 shows the footprints of the radiated field.

Fig. 4 and Table II clearly show that the optimized antenna gives a more uniform illumination with a higher average gain over the covering area than that of the reference antenna. The optimized antenna made it possible to obtain a directive gain of 25 dB on practically all the Brazilian territory and at least 30 dB on 80% of it. Fig. 5 is a contour graph showing the slight difference (in meters) between the z coordinates of the optimized reflector and reference one.

The shape of the optimized reflector can be obtained from the expansion coefficients shown in Fig. 6.

V. Conclusion

Optimization tools can give excellent results in a large class of problems if properly implemented. In the antenna optimization problem considered here, the GAs have shown a great effectiveness and ability to deal with a large number of variables (namely, the expansion coefficients). Moreover, as we saw, the modified genetic operators proposed here made it possible to obtain a very effective real-coded GA.

The optimization procedure carried out in this work has focused only on the reflector shape and we used as design parameters only the directive gain at the sampling points. However, in order to design high-performance antennas, one requires a procedure with additional degrees of freedom. Nevertheless, the obtained results demonstrated that the optimized antenna has much better radiation patterns than those of the reference paraboloid.

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