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## RELIABILITY MODELLING WITH DYNAMIC BAYESIAN NETWORKS

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**Abstract:** Nowadays, the complex manufacturing processes have to be dynamically modelled and controlled to optimise the diagnosis and the maintenance strategies. The work reported here presents a methodology for developing Dynamic Bayesian Networks (DBN) to formalise such complex dynamic models. A small valve system then is used to compare the reliability estimations obtained by the proposed DBN model and by the classical Markov Chain.

**Keywords:** Dynamic bayesian network, Markov models, reliability evaluation.

### 1. INTRODUCTION

One of the main challenges of the Extended Enterprise is to dynamically maintain and optimise the quality of the services delivered by industrial objects along their life cycle. The goal is thus to design decision-making aid systems for maintaining the system in operation. Nevertheless, most of current automated systems do not provide the means for intelligent interpretation of information copying with large process disturbances. Moreover, the state of the system may not be known exactly before making the decision. This imperfect perception argues in favour of using a probabilistic estimation of the system state. As described in (Boutillier, *et al.*, 1999), tools issued from Artificial Intelligence can be used to bring decision-making aid for manufacturing systems.

Works on system safety and Bayesian Networks (BNs) have recently been developed (Kang and Golay, 1999). Bobbio, *et al.*, (2001), explain how the Fault Tree can be achieved using BNs. Moreover, Weber, *et al.*, (2001), propose a model based decision allowing the fault diagnosis using the system functioning and dysfunctioning analyses. The solutions proposed in these last papers are based on a static probabilistic model of the system. To improve the decision making during the diagnosis, our goal is to define a dynamic model of the process behaviour. This model should allow computing state probability distributions by taking into account the component age and the last executed maintenance operations.

The purpose of this paper is to introduce Dynamic Bayesian Networks (DBNs) as an equivalent model to the Markov Chains (MCs) (Gertsbakh, 2000; Padhraic, 1997). The problems considered are those involving systems whose dynamics can be modelled as stochastic processes and where the decision maker's actions influence the system behaviour. The current system state and the applied action jointly determine the probability distribution over the next states. The proposed paper is a first study dedicated to the comparison of MCs and DBNs for the

estimation of system reliability. Section 2 presents the MC model and the fast growing of its state cardinality with respect to the complexity of the modelled system. The methodology proposed in this paper is an original formalisation of a system reliability model (section 4) by means of DBNs (section 3). A simulation of a classical system is developed in section 5 to compare MC and DBN models. Finally, section 6 presents the conclusions and perspectives.

## 2. PROBLEM STATEMENT

In order to take the uncertainty into account, it is possible to consider the process state as a random variable that takes its values from a finite state space corresponding to the possible process states. A MC allows modelling the dynamics of sequences taken by these states (Boutillier, *et al.*, 1999).

### 2.1. The Markov Chain notations

The notations concerning MC modelling are defined in this section. Let  $X$  a discrete random variable modelling a process with a finite number of mutually exclusive states  $\{s_1, \dots, s_M\}$ . The vector  $x$  then denotes a probability distribution over these states:

$$\begin{aligned} x &= [p_1 \quad \dots \quad p_m \quad \dots \quad p_M], \\ p_m &\geq 0 \quad \text{and} \quad \sum_{m=1}^M p_m = 1 \end{aligned} \quad (1)$$

Assuming that the system evolves in states, where the occurrence of an event marks the transition of a state ( $k$ ) to the next state ( $k+1$ ), then the process produces the sequence  $(x_0, x_1, \dots, x_{k-1}, x_k)$  that can be modelled as a MC if:

$$p(X = x_k | x_0, x_1, \dots, x_{k-1}) = p(X = x_k | x_{k-1}) \quad (2)$$

The Markov property makes it possible to specify the statistical relationship among states as a transition probability matrix  $\mathbf{P}_{MC}$ . If the transition probabilities  $p_{ij}$  are time independent then the MC is said to be homogeneous.

$$\mathbf{P}_{MC} = \begin{matrix} & \begin{matrix} x_k \end{matrix} \\ \begin{matrix} x_{k-1} \end{matrix} & \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1M} \\ p_{21} & & & \dots \\ \dots & & & \dots \\ p_{M1} & p_{11} & \dots & p_{MM} \end{bmatrix} \end{matrix} \quad (3)$$

### 2.2. Application to reliability

The reliability of a system with a low complexity level can be modelled as MC. This method leads to a graphic representation (Gertsbakh, 2000, pp. 116). For instance, considering the reliability of a component (entity), it is modelled by a discrete random variable  $X$  with states  $\{s_o, s_f\}$ ,  $s_o$  (operational state) and  $s_f$  (failure state) indicating if the component is up or down. To model the reliability, the transition probability matrix between the states  $s_o$  and  $s_f$  is defined as follows:

$$\mathbf{P}_{MC} = \begin{bmatrix} 1-\lambda & \lambda \\ 0 & 1 \end{bmatrix} \quad (4)$$

where  $\lambda$  represents the failure rate (considered as constant).

### 2.3. Problem to model complex process

The MC method is well suited to calculate the reliability of low complexity entity/system. However, within the framework of complex systems, the combinatorial explosion of states makes MC unmanageable. To decrease the complexity of the Markov model, the hypothesis (a) that there is no simultaneous occurrence of failure is assumed. This hypothesis simplifies considerably the Markov graph but leads to an approximation of the system reliability. Even more hypotheses are assumed in practice to reduce the complexity, as for example, the hypothesis (b) that assumes events statistically independent. In that case, methods based on Faults Tree (FT) are used. Unfortunately, this hypothesis is not respected when commune causes are taking into account, or when there are several failure causes for the same component. In the following, a method coupling in a unique representation these two approaches (MC, FT) without the hypotheses (a, b) is presented. This method is based on Dynamic Bayesian Networks.

## 3. BAYESIAN NETWORK THEORY

BNs are probabilistic networks based on graph theory. Each node represents a variable and arcs indicate direct probabilistic relations between the connected nodes. Variables are defined over several states. The DBNs allow taking time into account by defining different nodes to model variables with respect to different time slices.

### 3.1. The Bayesian Network notations

BNs are directed acyclic graphs used to represent uncertain knowledge in Artificial Intelligence (Jensen, 1996). A BN is defined as a pair:  $\mathcal{C}_j = ((N, A), \mathcal{P})$ , where  $(N, A)$  represents the graph; “ $N$ ” is a set of nodes; “ $A$ ” is a set of arcs;  $\mathcal{P}$  represents the set of conditional probability distributions that quantify the probabilistic dependencies. A discrete random variable  $X$  is represented by a node  $n \in N$  with a finite number of mutually exclusive states. States are defined on a state space  $\mathcal{S}_n : \{s_1^n, \dots, s_M^n\}$ . The vector  $x^n = [p_1 \dots p_M]$  denotes a probability distribution over these states as eq. (1), where  $p_m$  is the *a priori* probability of  $n$  being in state  $s_m^n$ . In the graph depicted in figure 1, the nodes  $n_i$  and  $n_j$  are linked by an arc. If the pair  $(n_i, n_j) \in A$  and  $(n_j, n_i) \notin A$  then  $n_i$  is considered as a parent of  $n_j$ . The parent set of a node  $n_j$  is defined as  $pa(n_j) = n_i$ .

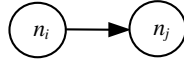


Fig. 1. A basic BN.

In this work, the set  $\mathcal{P}$  is represented with Conditional Probability Tables (CPT). Then, each node has an associated CPT. For instance, in figure 1, the nodes  $n_i$  and  $n_j$  are defined over the states  $\mathcal{S}_{n_i} : \{s_1^{n_i}, \dots, s_M^{n_i}\}$  and  $\mathcal{S}_{n_j} : \{s_1^{n_j}, \dots, s_L^{n_j}\}$ . Then, the CPT of  $n_j$  is defined by the conditional probabilities  $p(n_j | n_i)$  over each  $n_j$  state knowing its parents states ( $n_i$ ). This CPT is defined as a matrix:

$$\mathbf{P}(n_j | pa(n_j)) = \begin{bmatrix} p(n_j = s_1^{n_j} | n_i = s_1^{n_i}) & \dots & p(n_j = s_L^{n_j} | n_i = s_1^{n_i}) \\ \vdots & & \vdots \\ p(n_j = s_1^{n_j} | n_i = s_M^{n_i}) & \dots & p(n_j = s_L^{n_j} | n_i = s_M^{n_i}) \end{bmatrix} \quad (5)$$

Concerning the root nodes, *i.e.* without parent, the CPT just contains a row describing the *a priori* probability of each state.

Various inference algorithms can be used to compute marginal probabilities, the most classical one relying on the use of a junction tree (more explications can be found in (Jensen, 1996, pp. 76). Inference in BN allows taking into account any state variable observation (an event) for the updating of the probabilities of each variable. In other words, inference computes node probability distributions knowing the state of one or several

variables. Without any event observation, the computation is based on *a priori* probabilities. As observations are made, the knowledge is incorporated in the network and the probabilities over the process states are updated.

Knowledge is formalised as evidence. An *hard evidence* (instantiation) of the random variable  $X$  is an evidence that the state of the node  $n \in N$  is one of the states  $\mathcal{S}_n : \{s_1^n, \dots, s_M^n\}$ . For instance  $X$  is in state  $s_1^n : p(n = s_1^n) = 1$  and  $p(n = s_{m \neq 1}^n) = 0$ . Nevertheless knowledge can be uncertain. Then *soft evidence* is introduced (Valtora, *et al.*, 2002). A soft evidence for a node  $n$  is defined as any evidence that enables to update the prior probability values for the states of  $n$ . For instance  $X$  is in state  $s_1^n$  and  $s_M^n$  with the same probability and not in the other states:  $p(n = s_1^n) = 0.5$ ,  $p(n = s_M^n) = 0.5$  and  $p(n = s_{m \neq (1,M)}^n) = 0$ .

### 3.2. Dynamic Bayesian Network

A DBN is a BN including a temporal dimension. This new dimension is managed by time-indexed random variables.  $X_i$  is represented at time step  $k$  by a node  $n_{(i,k)} \in N$  with a finite number of states  $\mathcal{S}_{n_i} : \{s_1^{n_i}, \dots, s_M^{n_i}\}$ .  $x_k^{n_i}$  denotes the probability distribution over these states at time step  $k$ . Several time stages are represented by several sets of nodes  $N_0, \dots, N_k$ .  $N_k$  includes all the random variables relative to the time slice  $k$  (Hung, *et al.*, 1999; Bouillier, *et al.*, 1999, pp. 38-45).

An arc linking two variables belonging to different time slices represents a temporal probabilistic dependence between these variables. Then DBN allow to model random variables and their impacts on the future distribution of other variables. Defining these impacts as *transition-probabilities* between the states of the variable at time step  $k$  and time step  $k+1$ , these *transition-probabilities* lead to define CPTs relative to inter-time slices, equivalent to CPT defined in the previous section (eq. (5)). With this model, the future ( $k+1$ ) is conditionally independent of the past given the present ( $k$ ), which means that the CPT  $\mathbf{P}(n_{i,k+1} | pa(n_{i,k+1}))$  respects the Markov properties (Kjaerulff, 1995). Moreover, this CPT is equivalent to the Markovian model of the variable  $X_i$  described in the section 2.1 if  $pa(n_{i,k+1}) = n_{i,k}$  and  $\mathcal{S}_{n_{i,k}} = \mathcal{S}_{n_{i,k+1}}$ .

$$\mathbf{P}(n_{i,k+1} | n_{i,k}) = \mathbf{P}_{MC} \quad (6)$$

Starting from an observed situation at time step  $k=0$ , the probability distribution  $x_k^{n_i}$  over  $n_i$  states is computed by the DBN inference. To compute  $x_{k+T}^{n_i}$ , several solutions are proposed in the literature. One of them consists in developing  $T$  time slices, obtaining then a network size growing proportionally to  $T$  (Kjaerulff, 1995). Another solution, which keeps a compact network form, is based on iterative inferences. This solution is used in the following. The notion of time is introduced through inference. Indeed, it is possible to compute the probability distribution of any variable  $X_i$  at time step  $k+1$  based on the probabilities corresponding to time step  $k$ . The probability distributions at time step  $k+2, \dots$  are computed using successive inferences. Then a network with only two time slices is defined. The first slice contains the nodes corresponding to the current time step ( $k$ ), the second one those of the following time step ( $k+1$ ). Observations, introduced as hard evidence or probability distributions, are only realised in the current time slice. The time increment is carried out by setting the computed marginal probabilities of the node at time step  $k+1$  as observations for its corresponding node in the previous time slice.

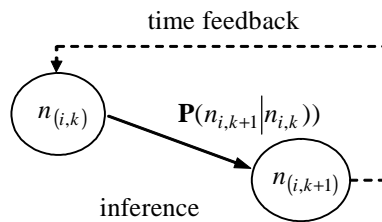


Fig. 2. A DBN for the random variable  $X_i$ .

## 4. DBN TO MODEL RELIABILITY

### 4.1. Dynamic Bayesian Networks to model entities

The reliability of low complexity component can be modelled as a DBN made of two nodes as presented in figure 2. An MC model of the reliability of a component  $X_i$  is easily translated into a DBN model. Thus independent components (entities) of the process are modelled using DBN equivalent to independent MC. For instance, as it is defined in section 2.2, a component is modelled by a discrete random variable  $X$  with states  $\{s_o, s_f\}$ . Then two nodes are defined to model the random variable at time slice  $(k)$  and  $(k+1)$ :  $n_{(i,k)}$  and  $n_{(i,k+1)}$ . These nodes, linked by an arc that represents the dependency of the component states at time step  $k+1$  to the component states at the time step  $k$ , are both described by the states  $\{s_o, s_f\}$ .

Equations (4) and (6) define the CPT  $\mathbf{P}(n_{i,k+1}|n_{i,k})$  linking the two time slices. The parameters are those defined to build the MC model of the component. To calculate  $p(n_{i,k+1} = s_1^{n_{i,k+1}})$  the probability that the variable  $X_i$  is in the state  $s_1^n$  at  $(k+1)$ , the following equation is used:

$$p(n_{i,k+1} = s_1^{n_{i,k+1}}) = (1-\lambda) \cdot p(n_{i,k} = s_1^{n_{i,k}}) \quad (7)$$

Equation (7) corresponds to the classical formula of the discrete model of the MC.

### 4.2. BN to model dependant failure modes

A Fault Tree (FT) allows describing the logic of the propagation of the failure throw the system. This method allows to model the reliability of the system assuming the hypothesis of independence of the events (failures) affecting the entities. The paper (Bobbio, *et al.*, 2001) showed the equivalence between FTs and BNs. The CPT is then defined automatically by OR/AND gate. These CPTs are given *a priori*, and the parameters are for most of them equal to 0 or 1. However, it is possible to introduce uncertainty by setting parameters different to 0 or 1.

Moreover, thanks to the CPTs, BNs can model the propagation on a system that has several failure modes. It is then possible to synthetically represent, with a factorised representation, a system composed by entities with several failure modes. The hypothesis of independence of events (failures) made for FT is not necessary. Indeed, BNs allow calculating exact repercussion of dependant variables to the system reliability.

## 5. APPLICATION

The method is applied to a classical example of reliability analysis. This example easily allows comparing the proposed method based on DBNs to the one using MCs.

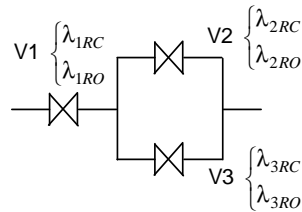


Fig. 3. Valve system.

Figure 3 describes the system. Three valves are used to distribute or not a fluid. Every valve has two failure modes: remains closed (RC) or remains opened (RO) while it is controlled. The failure rates are the following:

$$\begin{aligned} \lambda_{1RC} &= 1.10^{-3} & \lambda_{2RC} &= 2.10^{-3} & \lambda_{3RC} &= 3.10^{-3} \\ \lambda_{1RO} &= 2.10^{-3} & \lambda_{2RO} &= 3.10^{-3} & \lambda_{3RO} &= 4.10^{-3} \end{aligned}$$

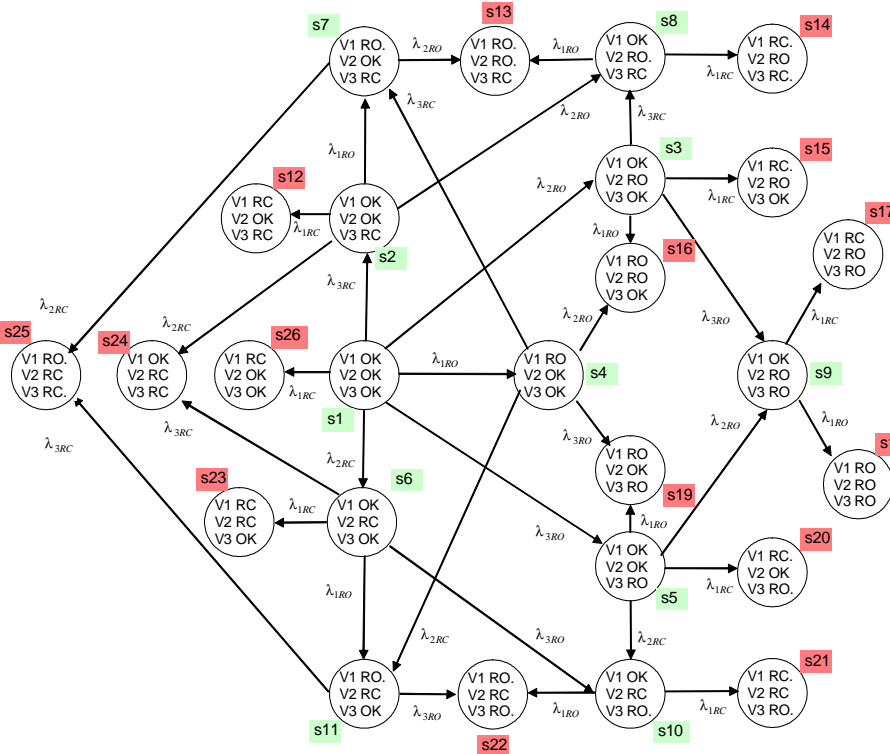


Fig. 4. MC model of the system.

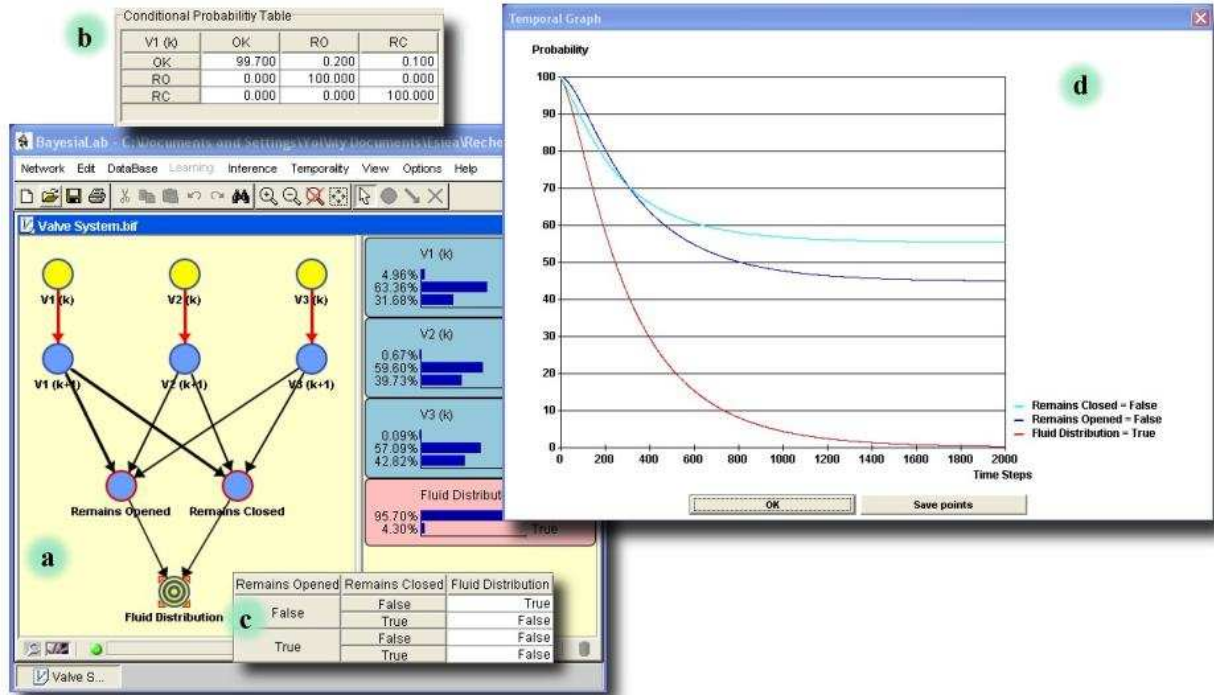


Fig. 5. (a) DBN model – (b) CPT for V1(k+1) – (c) Deterministic CPT for Fluid Distribution – (d) System reliability

Figure 4 depicts graphically the MC model of this system. As it is shown by the figure, 25 states  $s_1 \dots s_{25}$  are necessary to model this system: states  $s_1$  to  $s_{11}$  are states for which the system is available in spite of the degradation due to some failures; states  $s_{12}$  to  $s_{25}$  correspond to states where the system is unavailable due to the combination of failures. The transition matrix  $\mathbf{P}_{MC}$  defined the probability related to the different states  $s_1 \dots s_{25}$ . An equivalent model of this MC is realised by means of the DBN depicted in Figure 5.a. The state probabilities of the components  $V_i(k)$  (current time step) can be extracted without any difficulty by a simple shaping of values. These variables have three states: Available (OK), Remains Open (RO) or Remains Closed (RC). The system can then evolve according to the probability of every state and to the component failure rate values. The result of this degradation is modelled by the variable  $V_i(k+1)$  representing the state of the components at time step  $k+1$ . A CPT used to estimate the dynamic behaviour of the component reliability is illustrated in Figure 5.b.

The propagation through the Bayesian model allows taking into account the dependency between the failure modes for the computation of the system reliability. Inferences are realised thanks to the **BayesiaLab** software that uses an iterative procedure (<http://www.bayesia.com>). BayesiaLab is used to simulate the reliability behaviour of the system over 2000 time steps with the DBN depicted in figure 5.d.

The MC model is managed thanks to the **Supercab+** software (<http://www.cabinnovation.fr>). The system reliability has to be computed as:

$$\sum_{i=1}^{11} p(s_i) \quad (8)$$

Even if the results obtained by means of DBN are very close to those obtained with the MC model, they are in fact more precise. Indeed, the differences are due to the approximation made in the Markov model that assumes that simultaneous failures can not occurred, this hypothesis being not assumed in the DBN model.

## 6. CONCLUSION AND FURTHER WORK

The proposed method, based on the Dynamic Bayesian Networks theory, easily allows constructing DBN structures for the modelling of the temporal evolution of complex systems. The correspondence between Markov Chain, Fault Tree and DBN is presented and applied to the estimation of the system reliability.

The proposed method seems to be a solution to model the reliability of complex systems. Indeed, the number of states needed to model a complex system with MC increases in an exponential fashion (a state for each combination of elementary states). As the DBNs representation is based on the modelling of process entities, the obtained model is far more compact and readable than MC (compare for example the two models illustrated in Figure 4 and 5 that correspond to the small valve system). Furthermore, the dependency between several failure modes of a component and common modes are easily modelled by DBN. This paper shows then that DBNs constitute a very powerful tool for decision-making aid in maintenance.

In a future works, in order to achieve to perform this modelling technique we have to define how the learning algorithms of BN can contribute to model the dynamics of the system reliability and how the parameters behaviour can be then modelled.

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