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Robustness against unknown Networked induced Delays of Observer based FDI

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Abstract: In this paper, we are concerned by the problem of Fault diagnosis in Networked Control Systems (NCS). The effect of unknown networked induced delays on conventional observed based residual generator is studied. It is shown that the detection performances may be reduced due to the sensitivity of the residuals to the delays. With the assumption that the network delays belong to a given bounded set, in order to enhance the robustness of fault detection an adaptive evaluation procedure of the residuals is proposed. The detection thresholds which depend on the maximum influence of the network delays are estimated using an optimization approach.

Keywords: Fault detection, observer based residual generation, networked control systems, unknown delays, robustness.

1. INTRODUCTION

The new trends for the realization of fault diagnosis (FDI) and fault tolerant control (FTC) systems that implement supervision functionalities (performance evaluation, fault diagnosis) and reconfiguration mechanisms are to implement them thanks to cooperative functions that are also distributed on a networked architecture. The study and the design of such application, called *Networked Control Systems* (NECS) [9], represent an emerging important research field.

Conventional theories with ideal assumptions, such as non-delayed sensing and actuation must be re-evaluated. Unknown network induced delays [5], [6] and data dropouts [3], [4] are the main characteristics of the network that are to be considered in the design of FDI. However, whereas there are a great number of significant results for the control problems of NECS, studies related to FDI of NECS are just at the starting point.

In this paper we are interested on evaluating first the effects of the communication induced delays in the performance of the FDI system. It will be shown that the performances of FDI are altered by the delay. Depending on the

input signal applied to the system, delays can even induce false alarms. As a solution to the robustness problem an adaptive thresholding mechanism is proposed for residual evaluation.

The proposed approach for threshold adaptation is based on the optimisation of a performance index related to the maximum amplitude of the residual with respect to the possible variation of the delays inside a bounded region.

Simulations are performed for the validation of the proposed method.

2. INSTANTANEOUS EXECUTION OF FDI METHODS

In this section, we introduce the traditional model-based FDI methods. Let us consider the system:

$$\begin{cases} \dot{x}(t) = Ax(t) + \sum_{i=1}^m B_i u_i(t) + Ef(t) \\ y(t) = Cx(t) + Ff(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ the input vector, $y(t) \in \mathbb{R}^p$ the output vector and $f(t) \in \mathbb{R}^q$ represents fault vectors to be

identified. All the matrices in Eq. (1) are of appropriate dimensions.

Here, it is assumed the residual generation and evaluation algorithms are executed instantaneously at every sampling period kT .

Based on this assumption, if the control input is kept constant over each sampling interval T , and if we also consider that fault input is constant over this interval, the discrete time system can be described by:

$$\begin{cases} x((k+1)T) = \Phi x(kT) + \sum_{i=1}^m \Gamma_i u_i(kT) + \Xi f(kT) \\ y(kT) = Cx(kT) + Ff(kT) \end{cases} \quad (2)$$

where :

$$\Phi = e^{AT}, \Gamma_i = \int_0^T e^{As} B_i ds, \Xi = \int_0^T e^{As} E ds \quad (3)$$

The aim of failure diagnosis systems is to design a residual which is zero in the fault free case and non zero when a fault occurs. The problem to deal with is to design a linear filter whose inputs are the measurable outputs and control inputs which generates certain outputs called residual signals, one for each possible fault, not affected by other faults and not affected by disturbances.

In this paper, we consider a classical observer based residual generator. Therefore, our aim is to find an observer for system of Eq. (2) of the form:

$$\begin{cases} \hat{x}((k+1)T) = \Phi \hat{x}(kT) + \sum_{i=1}^m \Gamma_i u_i(kT) \\ \quad + L(y(kT) - \hat{y}(kT)) \\ \hat{y}(kT) = C\hat{x}(kT) \end{cases} \quad (4)$$

and the residual generator :

$$r(kT) = T(y(kT) - \hat{y}(kT)) \quad (5)$$

where $L \in \mathbb{R}^{n,p}$ and $T \in \mathbb{R}^{q,p}$ are matrices that are designed in order to fulfill fault detection and isolation requirements.

From (5) and (6), the estimation error $\varepsilon(kT) = x(kT) - \hat{x}(kT)$ and the output of the filter propagate as :

$$\begin{cases} \varepsilon((k+1)T) = (\Phi - LC)\varepsilon(kT) + (E - LF)f(kT) \\ r(kT) = TC\varepsilon(kT) \end{cases} \quad (6)$$

Actually, there are various approaches [2],[7] to determine the gain matrices L and T , but we do not discuss this topic in the paper. The residual generator given by Eqs. (4) and (5) will be considered in the next section to study the effect of network induced delays.

3. MODELLING OF NETWORK INDUCED DELAYS

If, it is now supposed that the system is controlled over a network, then we have to take into account the sensor to controller τ_{sc} delays and controller to actuator delays τ_{ca} as it illustrated by Fig. 1.

Note that the delays, in general, cannot be considered constant and known. Network induced delays may vary, depending on the network traffic, medium access protocol and the hardware.

Fig 1: A feedback control system with network induced delays

From the controller viewpoint the system of eq (1) can be described by the model :

$$\begin{cases} \dot{x}(t) = Ax(t) + \sum_{i=1}^m B_i u_i(t - \tau_i) + Ef(t) \\ y(t) = Cx(t) \end{cases} \quad (7)$$

where the delays τ_i depend on the sensor to controller and controller to actuator delays. This can be more clearly seen in Fig 2., in which kT are the sensor sampling time and the values $y(kT)$ arrives at the controller after a delay of τ_{sc} . When the controller receives the sensor value, it immediately computes the new control input value $u(k)$ and a 'ZOH' device holds this value constant until the next sensor reading. The control signal is received by the actuator after a delay of τ_{ca} .

The discrete time model of Eq (1) is now given by :

$$\begin{aligned} x((k+1)T) = \Phi x(kT) + \sum_{i=0}^m (\Gamma_{0i} u_i(kT)) \\ + \sum_{i=0}^m (\Gamma_{li} u_i((k-1)T)) + \Xi f(kT) \end{aligned} \quad (8)$$

with the definitions :

$$\Gamma_{0i} = \int_0^{T-\tau_i} e^{As} B_i ds \quad \Gamma_{li} = \int_{T-\tau_i}^T e^{As} B_i ds \quad (9)$$

Fig. 2. : Delay timing diagram for a networked control system.

Eq. (8) can also be rewritten as :

$$\begin{cases} x((k+1)T) = \Phi x(kT) + \sum_{i=1}^m (\Gamma u_i(kT)) \\ \quad - \sum_{i=1}^m (\Gamma_{li} \Delta u_i(kT)) + \Xi f(kT) \\ y(k) = Cx(k) + Ff(k) \end{cases} \quad (10)$$

with :

$$\Delta u_i(kT) = u_i(kT) - u_i((k-1)T) \quad (11)$$

The FDI system introduced in section 2 was designed a priori to achieve certain performances characteristic without considering the time delays.

Let us now examine the effects of network induced delays on to the residual generator given by Eq (5).

From Eqs. (10) and (5), the estimation error $e(kT) = x(kT) - \hat{x}(kT)$ and the residual vector $r(kT)$ propagate as :

$$\begin{cases} e((k+1)T) = (\Phi - LC)e(kT) \\ \quad + \sum_{i=1}^m \Gamma_{li}(\tau_i) \Delta u_i(kT) + (\Xi - LF)f(kT) \\ r(kT) = TCe(kT) \end{cases} \quad (12)$$

Hence, it appears that the robustness of the fault diagnosis system against network induced delays depend on the amplitude of the unknown term $\sum_{i=1}^m \Gamma_{li}(\tau_i) \Delta u_i(kT)$.

We consider that a fault is detectable if the evaluation function $\Psi(t) = |r(t)|$ given in the temporal domain exceeds pre-definite thresholds as follows :

$$\begin{cases} \Psi(r_i(kT)/f_j) \geq Th(kT) & \text{for } f_j \neq 0 \\ \Psi(r_i(kT)/f_j) < Th(kT) & \text{for } f_j = 0 \end{cases} \quad (13)$$

Thus, the detection thresholds $Th(f_j)$ depends upon the maximum amplitude of the uncertainties induced by the network delays :

$$Th(r_i(kT)/f_j) = \sup_{\tau} |r_i(kT)| = \sup_{\tau} |T\epsilon_i(kT)| \quad (14)$$

If we assume that the delays are bounded :

$0 \leq \tau \leq \tau^+$, then with :

$$|r^+(kT)| = \sup_{0 \leq \tau \leq \tau^+} |r(kT)| \quad (15)$$

the following inequalities hold :

$$0 \leq |r(kT)| \leq |r^+(kT)| \quad (16)$$

making thus possible to determine the optimal threshold for fault detection.

Several approaches have been proposed in the literature for adaptive thresholding [1] and interval analysis [8]. To determine the optimal threshold we suggest to consider an optimisation approach.

4. ADAPTIVE RESIDUAL EVALUATION

The aim is to determine the maximum amplitude of the residuals, considering the influence of the unknown network induced delays.

In the continuous time domain, over the sampling period $[kT, (k+1)T]$ the system model satisfy to :

$$\begin{cases} \dot{x}(t) = Ax(t) + \sum_{i=1}^m B_i u_i(t, \tau_i) + Ef(t) \\ y(t) = Cx(t) + Ff(t) \end{cases} \quad (17)$$

with the control input :

$$\begin{cases} u_i(t, \tau_i) = u_i(kT - 1), & kT \leq t \leq kT + \tau_i \\ u_i(t, \tau_i) = u_i(kT), & kT + \tau_i \leq t \leq (k+1)T \end{cases} \quad (18)$$

Over the same sampling period, the discrete time observer given by equation (5) is, in the continuous time domain, equivalent to :

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + \sum_{i=1}^m B_i u_i(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \quad (19)$$

where L is such as $\Phi - KC = \exp\{(A - LC)T\}$. But, since the control input applied to the observer

$$u_i(t, 0) = u_i(kT), \quad kT \leq t \leq (k+1)T$$

the estimation error $\varepsilon(t) = x(t) - \hat{x}(t)$ and the residual propagate as

$$\begin{cases} \dot{\varepsilon}(t) = (A - LC)\varepsilon(t) + \sum_{i=1}^m B_i(u_i(t, \tau_i) - u_i(t, 0)) \\ r(t) = TC\varepsilon(t) \end{cases} \quad (20)$$

Note that in comparison to Eq.(12), we have $-\Delta u_i(kT) = u_i(t, \tau_i) - u_i(t, 0)$.

The minimum threshold for fault detection, can be considered as a performance index to be maximized for the dynamic system described by Eq (20). Thus, the optimisation problem is to find the inputs $u_i^*(t)$ on the time interval $[kT, (k+1)T]$ such as the performance index $\Psi(t = kT) = |r(kT)|$ is maximised.

For that purpose, let us introduce the Hamiltonian :

$$H = \lambda^T [(A - LC)\varepsilon(t) + \sum_{i=1}^m B_i(u_i(t, \tau_i) - u_i(t, 0))] \quad (21)$$

where λ is the co-state vector.

The solution of the optimisation problem satisfy to the state and co-state equations :

$$\begin{aligned} \dot{\varepsilon}^*(t) &= \frac{\partial H}{\partial \lambda^*} = (A - LC)\varepsilon^*(t) \\ &+ \sum_{i=1}^n b_i(u_i^*(t) - u_i(t, 0)) \end{aligned} \quad (22)$$

$$\dot{\lambda}^*(t) = -\frac{\partial H}{\partial \varepsilon^*} = -(A - LC)\lambda^*(t) \quad (23)$$

where $*$ stands for the optimum and $u_i^*(t) = u_i(t, \tau_i^*)$ is given by the application of the Pontryagin minimum principle:

$$\begin{aligned} \tau_i^* &= \text{Arg}(\text{Min}(H)) \\ 0 &\leq \tau_i \leq \tau_i^+ \end{aligned} \quad (24)$$

In addition to the initial conditions $\varepsilon(kT)$, the terminal boundary conditions for this fixed terminal time problem is :

$$\left[\frac{\partial \Psi}{\partial \varepsilon} - \lambda(t) \right]_{t=(k+1)T} = 0 \quad (25)$$

which give :

$$\lambda((k+1)T) = TC \begin{bmatrix} \text{sgn}(\varepsilon_1((k+1)T)) \\ \vdots \\ \text{sgn}(\varepsilon_n((k+1)T)) \end{bmatrix} \quad (26)$$

Thus, the optimisation consists of solving state and co-state equations with boundary conditions while selecting at all time those values equations τ_i^* that maximize H . The optimisation problem is very simple since over the length of optimisation window only one commutation of the control input is applied. Therefore the solution to (21) is given by :

$$\tau^* = \text{Arg}(\text{Min}(\lambda^T \sum_{i=1}^m B_i \Delta u_i(t, \tau_i))) \quad (27)$$

$$0 \leq \tau \leq \tau^+$$

Obviously, the optimum for τ_i^* (either τ_i^+ or 0) depends only upon the sign of the terms in $\lambda^T \sum_{i=1}^m B_i u_i(t, \tau_i)$ which requires to know the sign of $\lambda(t)$.

This can be achieved when solving (23). Since the inputs over the considered time interval input, are of step type

The optimization procedure can be iterated over several sampling intervals.

5. APPLICATION

The process pictured in Fig.3. is considered for illustration purpose. The plant is composed of two tanks of section S supplied by two pumps with flow rates Q_1 and Q_2 . The two tanks are connected through a pipe. Output flow rate is delivered at the outlet of the second tank. Input control variable are the input flow-rates. Measurement output variables as well as state variables are the levels L_1 and L_2 in the two tanks.

Fig. 3. Physical description of the Benchmark

As an approximation, with $L_0=0$ it is supposed that the out flow-rates depends on the tank levels as follows:

$$Q_{ij} = R(L_i - L_j) \quad (28)$$

The system which is a 2^{nd} order one with 2 inputs and 2 outputs can be modeled in

the form of the state space representation (1) with the following matrices :

$$A = \begin{bmatrix} -\frac{1}{RS} & \frac{1}{RS} \\ \frac{1}{RS} & -\frac{2}{RS} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{S} & 0 \\ 0 & \frac{1}{S} \end{bmatrix} \quad (29)$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E = 0, \quad F = C.$$

Only sensor faults have been considered in the simulations.

For residual generation, two state observers driven by a single output (either L_1 or L_2 were designed in order to given a estimation of the other outputs. This dedicated scheme is a classical one for sensor fault detection and isolation.

In the simulations which were performed, the sampling period is 1s and the network induced delays are simulated as random variables constrained to belong to the interval $[0, 1s]$. Output signal #1 is of a sin wave type, while input signal #2 is of step type. The obtained results are reported on Fig.5 and Fig. 6. which show the results obtained with the 2 observers. Each figure gives respectively the output signal (L_1 or L_2) with the additive fault signal, the dedicated residuals when there is no network delays $r_1 = L_1 - \hat{L}_1$ or $r_2 = L_2 - \hat{L}_2$, and the residuals influenced by the delays . The bottom plots show clearly using for residual evaluation the adaptive threshold developed in section 4 will enhance the robustness of the detector.

Fig. 4: Simulator for FDI of a Networked control system

5. CONCLUSION

This paper presents an adaptive residual evaluation strategy suitable for fault diagnosis of networked Control systems. The network induced delays are considered as unknown bounded variables. To determine the adaptive

threshold an optimization procedure has been used.

Simulations results have shown the effectiveness of the proposed method.

Fig 5. Simulation results obtained with observer #1.

Fig 6. Simulation results obtained with observer #2.

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REFERENCES

- [1] Emami-Naeini A., Akhter M.M. and Rock S.M., "Effect of model uncertainty on failure detection: the threshold selector", *IEEE Trans. Aut. Control* 33, p. 1106-1115, 1988.
- [2] Frank, P., & Ding, S. Current development in the theory of FDI. *Proc. IFAC Safeprocess* (16-27), Budapest, Hungary., 2000.
- [3] Ling Q. and Lemmon M.D. Robust performance of soft real-time networked control systems with data dropouts», *Proc. Of the IEEE Conference on Decision and Control*, 2002.
- [4] Ling Q. and Lemmon M.D, «Soft real-time scheduling of networked systems with dropouts governed by a Markov chain», *Proc. Of the American Conference on Control*, 2003.
- [5] Nilsson. J Real-Time Control Systems with Delays. *PhD thesis, IT Lünd* (Sweden), 1998.
- [6] Nilsson J. et al, "Stochastic analysis and control of real-time systems with random time delays", *Automatica*, vol.34, pp.57-64, 1998.
- [7] Patton R.J., P. Frank, R. Clark (Ed.), Fault diagnosis in dynamic systems, *Prentice Hall, International series in systems and control engineering.*, 1989.
- [8] Puig V. , Quevedo J. , Escobet T. , De las Heras S., Passive robust Fault detection approaches using interval models., 15th Triennial World Congress of the International Federation of Automatic Control Barcelona, 21–26 July 2002.
- [9] W. Zhang, M. S. Branicky, and S. M. Philipps. Stability of networked control systems. *IEEE Control Systems Magazine*, 21(1):84–99, 2001