The infuence of domain geometry in the boundary behavior of large solutions of degenerate elliptic problems

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Abstract

In this paper we study the asymptotic boundary behavior of large solutions of the equation $\Delta u = d^\alpha u^p$ in a regular bounded domain Ω in $I\!R^N$, $N \geq 2$, where $d(x)$ denotes the distance from x to $\partial\Omega$, $p > 1$ and $\alpha > 0$. We precise the expansion which depends on the mean curvature of the boundary.

1 Introduction : notations and main results

Let Ω be a regular bounded domain in $I\!R^N$, $N \geq 2$, $p > 1$ and $\alpha > 0$. We denote by $d(x)$ the distance from x to $\partial\Omega$, the boundary of Ω. In this paper we consider the semilinear degenerate equation

$$\Delta u = d^\alpha u^p \qquad in \quad \Omega \tag{1}$$

and we are interesting in the large solutions of (1), that is solutions of (1) which blow up at the boundary :

$$u(x) \to +\infty \quad as \quad d(x) \to 0. \tag{2}$$

Note already that the maximum principle implies that the solutions $u \in C^2(\Omega)$ of (1)-(2) are positive in Ω.

Equation (1) registers in problems of the form

$$\Delta u = p(x) f(u) \qquad in \quad \Omega. \tag{3}$$

Those problems were first studied by Bieberbach [4] for the case $p(x) = 1$, $f(u) = e^u$ and $N = 2$, in the context of Riemannian surfaces of constant negative curvature, and the theory of automorphic functions. The case $p(x) > 0$ for all $x \in \overline{\Omega}$ has been, largely dealt with in the literature (see [7], [12], [8], [5] for example).
Existence of solutions of (1)-(2) was established by Lair and Wood [9]. The question of the uniqueness of solutions of (1)-(2) is more delicate. When $\alpha = 0$ and $p > 1$, it is well know that problem (1)-(2) has a unique solution which satisfies

[0]AMS Subject Classifications : 35J25, 35J70.
Keywords : elliptic equations, boundary behavior, blow-up.

$$\lim_{d(x)\to 0} u(x)d(x)^{\frac{2}{p-1}} = \left(\frac{2(p+1)}{(p-1)^2}\right)^{\frac{1}{p-1}}. \qquad (4)$$

This was first established by Loewner and Nirenberg [10] for the case $p = (N+2)/(N-2)$. Later we can find many extensions, see for example [1], [2] and [14] and the references cited there. The case $\alpha < 0$ and $p > 0$ is studied in [6]. In the general case $\alpha \geq 0$, Marcus and Véron proved the uniqueness of the solutions of (1)-(2) under the condition $1 < p < (N+1+\alpha)/(N-1)$. Our first theorem completes this result and gives the rate of the blow-up.

Theorem 1.1 *Let $u \in C^2(\Omega)$ be a solution of* $(1)-(2)$ *. Then it satisfies*

$$\lim_{d(x)\to 0} d(x)^{\frac{\alpha+2}{p-1}} u(x) = l \qquad (5)$$

where l is given by

$$l = \left[\frac{(\alpha+2)(\alpha+p+1)}{(p-1)^2}\right]^{\frac{1}{p-1}}. \qquad (6)$$

This theorem allows us to establish the uniqueness of solutions of (1)-(2) with no conditions on p and α.

Theorem 1.2 *Problem (1) possesses a unique large solution.*

In the second time we are interested in the influence of the geometry of Ω in the boundary behavior. When $\alpha = 0$, this problem was first studied by Bandle and Marcus [3] for the radially symetric solutions of (1)-(2) in a ball. Later their result was extended by del Pino and Letelier [13] for general solutions. They proved that a lower-order term, still explosive, appears in the expansion of u wich depends linearly of the mean curvature of the boundary of Ω. More precisely, if $1 < p < 3$ and $\alpha = 0$, then on a sufficiently small neighborhood of $\partial\Omega$ we have the expansion

$$u(x) = \left(\frac{2(p+1)}{(p-1)^2}\right)^{\frac{1}{p-1}} d(x)^{-\frac{2}{p-1}} \left\{1 + \frac{N+1}{p+3} H(\overline{x})d(x) + o(d(x))\right\}. \qquad (7)$$

Here, for all x in a neighborhood of $\partial\Omega$, $\overline{x}$ denotes the unique point of the boundary such that $d(x) = |x-\overline{x}|$ and $H(\overline{x})$ the mean curvature of the boundary at that point. Estimate (7) generalizes to our case $\alpha \geq 0$ in the following way.

Theorem 1.3 *Let $u \in C^2(\Omega)$ a large solution of* (1). *Then, on a sufficiently small neighborhood of $\partial\Omega$:*

$$u(x) = l d(x)^{-\frac{2+\alpha}{p-1}} \left\{1 + \frac{N-1}{\alpha+p+3} H(\overline{x})d(x) + o(d(x))\right\}. \qquad (8)$$

This theorem implies that on a sufficiently small neighborhood of $\partial\Omega$:

$$u(x) - ld(x)^{-\frac{2+\alpha}{p-1}} = \frac{N-1}{\alpha+p+3} H(\overline{x}) d(x)^{-\frac{\alpha+3-p}{p-1}} + o\left(d(x)^{-\frac{\alpha+3-p}{p-1}}\right) . \tag{9}$$

Therefore, we obtain that

- if $p > \alpha + 3$, then the first member of (9) tends to 0 at the boundary,

- if $p = \alpha + 3$, then $u(x) - ld(x)^{-\frac{2+\alpha}{p-1}} = \frac{N-1}{\alpha+p+3} H(\overline{x}) + o(1)$,

- if $p < \alpha + 3$, then the first member of (9) is not bounded and the blow-up depends on the mean curvature. Roughly, the "more curved" or "sharper" towards the exterior of Ω is around a given point of $\partial\Omega$, the higher the explosion rate at that point is.

That is a generalization of the results of Bandle and Marcus [3] for the radially symetric solutions of (1)-(2) in a ball $\Omega = B(0, R)$:

- if $p > 3$, then $u(r) - l(R-r)^{-\frac{2}{p-1}} \to 0$ when $r \to R$,

- if $p = 3$, then $u(r) - l(R-r)^{-\frac{2}{p-1}} \to \frac{C(N)}{R}$ when $r \to R$, which represents the mean curvature of the ball,

- if $p < 3$, then $u(r) - l(R-r)^{-\frac{2}{p-1}} \to \infty$ when $r \to R$.

Our paper is organized as follows :

1. Introduction
2. Asymptotic behavior and uniqueness
3. Boundary influence in the explosion rate.

2 Asymptotic behavior and uniqueness.

We begin this section by proving a classical estimate for all solution u of (1). (see [12]).

Proposition 2.1 : *(Osserman estimate): There exist two positive constants $a = a(\partial\Omega)$ and $C = C(\Omega, \alpha, p)$ such that for all solution $u \in C^2(\Omega)$ of equation (1), we have :*

$$u(x) \leq C d(x)^{-\frac{2+\alpha}{p-1}} \tag{10}$$

for all $x \in \Omega$ such that $d(x) < a$.

Proof : Since Ω is regular there exist $\tilde{a} = \tilde{a}(\Omega) > 0$ and $M = M(\Omega) > 0$ such that

$$|\Delta d(x)| \leq M, \qquad |\nabla d(x)| = 1 \tag{11}$$

for all $x \in \Omega$ such that $d(x) < \tilde{a}$. Set $a = \min(1, \frac{\tilde{a}}{2})$. Let $x_0 \in \Omega$ such that $d(x_0) < a$ and $r_0 = d(x_0)/2$. We denote by B_0 the ball centered at x_0 of radius r_0 and we define the function w in B_0 as follows :

$$w(x) = \lambda d(x)^{-\frac{\alpha}{p-1}} (r_0^2 - |x - x_0|^2)^{-\frac{2}{p-1}} \tag{12}$$

with $\lambda > 0$ to determine such that

$$-\Delta w + d^\alpha w^p \geq 0 \quad in \quad B_0. \tag{13}$$

A straightforward computation gives :

$$-\Delta w + d^\alpha w^p = \lambda d^{-\frac{\alpha}{p-1}}(r_0^2 - |x-x_0|^2)^{-\frac{2p}{p-1}} \times$$

$$\Big[-\frac{\alpha(\alpha+p-1)}{(p-1)^2}(r_0^2-|x-x_0|^2)^2 d^{-2} + \frac{\alpha}{p-1}(r_0^2-|x-x_0|^2)^2 d^{-1}\Delta d$$

$$+\frac{8\alpha}{(p-1)^2}(r_0^2-|x-x_0|^2)d^{-1}\nabla d.(x-x_0) - \frac{8(p+1)}{(p-1)^2}|x-x_0|^2$$

$$-\frac{4N}{p-1}(r_0^2-|x-x_0|^2)+\lambda^{p-1}\Big].$$

Since $|x-x_0| < d(x_0) \leq 1$, $d(x) \geq d(x_0)/2$ and $r_0^3 < r_0^2$, there exists a constant $L = L(\alpha, p, M) > 0$ such that

$$-\Delta w + d^\alpha w^p \geq \lambda d^{-\frac{\alpha}{p-1}}(r_0^2-|x-x_0|^2)^{-\frac{2p}{p-1}}(-Lr_0^2+\lambda^{p-1})$$

in B_0. Therefore, we choose $\lambda = L^{\frac{1}{p-1}} r_0^{\frac{2}{p-1}}$ and we obtain (13). Note that $w(x) = +\infty$ if $x \in \partial B_0$ because $-2/(p-1) < 0$. The comparison principle implies $u \leq w$ in B_0 and in particular

$$u(x_0) \leq w(x_0) = L^{\frac{1}{p-1}}\left(\frac{d(x_0)}{2}\right)^{\frac{2}{p-1}}(d(x_0))^{-\frac{\alpha}{p-1}}\left(\frac{d(x_0)}{2}\right)^{-\frac{4}{p-1}}$$

which gives inequality (10).

We now establish an estimate from below for the solutions of (1)-(2). The results of [1] and [2] can't be used because the distance function d is not positive in $\overline{\Omega}$. Nevertheless we can adapt them as follows.

Proposition 2.2 *Let $u \in C^2(\Omega)$ be a solution of* $(1)-(2)$. *Then*

$$\liminf_{d(x)\to 0} d(x)^{\frac{\alpha+2}{p-1}} u(x) \geq l \tag{14}$$

where l is defined in (6).

Proof : Let $\varepsilon > 0$, $\tilde{a}$ be as the proof of proposition 2.1 and $\beta \in (0,1)$. We define

$$\underline{u}(x) = \beta l((d(x)+\varepsilon)^{-\frac{\alpha+2}{p-1}} - (\overline{a}+\varepsilon)^{-\frac{\alpha+2}{p-1}})$$

where $\overline{a}$ will be determined such that $\overline{a} < \tilde{a}$. We have $\underline{u} > 0$ on $\partial\Omega$ and $\underline{u}(x) = 0$ for all x such that $d(x) = \overline{a}$. Moreover a straighforward computation yields

$$-\Delta\underline{u} + d^\alpha \underline{u}^p = \beta\left[\Delta d\left(\frac{\alpha+2}{p-1}\right) l(d+\varepsilon)^{-\frac{\alpha+p+1}{p-1}} - l^p(d+\varepsilon)^{-\frac{\alpha+2p}{p-1}}\right.$$

$$+d^\alpha\beta^{p-1}l^p((d+\varepsilon)^{-\frac{\alpha+2}{p-1}}-(\overline{a}+\varepsilon)^{-\frac{\alpha+2}{p-1}})^p\Big]$$

in $0<d(x)<\overline{a}$. Using inequality (11), we obtain

$$-\Delta\underline{u}+d^\alpha\underline{u}^p\le\beta l^p(d+\varepsilon)^{-\frac{\alpha+2p}{p-1}}\left[M\left(\frac{\alpha+2}{p-1}\right)l^{1-p}(d+\varepsilon)\right.$$

$$\left.-1+\beta^{p-1}\left(\frac{d}{d+\varepsilon}\right)^\alpha\right]$$

which implies

$$-\Delta\underline{u}+d^\alpha\underline{u}^p\le\beta l^p(d+\varepsilon)^{-\frac{\alpha+2p}{p-1}}[\overline{M}(d+\varepsilon)-(1-\beta^{p-1})]$$

with $\overline{M}=M(\frac{\alpha+2}{p-1})l^{1-p}$. We now choose $\overline{a}=\frac{1}{2}\min(\tilde{a},\frac{1-\beta^{p-1}}{\overline{M}})$ and impose $\varepsilon<\frac{1}{2}(\frac{1-\beta^{p-1}}{\overline{M}})$.

Then $\overline{u}$ is a subsolution of (1) in $0<d(x)<\overline{a}$. By the maximum principle we derive $\underline{u}\le u$ in $0<d(x)<\overline{a}$. Letting ε tend to 0, this implies for all $\beta\in(0,1)$ and x such that $d(x)<\overline{a}$:

$$\beta l\left[1-\left(\frac{d(x)}{\overline{a}}\right)^{\frac{\alpha+2}{p-1}}\right]\le d(x)^{\frac{\alpha+2}{p-1}}u(x).$$

Therefore for all $\beta\in(0,1)$:

$$\beta l\le\liminf_{d(x)\to0}d(x)^{\frac{\alpha+2}{p-1}}u(x)$$

which ends the proof.

Because of proposition 2.2, we can describe the asymptotic behavior of radially symetric solutions of (1)-(2).

Proposition 2.3 *Let $R>0$ and $v\in C^2(0,R)$ a solution of*

$$-v''-\frac{N-1}{r}v'+(R-r)^\alpha v^p=0 \tag{15}$$

in $(0,R)$ such that

$$\lim_{r\to R}v(r)=+\infty.$$

Then

$$\lim_{r\to R}(R-r)^{\frac{\alpha+2}{p-1}}v(r)=l \tag{16}$$

where l is defined in (6).

We omit the proof of this proposition because it follows the idea of [14]: the function $w(t) = (R-r)^{\frac{\alpha+2}{p-1}}v(r)$ with $R-r = e^{-t}$ is bounded and satisfies a second order differentiel equation in a neighborhood of infinity and the ω-limit set of a trajectory of that equation is $\{0\}$ or $\{l\}$. Therefore proposition 2.2 implies proposition 2.3. Those results allows us to prove theorem 1.1.

Proof of theorem 1.1 : In view of (14) we must only prove that

$$\limsup_{d(x)\to 0} d(x)^{\frac{\alpha+2}{p-1}}u(x) \leq l. \tag{17}$$

Still the results of [1], [2] or [14] don't apply directly but we can adapt them. Let $y \in \partial\Omega$. Since $\partial\Omega$ is smooth, there exists a ball B_y centered at a point Y of radius R_y such that $B_y \subset \Omega$ and $\overline{B_y} \cap \partial\Omega = \{y\}$. We introduce the function V defined by $V(x) = v(|x|)$ for all $x \in B_{R_y}$ where v is a function as in proposition 2.3 with $R = R_y$. The function v exists because it is the radial solution of (1)-(2) for $\Omega = B$ (see [9]). Let $k > 1$. Finally we introduce the function V_k defined by $V_k(x) = k^{\frac{2}{p-1}}V(k(x-Y))$ for all $x \in B(Y, \frac{R_y}{k})$. Note that $B(Y, \frac{R_y}{k}) \subset B_y$ and V_k is solution of

$$-\Delta V_k + (R_y - k|x-Y|)^\alpha V_k^p = 0$$

in $B(y, \frac{R_y}{k})$ and satisfies

$$\lim_{|x-Y|\to\frac{R_y}{k}} V_k(x) = +\infty.$$

Since $x \in B(Y, \frac{R_y}{k})$ implies $d(x) \geq R_y - |x-Y| \geq R_y - k|x-Y|$, the comparison principle involves $u \leq V_k$ in $B(Y, \frac{R_y}{k})$. Letting k tend to 1, we obtain

$$u(x) \leq v(|x-Y|) \qquad in \quad B_y. \tag{18}$$

Because of proposition 2.3, for all $\varepsilon > 0$ there exists $\eta > 0$ such that

$$|s^{\frac{\alpha+2}{p-1}}v(R_y - s) - l| < \varepsilon \quad \forall s \in (0, \eta). \tag{19}$$

Let $\tilde{\eta} > 0$ be sufficiently small so that for all $x \in \Omega$ with $d(x) < \tilde{\eta}$ there exists a unique $y \in \partial\Omega$ such that $|x-y| = d(x)$. Then for all $x \in \Omega$ such that $d(x) < \min(\eta, \tilde{\eta})$, both inequalities (18) and (19) imply

$$d(x)^{\frac{\alpha+2}{p-1}}u(x) \leq d(x)^{\frac{\alpha+2}{p-1}}v(R_y - d(x)) < l + \varepsilon$$

and inequality (17) holds.

Proof of theorem 1.2 : Large solutions of (1) satisfy (5). Then two large solutions u_1 and u_2 of (1) are such that

$$\lim_{d(x)\to 0} \frac{u_1(x)}{u_2(x)} = 1.$$

and the result follows as in [1] or [11].

3 Boundary influence in the explosion rate.

In this section we prove theorem 1.3. As in [13] we construct suitable sub- and supersolutions of (1) in a neighborhood of $\partial\Omega$ which are inspired of the radial study that we omit here.

Since Ω is regular there exists $\overline{b} > 0$ such that d is a function of class C^2 in $\{x \in \Omega \ / \ d(x) < \overline{b}\}$, $|\nabla d(x)| = 1$ and

$$\Delta d(x) = -(N-1)H(\overline{x}) + o(1) \quad as \quad d(x) \to 0. \tag{20}$$

Let $b_0 \in (0, \overline{b})$, $b \in (0, b_0)$ and $\varepsilon > 0$. We introduce the function Ψ defined in $E_{b,b_0} = \{x \in \Omega \ / \ b < d(x) < b_0\}$ by

$$\Psi(x) = l(d(x)-b)^{-\frac{\alpha+2}{p-1}} + \frac{l}{\alpha+p+3}((N-1)H(\overline{x}) + \varepsilon)(d(x)-b)^{-\frac{\alpha+3-p}{p-1}}.$$

We claim that if b_0 is chosen sufficiently small, independently of ε and b, then Ψ is a supersolution in E_{b,b_0}. Indeed, a straightforward computation using (20) gives :

$$\Delta\Psi = l^p(d(x)-b)^{-\frac{\alpha+2p}{p-1}} + l(d(x)-b)^{-\frac{\alpha+p+1}{p-1}} \left[\frac{\alpha+2}{p-1}(N-1)H(\overline{x})\right.$$

$$+\frac{(\alpha+3-p)(\alpha+2)}{(\alpha+p+3)(p-1)^2}((N-1)H(\overline{x}) + \varepsilon) + o(1)$$

$$\left.+\frac{\alpha+3-p}{(p-1)(\alpha+p+3)}((N-1)H(\overline{x}) + \varepsilon)((N-1)H(\overline{x}) + o(1))(d(x)-b)\right].$$

On the other hand, we have

$$\begin{aligned} d(x)^\alpha\Psi^p &\geq (d(x)-b)^\alpha\Psi^p \\ &\geq l^p(d(x)-b)^{-\frac{\alpha+2p}{p-1}}\left[1 + \tfrac{p}{\alpha+p+3}((N-1)H(\overline{x}) + \varepsilon)(d(x)-b) + o(d(x)-b)\right]. \end{aligned}$$

Then

$$-\Delta\Psi + d^\alpha\Psi^p \geq l(d(x)-b)^{-\frac{\alpha+p+1}{p-1}} \times$$

$$\left[-\frac{\alpha+2}{p-1}(N-1)H(\overline{x}) - \frac{(\alpha+3-p)(\alpha+2)}{(\alpha+p+3)(p-1)^2}((N-1)H(\overline{x}) + \varepsilon)\right.$$

$$-\frac{\alpha+3-p}{(p-1)(\alpha+p+3)}((N-1)H(\overline{x}) + \varepsilon)((N-1)H(\overline{x}) + o(1))(d(x)-b)$$

$$\left.+\frac{l^{p-1}p}{\alpha+p+3}((N-1)H(\overline{x}) + \varepsilon) + o(1)\right]$$

Since

$$-\frac{\alpha+2}{p-1}-\frac{(\alpha+3-p)(\alpha+2)}{(\alpha+p+3)(p-1)^2}+\frac{l^{p-1}p}{\alpha+p+3}=0\ ,$$

and since the coefficient of ε is $(\alpha+2)/(p-1)$, it implies that there exists $b_0=b_0(\varepsilon)\in(0,\overline{b})$ such that for all $0<b<b_0$:

$$-\Delta\Psi+d^\alpha\Psi^p\geq 0\quad in\quad E_{b,b_0}.$$

Consider the solution u of (1)-(2). We claim that there exists a positive number K independent of $b\in(0,b_0)$ such that :

$$\Psi(x)+K\geq u(x) \tag{21}$$

for all $x\in\Omega$ with $d(x)=b_0$. In fact, if we define

$$M_0=\max_{d(x)=b_0}u(x)\ ,$$

we can compute for all x such that $d(x)=b_0$:

$$\Psi(x)=l(b_0-b)^{-\frac{\alpha+2}{p-1}}+\frac{l}{\alpha+p+3}((N-1)H(\overline{x})+\varepsilon)(b_0-b)^{-\frac{\alpha+3-p}{p-1}}.$$

Since $\partial\Omega$ is regular, there exists a real $b_1\in(0,b_0)$ such that

$$\left|\frac{1}{\alpha+p+3}((N-1)H(\overline{x})+\varepsilon)(b_0-b)\right|\leq\frac{1}{2}$$

for all $b\in(b_1,b_0)$, where $\overline{x}$ is such that $d(x)=|x-\overline{x}|$. Therefore

$$1+\frac{1}{\alpha+p+3}((N-1)H(\overline{x})+\varepsilon)(b_0-b)\geq\frac{1}{2}$$

and then

$$\Psi(x)\geq\frac{l}{2}(b_0-b)^{-\frac{\alpha+2}{p-1}}\geq\frac{l}{2}(b_0-b_1)^{-\frac{\alpha+2}{p-1}}$$

for all $b\in(b_1,b_0)$, where $\overline{x}$ is such that $d(x)=|x-\overline{x}|$. On the other hand, for all $b\in(0,b_1]$ and $d(x)=b_0$:

$$\begin{aligned}\Psi(x)\ &=l(b_0-b)^{-\frac{\alpha+2}{p-1}}+\tfrac{l}{\alpha+p+3}((N-1)H(\overline{x})+\varepsilon)(b_0-b)^{-\frac{\alpha+3-p}{p-1}}\\ &\geq lb_0^{-\frac{\alpha+2}{p-1}}-C(b_0-b_1)^{-\frac{\alpha+3-p}{p-1}}\end{aligned}$$

with $C>0$ and because the assumption if we assume $\alpha+3-p>0$ (we omit the proof in the case $\alpha+3-p\leq 0$ which is simplier). Finally we obtain for all $b\in(0,b_0)$:

$$\Psi(x)\geq L=\min\left(\frac{l}{2}(b_0-b_1)^{-\frac{\alpha+2}{p-1}},lb_0^{-\frac{\alpha+2}{p-1}}-C(b_0-b_1)^{-\frac{\alpha+3-p}{p-1}}\right)\ ,$$

then, for all x such that $d(x)=b_0$, $u\leq M_0\leq\max(1,M_0-L)+L\leq\max(1,M_0-L)+\psi$ which implies (21).

On the other hand the function $\Psi + K$ is itself a supersolution of equation (1) in E_{b,b_0}. Therefore the comparison principle implies (21) in E_{b,b_0}. Letting b tend to 0, we obtain

$$u(x) \leq l d(x)^{-\frac{\alpha+2}{p-1}} + \frac{l}{\alpha+p+3}((N-1)H(\overline{x}) + \varepsilon) d(x)^{-\frac{\alpha+3-p}{p-1}} + K$$

for all $x \in \Omega$ such that $0 < d(x) < b_0$. In the same way, by considering subsolutions in the form

$$\phi(x) = l(d(x)+b)^{-\frac{2+\alpha}{p-1}} + \frac{l}{\alpha+p+3}((N-1)H(\overline{x}) - \varepsilon)(d(x)+b)^{-\frac{\alpha+3-p}{p-1}} - \overline{K}$$

we obtain expansion (8).

Aknowledgements : the authors are pleased to aknowledge Catherine Bandle for their profitable discussions.

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