Statistical Approach for Unpolarized Fragmentation Functions for the Octet Baryons
Claude Bourrely, Jacques Soffer

To cite this version:

HAL Id: hal-00127003
https://hal.archives-ouvertes.fr/hal-00127003
Submitted on 27 Jan 2007

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Statistical Approach for Unpolarized Fragmentation Functions for the Octet Baryons

Claude Bourrely * and Jacques Soffer †

Centre de Physique Théorique†, CNRS Luminy case 907, F-13288 Marseille Cedex 90, France

A statistical model for the parton distributions in the nucleon has proven its efficiency in the analysis of deep inelastic scattering data, so we propose to extend this approach to the description of unpolarized fragmentation functions for the octet baryons. The characteristics of the model are determined by using some data on the inclusive production of proton and $\Lambda$ in unpolarized deep inelastic scattering and a next-to-leading analysis of the available experimental data on the production of unpolarized octet baryons in $e^+e^-$ annihilation. Our results show that both parton distributions and fragmentation functions are compatible with the statistical approach, in terms of a few free parameters, whose interpretation will be discussed.

PACS numbers: PACS numbers: 12.40.Ee, 13.87.Fh, 13.85.Ni, 13.60.Rj

I. INTRODUCTION

Following the first evidence of partonic substructure of the nucleon, by means of deep inelastic scattering (DIS), a large amount of experimental data have been collected in order to understand the parton structure of the nucleon. It is currently known that the $Q^2$ evolution behavior of the parton distribution functions (PDF) is well described by perturbative QCD. However, the parton distributions at an initial scale reflect the nonperturbative quark and gluon dynamics of QCD bound states, which cannot be determined from first principles. For this reason, many parametrizations have been proposed [1–5], but most of these expressions involve a large number of free parameters, with no clear physical meaning. However, some efforts have been recently made [6] to parametrize unpolarized and polarized nucleon PDF based on a statistical approach [7, 8]. The nucleon PDF, which involve in this framework a small number of free parameters, can well describe all available experimental unpolarized and polarized DIS data, so this is perhaps an indication that the PDF retain some important statistical features of the nucleon. The statistical approach of the nucleon PDF allows to make predictions, which were tested recently in a satisfactory way, by various experimental data in DIS [9]. In order to extend further this framework to new areas, it is natural to envisage, for example, its application to the description of the PDF of the other octet baryons. Unfortunately, these PDF are not directly accessible because, due to their short lifetimes, the hyperons cannot be used as a target in a DIS experiment.

However, it is well known that the quark distributions in a hadron $h$, $q_h(x,Q^2)$ are related to the corresponding quark to hadron fragmentation function (FF) $D_h^q(x,Q^2)$, by means of the so called Gribov-Lipatov relations [10]. This connection between two basic quantities of the hadron structure has been used in several recent works [11, 12], as an attempt to improve our present poor knowledge on the hadron FF. So this is our motivation to extend the statistical approach to a global description of the octet baryon FF and to check its validity against the available experimental data.

* Electronic address: Claude.Bourrely@cpt.univ-mrs.fr
† Electronic address: Jacques.Soffer@cpt.univ-mrs.fr
‡ Unité propre de Recherche 7061
The paper is organized as follows. In Sect. 2, we review the main points of the framework and we give the basic procedure for the construction of the octet baryons FF in the statistical approach, in terms of Fermi-Dirac distributions. In Sect. 3, we determine all free parameters of the model by using some data on the inclusive production of proton and Λ in unpolarized deep inelastic scattering and a next-to-leading (NLO) fit to the available experimental data on the production of unpolarized octet baryons in $e^+e^-$ annihilation. The results of the fit for the cross sections and the obtained FF are also presented and we compare our approach with some previous works on baryon FF [13-16]. Finally in Sec. 4, we give our conclusions.

II. THE STATISTICAL APPROACH FOR UNPOLARIZED FF

In the statistical approach, a hadron can be viewed as a gas of massless partons (quarks, antiquarks, gluons) in equilibrium at a given temperature, in a finite size volume. In hadron production, when a parton fragments into a hadron, it picks up other partons from the QCD vacuum in order to form a specific hadron. The formation probability of the hadron is characterized by its statistical properties. Therefore we believe that statistical features which were proposed to build up the nucleon PDF in Ref. [6], can be used also to construct the FF for the octet baryons. We assume that the parton ($p$) to hadron ($h$) FF $D_{ph}^h(x)$, at an input energy scale $Q_0^2$, is proportional to

$$[\exp[(x - X_0)/\bar{x}] \pm 1]^{-1},$$

where the plus sign for quarks and antiquarks, corresponds to a Fermi-Dirac distribution and the minus sign for gluons, corresponds to a Bose-Einstein distribution. Here $X_0$ is a constant which plays the role of the thermodynamical potential of the quark hadronization into a hadron and $\bar{x}$ is the universal temperature, which is assumed to be equal for all octet baryons. It is reasonable to take its value to be the same as for the nucleon PDF, i.e. $\bar{x} = 0.099$, according to Ref. [6]. The statistical approach for the PDF allows to construct quark distributions of a given helicity and it is also possible to relate simply quark to antiquark distributions, resulting from chiral properties of QCD, as explained in Ref. [6]. All these physical quantities were determined precisely due to the existence of a huge amount of data in unpolarized and polarized DIS. In the case of the octet baryon FF, the situation is different and the scarcity of the polarized data does not allow such a clear separation, so we will restrict ourselves to the determination of the unpolarized quark FF, although the extension to the polarized case can be easily done. For the quarks $q = u, s, d$ the FF are then expressed as

$$D_q^B(x, Q_0^2) = \frac{A_q^B x_q^B x_q^B}{\exp[(x - X_q^B)/\bar{x}] + 1},$$

where $X_q^B$ is the potential corresponding to the fragmentation $q \rightarrow B$ and $Q_0^2$ is an initial scale, given below in Table 1. We will ignore the antiquark FF $D_{\bar{q}}^B$, which are considered to be strongly suppressed. The heavy quark FF $D_Q^B(x, Q_0^2)$ for $Q = c, b, t$, which are expected to be large only in the small $x$ region ($x \leq 0.1$ or so), are parametrized by a diffractive term with a vanishing potential

$$D_Q^B(x, Q_0^2) = \frac{\tilde{A}_Q x_Q^B}{\exp(x/\bar{x}) + 1}.$$
The initial scale $Q_0^2$, which is flavor dependent in this case, is given below in Table 1. This FF for $Q \to B$ depends on $b$ and a normalization constant $A_B^2$ for each baryon $B$. For the other quarks, we make some reasonable assumptions in order to reduce the number of parameters in addition to $b$, the universal power of $x$ in Eq. (2). First we have the obvious constraints, namely, $D_B^B = D_B^B$ for $B = p, \Lambda$. Moreover we assume that we need only four potentials, two for the proton $X_\nu^p = X_\nu^p$ and $X_\nu^p$ and two for the hyperons $X_u^Y = X_d^Y$ and $X_s^Y$ where $Y = \Lambda, \Sigma^\pm, \Xi^-$. Finally for the gluon to baryon FF $D_g^B(x, Q^2)$, which is hard to determine precisely, we take a Bose-Einstein expression with a vanishing potential

$$D_g^B(x, Q_0^2) = \frac{A_g^B x^{\hat{s} + 1}}{\exp(x/\hat{s}) - 1}. \quad (4)$$

We assume it has the same small $x$ behavior as the heavy quarks and it is the same for all baryons. The normalization constants $A_q^B, A_q^B$ and $A_B^g$ will have also to be determined by fitting the data, a procedure we present now.

III. DETERMINATION OF THE PARAMETERS FROM THE DATA ANALYSIS

The experimental data which was used is twofold. First, hadron production in DIS gives access to a direct determination of $D_q^2(x, Q^2)$ [17] and $D_q^A(x, Q^2)$ [18], in a limited range of $x$ and $Q^2$. We have also determined the free parameters of the model by making an analysis of the differential cross section for the semi-inclusive hadron production process $e^+e^- \to h + X$, for $p, \Lambda, \Sigma, \Xi$. Here we note that in these experiments, they usually do not distinguish between $B$ and $\bar{B}$, so we will include both contributions in our calculations by making the natural assumption $D_q^B(x, Q^2) = D_q^B(x, Q^2)$. The differential cross section can be expressed as [19–22]

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{dx_E} = \sum_q \int_{x_E} \frac{d \eta}{\eta} D_q^h(x_E, \eta, \mu_F^2) \frac{1}{\sigma_{tot}} \left[ C_T^q(\eta, Q^2, \mu_F^2) + C_L^q(\eta, Q^2, \mu_F^2) \right], \quad (5)$$

where $x_E = 2E_h/\sqrt{s}$, which is the energy $E_h$ of the produced hadron scaled to the beam energy $Q/2 \equiv \sqrt{s}/2$. The subscripts $T$ and $L$ denote the contributions due to transverse and longitudinal polarizations, respectively. The summation index $q$ includes quarks, antiquarks and gluon contributions. In the following we will set the renormalization and the factorization scales equal to $\mu_F^2 = \mu_R^2 = Q^2$. The definitions of the functions $C_T^q$ and $C_L^q$ follow those of Ref. [21]. In Eq. (5), $\sigma_{tot}$ is the total cross section for the process.

$$\sigma_{tot} = N_c \sum_q \frac{4\pi \alpha^2}{3s} e_q^2(s) \left( 1 + \frac{\alpha_s(s)}{\pi} \right), \quad (6)$$

where $N_c$ is the color number and we shall take $N_c = 3$, $\alpha$ is the QED fine structure constant and $\alpha_s(s)$ is the strong coupling constant. The sum in the above equation should be over all active quarks and antiquarks. The electroweak charges in Eq. (6) can be expressed as

$$\tilde{e}_q^2(s) = e_q^2 + 2e^e e^e \rho_1(s) + (a^2 + \epsilon_2^e)(a^2 + \epsilon_2^u) \rho_2(s), \quad (7)$$

\footnote{Due to the fact that the input scale of the $t$ quark is above the highest energy data investigated in this work, it does not contribute to our analysis.}

\footnote{When experimental data are presented in the momentum scaling variable $z_2 = 2p_2/Q$, we made a conversion to $x_E$ in the following figures.}
with

\[ \rho_1(s) = \frac{1}{4 \sin^2 \theta_W \cos^2 \theta_W (M_Z^2 - s)} \frac{s(M_Z^2 - s)}{s(M_Z^2 - s) + M_Z^2 Y^2_Z}, \]

(8)

\[ \rho_2(s) = \frac{1}{16 \sin^4 \theta_W \cos^4 \theta_W (M_Z^2 - s)^2 + M_Z^2 Y^2_Z}, \]

(9)

\[ a_e = -1, \quad a_q = T_{3q}, \]

(10)

\[ v_e = -\frac{1}{2} + 2 \sin^2 \theta_W, \quad v_q = T_{3q} - 2e_q \sin^2 \theta_W, \]

(11)

where \( e_q \) is the electric charge of the quarks and \((v_e), (a_q)\) the electroweak vector and axial couplings of the electron and the quarks to the \( Z \), respectively. \( T_{3q} \) is the third component of the strong isospin, \( T_{3q} = 1/2, -1/2 \) for up-type quark and down-type quark, respectively. \( \theta_W \) is the weak-mixing angle, and \( M_Z, \Gamma_Z \) are the mass and width of the \( Z \).

The FF have been evolved at NLO following a method defined in Ref. [6], where for the DGLAP equations we used the timelike splitting functions calculated in Refs. [23]. For baryon production in \( e^+e^- \) collisions, we made a NLO fit with formula (5) of the experimental data from Refs. [24]-[42], where we restricted \( x_E \geq 0.1 \). In addition, the input scales choice [21] and \( \Lambda(\overline{MS}) \) are given in Table I.

<table>
<thead>
<tr>
<th>quark</th>
<th>u, d, s</th>
<th>c</th>
<th>b</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_0 )</td>
<td>0.632</td>
<td>1.4</td>
<td>4.5</td>
<td>175</td>
</tr>
<tr>
<td>( \Lambda(\overline{MS}) )</td>
<td>0.299</td>
<td>0.246</td>
<td>0.168</td>
<td>0.068</td>
</tr>
</tbody>
</table>

Now, let us report the values of the free parameters we have obtained from the NLO fit:

\[ X_u^p = 0.648, \quad X_s^p = 0.247, \quad X_u^A = 0.296, \quad X_s^A = 0.476 \]
\[ b = 0.200, \quad \bar{b} = -0.472, \quad A_y^B = 0.051. \]

(12)

These parameters have similar values to those obtained for the nucleon PDF [6], in particular, for the thermodynamical potentials, so the intrinsic properties of the quarks when observed in DIS or in fragmentation processes seem to be preserved. Notice that in the nucleon PDF the \( u \) quark which is dominant has the larger potential and here we have analogously, \( X_u^p > X_s^p \) and \( X_u^A > X_s^A \), a situation which is natural to expect. The other parameters \( A_{u1}^B, A_{d1}^B \), for the quark to baryon FF, and \( A_{Q1}^B \) for the heavy quarks are given in Table II, together with the quark content \( q_1, q_2 \) and \( Q \) for each baryon \( B \). Clearly these normalization constants are decreasing when going from the proton to the heavier hyperons, following the magnitude of the corresponding measured cross sections.

Finally, let us comment on our results. With the data set on proton, \( \Lambda, \Sigma^+, \Sigma^- \) and \( \Xi^- \) production, we get a \( \chi^2 = 227.5 \) for 206 experimental points \( (x_E \geq 0.1) \), i.e. \( \chi^2/\text{point} = 1.1 \) for our NLO fit. This agreement of our results with experimental data is satisfactory and it confirms that the statistical approach is also successful in the description of the octet baryons FF.
TABLE II: Values of the normalization constants of the FF for the octet baryons

<table>
<thead>
<tr>
<th>Baryon</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$A_{20}^H$</th>
<th>$A_{20}^A$</th>
<th>$A_{22}^A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(uud)$</td>
<td>$u = d$</td>
<td>$s$</td>
<td>0.264</td>
<td>1.168</td>
<td>2.943</td>
</tr>
<tr>
<td>$\Lambda(uds)$</td>
<td>$u = d$</td>
<td>$s$</td>
<td>0.428</td>
<td>1.094</td>
<td>0.720</td>
</tr>
<tr>
<td>$\Sigma^+(uus)$</td>
<td>$u$</td>
<td>$s$</td>
<td>0.033</td>
<td>0.462</td>
<td>0.180</td>
</tr>
<tr>
<td>$\Sigma^-(dds)$</td>
<td>$d$</td>
<td>$s$</td>
<td>0.030</td>
<td>0.319</td>
<td>0.180</td>
</tr>
<tr>
<td>$\Xi^-(dss)$</td>
<td>$d$</td>
<td>$s$</td>
<td>0.023</td>
<td>0.082</td>
<td>0.072</td>
</tr>
</tbody>
</table>

In Figs. 1 and 2 we display the available DIS data, which yield directly $D_u^p(x, Q^2)$ and $D_u^\Lambda(x, Q^2)$, and the result of our fit. In Figs. 3-6 we show a comparison of the calculated cross sections for various baryons with experimental data. In Fig. 3, we give our results for the proton cross section in electron-positron annihilation at $\sqrt{s} = 22, 29, 34$ GeV and at the Z-pole i.e. $\sqrt{s} = 91.2$ GeV. The agreement with data is very good even in the low $x_E$ region ($x_E \leq 0.1$) as shown on the same figure, although these data points were not included in our fit.

For the $\Lambda$ production, our results are displayed in Fig. 4 for the energies $\sqrt{s} = 14, 22, 29, 33.3, 34.8, 42.1, 91.2$ GeV. It is clear from Figs. 3 and 4, which display a sizeable energy domain, that the scaling violations in this range are very small. In order to improve the small $x_E$ behavior, we need to modify the evolution of the FF in this region, and also include finite mass corrections and modifications of the splitting functions (see a discussion in Ref. [13]), but such corrections are outside the scope of the present paper. In Fig. 5 we give our results for $\Sigma^+$ production at $\sqrt{s} = 91.2$ GeV and in Fig. 6 our results for $\Xi^-$ production at the Z-pole. For these strangeness production processes, our results show a satisfactory agreement. The corresponding quark to baryon FF are presented in Fig. 7. For all hyperons the strange quark FF dominates largely over the $u, d$ quarks, which seems natural. For $\Lambda$, we are in agreement with a model for $SU(3)$ symmetry breaking in Ref. [15], which leads to $D_u^\Lambda \sim 0.07D_s^\Lambda$. In Ref. [14] one also finds $D_u^A$ smaller than $D_s^A$ and $D_\Lambda^A$ is strongly suppressed. This is in contrast with the situation of Ref. [13], where $u, d$ and $s$ are assumed to contribute equally. However the heavy quarks have a pattern similar to Ref. [13], with a sizeable contribution only for $x \leq 0.1$ and a fast dropping off for large $x$. This is also at variance with Ref. [16], where $D_u^A/D_s^A$ decreases from 1 to 0.2 when $x$ goes from 0 to 1. For the proton it is surprising to see that the $u$-quark FF dominates only at large $x$, whereas the strange and heavy quarks contribute substantially for $x \leq 0.3$ or so. Finally we notice that for the $\Lambda$, the heavy quarks have a pattern similar to Ref. [13], with a sizeable contribution only for $x \leq 0.1$ and a fast dropping off for large $x$.

IV. CONCLUSIONS

With the motivation to check whether the octet baryons FF have similar statistical features as the nucleon PDF previously studied, we extended the statistical approach to analyze some data on the inclusive production of the octet baryons. We found that these FF can be well described with a small number of free parameters, whose interpretation was discussed. We obtained a satisfactory description of the unpolarized experimental data, suggesting that the statistical approach, with Fermi-Dirac type FF, works equally well, compared to other parametrization forms. The semi-inclusive DIS data for proton and $\Lambda$ production, which give strong constraints, have allowed a flavor separation between $u, d$ and $s$ quarks FF, an interesting result which remains to be more seriously checked in the future.

In our present analysis, we did not introduce polarized FF, although our formalism can be
easily extended to this case. Actually, it is impossible to extract some reliable information on the polarized FF for the moment, due to the scarcity of the data, but we hope this will be possible in the future.

Acknowledgments

We thank J.-J. Yang for his contribution at the earlier stage of this work. We are grateful to W. Vogelsang and M. Stratmann for an helpful correspondence and useful comments.

[41] OPAL Collaboration, G. Alexander et al., Z. Phys C 73, 569 (1997); ibid. 73, 587 (1997).
FIG. 1: The $u$ quark to proton FF $D_u^p(x,Q^2)$ as a function of $x$ at $Q^2 = 25$ GeV$^2$. The experimental data are from Ref. [17].
FIG. 2: The FF for u quark to Λ, $D_u^A(x, Q^2)$, as a function of $x$ at $Q^2 = 2.5\text{GeV}^2$. The experimental data are from Ref. [18].
FIG. 3: Cross sections for proton production in $e^+e^-$ annihilation at several energies as function of $x_E$. The experimental data are from Refs. [24–29].
FIG. 4: Cross sections for $\Lambda$ production in $e^+e^-$ annihilation at several energies, as function of $x_E$. The experimental data are from Refs. [29–40].
FIG. 5: Cross sections for $\Sigma^\pm$ production in $e^+e^-$ annihilation at the Z-pole as function of $x_E$. The experimental data are from Ref. [41].
FIG. 6: Cross sections for $\Xi^-$ production in $e^+e^-$ annihilation at the Z-pole as function of $x_E$. The experimental data are from Refs. [37, 39, 42].
FIG. 7: The quark to octet baryons FF $D_Q^B(x, Q^2)$ and $D_Q^{R}(x, Q^2)$ ($B = p, \Lambda, \Sigma^\pm, \Xi^-$, $q = u, d, s$ and $Q = c, b, t$), as a function of $x$ at $Q = 91.2$ GeV. Note that we used different vertical scales in the upper and lower parts of the figure.