Asymptotic Estimates for Perturbed Scalar Curvature Equation
Samy Skander Bahoura

To cite this version:
hal-00100324

HAL Id: hal-00100324
https://hal.archives-ouvertes.fr/hal-00100324
Preprint submitted on 26 Sep 2006

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
ASYMPTOTIC ESTIMATE FOR PERTURBED SCALAR CURVATURE EQUATION.

SAMY SKANDER BAHOURA

ABSTRACT. We consider the equation \( \Delta u_\varepsilon = V_\varepsilon u_\varepsilon^{(n+2)/(n-2)} + \varepsilon W_\varepsilon u_\varepsilon^\alpha \) with \( \alpha \in \left[ \frac{n}{n-2} - \frac{n+2}{n-2} \right] \) and we give some minimal conditions on \( \nabla V \) and \( \nabla W \) to have an uniform estimate for their solutions when \( \varepsilon \to 0 \).

1. INTRODUCTION AND RESULTS.

We denote \( \Delta = -\sum_i \partial_{ii} \) the geometric Laplacian on \( \mathbb{R}^n, n \geq 3 \).

Let us consider on open set \( \Omega \) of \( \mathbb{R}^n, n \geq 3 \), the following equation:

\[
\Delta u_\varepsilon = V_\varepsilon u_\varepsilon^{(n+2)/(n-2)} + \varepsilon W_\varepsilon u_\varepsilon^\alpha \quad (E_\varepsilon)
\]

where \( V_\varepsilon \) and \( W_\varepsilon \) are two regular functions and \( \alpha \in \left[ \frac{n}{n-2} - \frac{n+2}{n-2} \right] \).

We assume:

\[
0 < a \leq V_\varepsilon(x) \leq b, \quad ||\nabla V_\varepsilon||_{L^\infty} \leq A \quad (C_1)
\]

\[
0 < c \leq W_\varepsilon(x) \leq d, \quad ||\nabla W_\varepsilon||_{L^\infty} \leq B \quad (C_2)
\]

**Problem:** Can we have an \( \sup \times \inf \) estimate with the minimal conditions \((C_1)\) and \((C_2)\) ?

Note that for \( W \equiv 0 \), the equation \((E_\varepsilon)\) is the wellknown scalar curvature equation on open set of \( \mathbb{R}^n, n \geq 3 \). In this case, there is many results about this equation, see for example [B] and [C-L 1].

When \( \Omega = S_n \cdot Y. Li \), give a flatness condition to have the boundedness of the energy and the existence of the simple blow-up points, see [L1] and [L2].

In [C-L 2], Chen and Lin gave a contexmple of solutions of the scalar curvature equation with unbounded energy. The conditions of Li are minimal in heigh dimension.

Note that, in [C-L 1] and [C-L 3], there is some results concerning Harnack inequalities of type \( \sup \times \inf \) with the "Li-flatness" conditions for the following equation:

\[
\Delta u = V u^{(n+2)/(n-2)} + g(u)
\]

where \( g \) is a regular function ( at least \( C^1 \) ) such that \( g(t)/[t^{(n+2)/(n-2)}] \) is deacrising and tends to 0 when \( t \to +\infty \). They extend Li result ([L1]) to any open set of the euclidian space.

We can find in [A], some existence results for the prescribed scalar curvature equation.

In our work we have no assumption on the energy. We use the blow-up analysis and the moving-plane method, developed by Gidas-Ni-Nirenberg, see [G-N-N]. This method was used by different authors to have a priori estimates, look for example, [B], [B-L-S] ( in dimension 2), [C-L 1], [C-L 3], [L 1] and [L 2].

We set \( \delta = [(n+2) - \alpha (n-2)]/2, \delta \in ]0, 1[ \). We have:
Theorem 1. For all \(a, b, c, d, A, B > 0\), for all \(\alpha \in (\frac{n}{n-2}, \frac{n+2}{n-2})\) and all compact set \(K\) of \(\Omega\), there is a positive constant \(c = c(a, b, c, d, A, B, \alpha, K, \Omega, n)\) such that:

\[
\epsilon^{(n-2)/2(1-\delta)}(\sup_{K} u_{\epsilon})^{1/3} \times \inf_{\Omega} u_{\epsilon} \leq c
\]

for all \(u_{\epsilon}\) solution of \((E_{\epsilon})\) with \(V_{\epsilon}\) and \(W_{\epsilon}\) satisfying the conditions \((C_{1})\) and \((C_{2})\).

Now, suppose we that \(V_{\epsilon}\) satisfies:

\[
0 < a \leq V_{\epsilon}(x) \leq b \quad \text{and} \quad ||\nabla V_{\epsilon}||_{L^{\infty}(\Omega)} \leq k \epsilon
\]

We have:

Theorem 2. For all \(a, b, c, d, k, B > 0\), for all \(\alpha \in (\frac{n}{n-2}, \frac{n+2}{n-2})\) and all compact set \(K\) of \(\Omega\), there is a positive constant \(c = c(a, b, c, d, k, B, \alpha, K, \Omega, n)\) such that:

\[
\sup_{K} u_{\epsilon} \times \inf_{\Omega} u_{\epsilon} \leq c
\]

for all \(u_{\epsilon}\) solution of \((E_{\epsilon})\) with \(V_{\epsilon}\) and \(W_{\epsilon}\) satisfying the conditions \((C_{3})\) and \((C_{2})\).

Note that in [B], we have some results as the previous but for prescribed scalar curvature equation with subcritical exponent tending to the critical. Here, we have a \(\sup \times \inf\) inequality for the scalar curvature equation, with critical exponent, perturbed by a nonlinear term. We can see the influence of this non-linear term.

2. PROOFS OF THE THEOREMS.

Proof of the theorem 1.

Without loss of generality, we suppose \(\Omega = B_{1}\) the unit ball of \(\mathbb{R}^{n}\). We want to prove an a priori estimate around 0. We can also suppose \(\epsilon \to 0\), the case \(\epsilon \not\to 0\) is solved in [B].

Let \((u_{i})\) and \((V_{i})\) be a sequences of functions on \(\Omega\) such that:

\[
\Delta u_{i} = V_{i}^{a} u_{i}^{(n+2)/(n-2)} + \epsilon_{i} W_{i}^{a} u_{i}, \quad u_{i} > 0, \quad 0 < a \leq V_{i}(x) \leq b, \quad 0 < a \leq W_{i}(x) \leq d, \quad ||V_{i}||_{L^{\infty}} \leq A \quad \text{and} \quad ||W_{i}||_{L^{\infty}} \leq B.
\]

We argue by contradiction and we suppose that the \(\sup \times \inf\) is not bounded.

We have:

\[
\forall c, R > 0 \exists u_{c,R} \text{ solution of } (E_{1}) \text{ such that:}
\]

\[
\epsilon^{(n-2)/2(1-\delta)} R^{n-2} \left( \sup_{B(0,R)} u_{c,R} \right)^{1/3} \times \inf_{\Omega} u_{c,R} \geq c, \quad (H)
\]

Proposition 5 (blow-up analysis)

There is a sequence of points \((y_{i})_{i}, y_{i} \to 0\) and two sequences of positive real numbers \((l_{i})_{i}, (L_{i})_{i}, l_{i} \to 0, L_{i} \to +\infty\), such that if we set \(v_{i}(y) = \frac{u_{i}(y + y_{i})}{u_{i}(y_{i})}\), we have:

\[
0 < v_{i}(y) \leq \beta_{i} \leq 2^{(n-2)/2}, \quad \beta_{i} \to 1.
\]

\[
v_{i}(y) \to \left( \frac{1}{1 + |y|^{2}} \right)^{(n-2)/2}, \quad \text{uniformly on all compact set of } \mathbb{R}^{n}.
\]

\[
i^{(n-2)/2} \epsilon_{i}^{(n-2)/2(1-\delta)} [u_{i}(y_{i})]^{1/3} \times \inf_{B_{1}} u_{i} \to +\infty.
\]

Proof of the proposition:

We use the hypothesis \((H)\), we take two sequences \(R_{i} > 0, R_{i} \to 0\) and \(c_{i} \to +\infty\), such that,
\( \varepsilon_i^{(n-2)/2(1-\delta)} R_i^{(n-2)} (\sup_{B(0, R_i)} u_i)_1^{1/3} \times \inf_{B_i} u_i \geq c_i \to +\infty, \)

Let \( x_i \in B(x_0, R_i) \) be a point such that \( \sup_{B(0, R_i)} u_i = u_i(x_i) \) and \( s_i(x) = (R_i - |x - x_i|)^{(n-2)/2} u_i(x), x \in B(x_i, R_i) \). Then, \( x_i \to 0. \)

We have:

\[
\max_{B(x_i, R_i)} s_i(x) = s_i(y_i) \geq s_i(x_i) = R_i^{(n-2)/2} u_i(x_i) \geq \sqrt{c_i} \to +\infty.
\]

We set:

\[
l_i = R_i - |y_i - x_i|, \quad \bar{u}_i(y) = u_i(y_i + y), \quad v_i(z) = \frac{u_i[y_i + \left( z/|u_i(y_i)|^{2/(n-2)} \right)]}{u_i(y_i)}.
\]

Clearly we have, \( y_i \to x_0 \). We also obtain:

\[
L_i = \frac{l_i}{(c_i)^{1/2(n-2)}} [u_i(y_i)]^{2/(n-2)} = \frac{[s_i(y_i)]^{2/(n-2)}}{c_i^{1/2(n-2)}} \geq \frac{c_i^{1/(n-2)}}{c_i} = c_i^{1/2(n-2)} \to +\infty.
\]

If \( |z| \leq L_i \), then \( y = [y_i + z/|u_i(y_i)|^{2/(n-2)}] \in B(y_i, \delta_i l_i) \) with \( \delta_i = \frac{l_i}{(c_i)^{1/2(n-2)}} \) and \( |y - y_i| < R_i - |y_i - x_i| \), thus, \( |y - x_i| < R_i \) and, \( s_i(y) \leq s_i(y_i) \). We can write:

\[
u_i(y)(R_i - |y - y_i|)^{(n-2)/2} \geq u_i(y_i)(l_i)^{(n-2)/2}.
\]

But, \( |y - y_i| \leq \delta_i l_i, R_i > l_i \) and \( R_i - |y - y_i| \geq R_i - \delta_i l_i > l_i - \delta_i l_i = l_i(1 - \delta_i) \). We obtain,

\[
0 < v_i(z) = \frac{u_i(y)}{u_i(y_i)} \leq \left[ \frac{l_i}{l_i(1 - \delta_i)} \right]^{(n-2)/2} \leq 2^{(n-2)/2}.
\]

We set, \( \beta_i = \left( \frac{1}{1 - \delta_i} \right)^{(n-2)/2} \), clearly, we have, \( \beta_i \to 1. \)

The function \( v_i \) satisfies:

\[
\Delta v_i = \tilde{V}_i \varepsilon_i^{(n+2)/(n-2)} + \varepsilon_i \tilde{W}_i \left[ u_i(y_i) \right]^{n/(n-2)} - \alpha,
\]

where, \( \tilde{V}_i(y) = V_i[y + y/|u_i(y_i)|^{2/(n-2)}] \) and \( \tilde{W}_i(y) = W_i[y + y/|u_i(y_i)|^{2/(n-2)}] \).

Without loss of generality, we can suppose that \( \tilde{V}_i \to V(0) = n(n - 2). \)

We use the elliptic estimates, Ascoli and Ladeyenskaya theorems to have the uniform convergence of \( \{v_i\} \) to \( v \) on compact set of \( \mathbb{R}^n \). The function \( v \) satisfies:

\[
\Delta v = n(n - 2) v^{N-1}, \quad v(0) = 1, \quad 0 \leq v \leq 1 \leq 2^{(n-2)/2},
\]

By the maximum principle, we have \( v > 0 \) on \( \mathbb{R}^n \). If we use Caffarelli-Gidas-Spruck result, (see [C-G-S]), we obtain, \( v(y) = \left( \frac{1}{1 + |y|^2} \right)^{(n-2)/2} \). We have the same properties that in [B].

**Polar Coordinates** (Moving-Plane method)

Now, we must use the same method than in the Theorem 1 of [B]. We will use the moving-plane method.

We must prove the lemma 2 of [B].

We set \( t \in ] - \infty, - \log 2 [ \) and \( \theta \in S_{n-1} : \)
Here we work on operator on $S$ where $w_i(t, \theta) = e^{(n-2)t/2}u_i(y_i + e^t \theta)$, $V_i(t, \theta) = V_i(y_i + e^t \theta)$ and $\bar{W}_i(t, \theta) = W_i(y_i + e^t \theta)$.

We consider the following operator $L = \partial_t - \Delta_{\sigma} - \frac{(n-2)^2}{4}$, with $\Delta_{\sigma}$ the Laplace-Baltrami operator on $S_{n-1}$.

The function $w_i$ is solution of:

$$-Lw_i = \bar{V}_i w_i^{N-1} + \epsilon_i e^{[(n+2)-(n-2)\alpha]t/2}w_i^{\alpha}. $$

For $\lambda \leq 0$ we set:

$$t^\lambda = 2\lambda - t \ w_i^\lambda(t, \theta) = w_i(t^\lambda, \theta), \ V_i^\lambda(t, \theta) = V_i(t^\lambda, \theta) \text{ et } \bar{W}_i^\lambda(t, \theta) = \bar{W}_i(t^\lambda, \theta).$$

**Remark:** Here we work on $[\lambda, t_i] \times S_{n-1}$, with $\lambda \leq -\frac{2}{n-2} \log u_i(y_i) + 2$ and $t_i \leq \log \sqrt{t_i}$, where $t_i$ is choosen as in the proposition.

First, like in [B], we have the following lemma:

**Lemma 1:**

Let $A_\lambda$ be the following property:

$$A_\lambda = \{ \lambda \leq 0, \exists (t_\lambda, \theta_\lambda) \in [\lambda, t_i] \times S_{n-1}, \ w_i^\lambda(t_\lambda, \theta_\lambda) - w_i(t_\lambda, \theta_\lambda) \geq 0 \}. $$

Then, there is $\nu \leq 0$, such that for $\lambda \leq \nu$, $A_\lambda$ is not true.

Like in the proof of the Theorem 1 of [B], we want to prove the following lemma:

**Lemma 2:**

For $\lambda \leq 0$ we have:

$$w_i^\lambda - w_i \leq 0 \Rightarrow -L(w_i^\lambda - w_i) \leq 0,$$

on $[\lambda, t_i] \times S_{n-1}$.

Like in [B], we have:

**A useful point:**

$\xi_i = \sup \{ \lambda \leq \lambda_i = 2 + \log \eta_i, w_i^\lambda - w_i < 0, \text{ on } [\lambda, t_i] \times S_{n-1} \}$. The real $\xi_i$ exists.

First, we have:

$$w_i(2\xi_i - t_i, \theta) = w_i([\xi_i - t + \xi_i - \log \eta_i - 2] + (\log \eta_i + 2)],$$

the definition of $w_i$ and the fact that, $\xi_i \leq t$, we obtain:

$$w_i(2\xi_i - t_i, \theta) = e^{[(n-2)(\xi_i - t + \xi_i - \log \eta_i - 2)]/2}e^{-2t_i[\theta e^{\xi_i}(\xi_i - t + (\log \eta_i - 2)] \leq 2^{(n-2)/2}e^{n-2} = \bar{c}. $$

**Proof of the Lemma 2:**

We know that:

$$-L(w_i^{\xi_i} - w_i) = [\bar{V}_i w_i^{\xi_i} N^{-1} - \bar{V}_i w_i^{N-1}] + \epsilon_i e^{\delta t_i} w_i^{\alpha} - e^{\delta t_i} w_i^{N-1},$$

with $\delta = [(n + 2) - (n - 2)\alpha]/2$.

We denote by $Z_1$ and $Z_2$ the following terms:

$$Z_1 = (\bar{V}_i^{\xi_i} - \bar{V}_i)(w_i^{\xi_i})^{N-1} + \bar{V}_i ([w_i^{\xi_i}]^{N-1} - w_i^{N-1}),$$

and
\[ Z_2 = \epsilon_i (W_i^{\xi_i} - W_i) \left( u_i^{\xi_i} \right)^{\alpha} e^{\delta t^{\xi_i}} + \epsilon_i e^{\delta t^{\xi_i}} W_i \left[ \left( u_i^{\xi_i} \right)^{\alpha} - w_i^{\alpha} \right] + \epsilon_i W_i w_i^{\alpha} (e^{\delta t^{\xi_i}} - e^{\delta t}). \]

But, using the same method as in the proof of the theorem 1 of [B], we have:

\[ w_i^{\xi_i} \leq w_i \text{ et } w_i^{\xi_i} (t, \theta) \leq w_i \text{ pour tout } (t, \theta) \in [\xi_i, \log 2] \times S_{n-1}, \]
where \( \bar{c} \) is a positive constant not depending on \( i \) for \( \xi_i \leq \log n_i + 2; \)

\[ |\bar{V}_i^{\xi_i} - \bar{V}_i| \leq A(e^t - e^{\xi_i}) \text{ et } |\bar{V}_i^{\xi_i} - \bar{V}_i| \leq B(e^t - e^{\xi_i}), \]

Then,

\[ Z_1 \leq A \left( w_i^{\xi_i} \right)^{N-1} (e^t - e^{\xi_i}) \text{ et } Z_2 \leq \epsilon_i B \left( w_i^{\xi_i} \right)^{\alpha} (e^t - e^{\xi_i}) + \epsilon_i \bar{c} \times (e^{\delta t^{\xi_i}} - e^{\delta t}). \]

and,

\[ -L(w_i^{\xi_i} - w_i) \leq (w_i^{\xi_i})^\alpha \left[ \left( A w_i^{\xi_i} \right)^{N-1-\alpha} + \epsilon_i B \right] (e^t - e^{\xi_i}) + \epsilon_i \bar{c} \times (e^{\delta t^{\xi_i}} - e^{\delta t}). \]

But, \( w_i^{\xi_i} \leq \bar{c}, \) we obtain:

\[ -L(w_i^{\xi_i} - w_i) \leq (w_i^{\xi_i})^\alpha \left[ \left( A \bar{c} \right)^{N-1-\alpha} + \epsilon_i B \right] (e^t - e^{\xi_i}) + \epsilon_i \bar{c} \times (e^{\delta t^{\xi_i}} - e^{\delta t}). \] (1)

We must see the sign of:

\[ \hat{Z} = \left[ \left( A \bar{c} \right)^{N-1-\alpha} + \epsilon_i B \right] (e^t - e^{\xi_i}) + \epsilon_i \bar{c} \times (e^{\delta t^{\xi_i}} - e^{\delta t}). \]

But \( \alpha \in \mathbb{Z}, \) \( \frac{n+2}{n-2}, \frac{n+2}{n-2} \alpha \in ]0, 1[, \)

For \( t \leq t_i < 0, \) we have:

\[ e^t \leq e^{(1-\delta)t_i} e^{\delta t}, \text{ for all } t \leq t_i, \]

and the fact that \( l^{\xi_i} \leq t (\xi_i \leq t), \) by integration of the previous two members, we obtain:

\[ e^t - e^{\xi_i} \leq \frac{e^{(1-\delta)t_i}}{\delta} (e^{\delta t} - e^{\delta t^{\xi_i}}), \text{ for all } t \leq t_i, \]

We can write:

\[ (e^{\delta t^{\xi_i}} - e^{\delta t}) \leq \frac{\delta}{e^{(1-\delta)t_i}} (e^{\xi_i} - e^t). \]

Then,

\[ -L(w_i^{\xi_i} - w_i) \leq (w_i^{\xi_i})^\alpha \left[ -\frac{\epsilon_i \bar{c}}{e^{(1-\delta)t_i}} + A \bar{c}^{N-1-\alpha} + \epsilon_i B \right] (e^t - e^{\xi_i}) \]

The term \( \frac{\epsilon_i \bar{c}}{e^{(1-\delta)t_i}} - A \bar{c}^{N-1-\alpha} - \epsilon_i B \) is positive if:

\[ \epsilon_i e^{-(1-\delta)t_i} \rightarrow +\infty, \]

then,

\[ \epsilon_i^{(n-2)/2(1-\delta)} e^{-(n-2)/2t_i} \rightarrow +\infty. \]

If we take, \( t_i = -\frac{2}{3(n-2)} \log u_i(y_i), \) we have:

\[ \epsilon_i^{(n-2)/2(1-\delta)} |u_i(y_i)|^{1/3} \rightarrow +\infty. \]

It is given by our Hypothesis in the proposition.

But the Hopf Maximum principle, gives:

\[ \min_{\theta \in S_{n-1}} w_i(t_i, \theta) \leq \max_{\theta \in S_{n-1}} w_i(2\xi_i - t_i, \theta), \]

then,
Proof of the Theorem 2.

The proof is similar than the proof of the theorem 1. Only the end of the proof is different.

Step 1: The blow-up analysis give:

There is a sequence of points \((y_i)\) \(y_i \to 0\) and two sequences of positive real numbers \((l_i)\), \((L_i)\), \(l_i \to 0\), \(L_i \to +\infty\), such that if we set \(v_i(y) = \frac{u_i(y + y_i)}{u_i(y_i)}\), we have:

\[
0 < v_i(y) \leq \beta_i \leq 2^{(n-2)/2}, \quad \beta_i \to 1.
\]

Step 2: Application of the Hopf maximum principle.

We have the same notation that in the proof of the theorem 1. First, we take \(t_i = \sqrt{l_i}\) as in the Step 1 and we look to the end of the proof of the theorem 1. We replace \(A\) by \(k\epsilon_i\). We want to proof that:

\[
e^{(n-2)t_i} u_i(y_i) \min_{B_2(0)} u_i \leq c,
\]

Contradiction.

Proof of Theorem 2.

The proof is similar than the proof of the theorem 1. Only the end of the proof is different.

Step 1: The blow-up analysis give:

There is a sequence of points \((y_i)\), \(y_i \to 0\) and two sequences of positive real numbers \((l_i)\), \((L_i)\), \(l_i \to 0\), \(L_i \to +\infty\), such that if we set \(v_i(y) = \frac{u_i(y + y_i)}{u_i(y_i)}\), we have:

\[
0 < v_i(y) \leq \beta_i \leq 2^{(n-2)/2}, \quad \beta_i \to 1.
\]

\[
v_i(y) \to \left(\frac{1}{1 + |y|^2}\right)^{(n-2)/2}, \quad \text{uniformly on all compact set of } \mathbb{R}^n.
\]

\[
l_i \frac{(n-2)}{2} u_i(y_i) \to +\infty, \quad \text{uniformly on all compact set of } \mathbb{R}^n.
\]

Step 2: Application of the Hopf maximum principle.

We have the same notation that in the proof of the theorem 1. First, we take \(t_i = \sqrt{l_i}\) as in the Step 1 and we look to the end of the proof of the theorem 1. We replace \(A\) by \(k\epsilon_i\). We want to proof that:

\[
w_i - w_i \leq 0 \Rightarrow -L(w_i - w_i) \leq 0,
\]

on \([\xi, t_i] \times S_{n-1}\). We have the same definition for \(\xi_i\) (as in the proof of the theorem 1).

For \(t \leq t_i < 0\), we have:

\[
e^t \leq e^{(1-\delta)t_i} e^{\delta t_i}, \quad \text{for all } t \leq t_i,
\]

and the fact that \(t_i \leq t (\xi_i \leq t)\), by integration of the previous two members, we obtain:

\[
e^t - e^{\epsilon t_i} \leq \frac{e^{(1-\delta)t_i}}{\delta}(e^{\delta t_i} - e^{\epsilon t_i}), \quad \text{for all } t \leq t_i.
\]

We can write:

\[
(e^{\epsilon t_i} - e^{\delta t_i}) \leq \frac{\delta}{e^{(1-\delta)t_i}} (e^{\epsilon t_i} - e^t).
\]

Then,

\[
-L(w_i - w_i) \leq (w_i - w_i) \alpha - \frac{\epsilon_i \delta c}{e^{(1-\delta)t_i}} + k\epsilon_i e^{N-1-\alpha} + \epsilon_i B[e^t - e^{\epsilon t_i}].
\]

The term \(\frac{\delta c}{e^{(1-\delta)t_i}} - k e^{N-1-\alpha} - B\) is positive because \(t_i \to -\infty\) and \(\delta \in [0, 1]\).

But the Hopf Maximum principle, gives:

\[
\min_{\theta \in S_{n-1}} w_i(t_i, \theta) \leq \max_{\theta \in S_{n-1}} w_i(2\xi_i - t_i, \theta),
\]

then,

\[
e^{(n-2)t_i} u_i(y_i) \min_{B_2(0)} u_i \leq c,
\]

and,
Contradiction with the step 1.

Références:


[B] S.S Bahoura. Majorations du type \( \sup u \times \inf u \leq c \) pour l’équation de la courbure scalaire prescrite sur un ouvert de \( \mathbb{R}^n \), \( n \geq 3 \). J.Math.Pures Appl.(9) 83 (2004), no.9, 1109-1150.


6, RUE FERDINAND FLOCON, 75018 PARIS, FRANCE.
E-mail address: samybahoura@yahoo.fr, bahoura@ccr.jussieu.fr