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Abstract : A new method for the design of reduced-order observers for descriptor systems with unknown inputs is presented. The approach is based on the generalized constrained Sylvester equation. Sufficient conditions for the existence of the observer are given.

1. Introduction

The problem of observers design for standard systems with unknown inputs has received considerable attention in the last two decades ([1]-[4] and references therein). This problem is of great importance in theory and practice since there are many situations where disturbances or partial inputs are inaccessible. In [5] a technique for computing an efficient solution for the unknown input observer design is given. This solution uses the constrained Sylvester equation. The usage of constrained and coupled Sylvester equation in automatic control is well-known [6], [7], and [8]. Recently, a great deal of work has been devoted to the observer design for descriptor systems and many approaches to design such observers exist [9]-[16]. In [9] a method based on the singular value decomposition and the concept of matrix generalized inverse to design a reduced-order observer has been proposed. In [11] the generalized Sylvester equation was used to develop a procedure for designing reduced-order observers. In [12] a method based on the generalized inverse was presented. Observers for continuous descriptor under less restrictive conditions and using only a straightforward matrix manipulation have been presented in [17], [18]. Observers for discrete-time descriptor systems have been developed in [14], [16].

However, only few results have been presented to design observers for descriptor systems with unknown inputs [19], [20], and [21]. Descriptor systems are very sensitive to slight input changes, and the presence of unmeasurable disturbances or unknown inputs is very detrimental to the design of observers. This fact justifies the importance of the observers design for descriptor systems in presence of unknown inputs. On the other hand, many practical systems can be described by descriptor models, and the fault diagnosis of these systems may be based on the unknown input observer design.

In [19] and [20] only square singular systems have been considered under the regularity condition. In addition, the strong observability [19] and the modal observability [20] have been assumed. In [21], a coordinate transformation is used to design a reduced-order observer.

In this paper, we present a new method to design a reduced-order observer for continuous-time descriptor systems subject to unknown inputs and unknown measurement disturbances. As in [21] systems considered are in a general form and less restrictive conditions are required.

2. Statement of the problem

Consider the linear time-invariant descriptor system

$$E^* \dot{x} = A^* x + B^* u + F^* w \tag{1.a}$$

$$y^* = C^* x + G^* w \tag{1.b}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^k$, $w \in \mathbb{R}^q$ and $y^* \in \mathbb{R}^p$ are the state vector, the control input vector, the unmeasurable input vector and the output vector respectively. $E^* \in \mathbb{R}^{mxn}$, $A^* \in \mathbb{R}^{mxn}$, $B^* \in \mathbb{R}^{mxk}$, $F^* \in \mathbb{R}^{mxq}$, $G^* \in \mathbb{R}^{pxq}$, and $C^* \in \mathbb{R}^{pxn}$ are known constant matrices. We assume that rank $E^* = r < n$, and without loss of generality rank $[C^* \quad G^*] = p$.

Assumptions. In the sequel we assume that

i) rank $\begin{bmatrix} F^* \\ G^* \end{bmatrix} = q \leq p$

ii) rank $\begin{bmatrix} E^* & A^* & F^* & 0 \\ 0 & E^* & 0 & F^* \\ 0 & C^* & G^* & 0 \\ 0 & 0 & 0 & G^* \end{bmatrix}$ - rank $\begin{bmatrix} E^* & F^* \\ 0 & G^* \end{bmatrix} = n + q$

These conditions are not restrictive. Condition i) can always be met by redefining the unknown input. If rank $\begin{bmatrix} F^* \\ G^* \end{bmatrix} = s < q$, then we have $\begin{bmatrix} F^* \\ G^* \end{bmatrix} w = \begin{bmatrix} F_1^* \\ G_1^* \end{bmatrix} v$, where $\begin{bmatrix} F_1^* \\ G_1^* \end{bmatrix}$ is of full column rank, and v can be considered as a new unknown input. Condition ii) generalizes the condition of the impulse observability of singular square systems (i.e. m = n and det $E^* = 0$) when $F^* = G^* = 0$.

For m = n, $E^* = I$, and $G^* = 0$, system (1) becomes a standard one with unknown inputs, in this case condition ii) can be written as

rank $\begin{bmatrix} I & F^* \\ C^* & 0 \end{bmatrix} = n + q$ or equivalently rank C^*F^* = rank $F^* = q$

which is the condition generally assumed in the standard observer for unknown input systems [1]-[4].

Now, since rank $E^* = r$, there exists a non-singular matrix P such that

$P\ E^* = \begin{bmatrix} E \\ 0 \end{bmatrix}$, $P\ A^* = \begin{bmatrix} A \\ A_1 \end{bmatrix}$, $P\ B^* = \begin{bmatrix} B \\ B_1 \end{bmatrix}$ and $P\ F^* = \begin{bmatrix} F \\ F_1 \end{bmatrix}$

with $E \in \mathbb{R}^{r.n}$ and rank E = r. Then system (1) is restricted system equivalent (r.s.e) to

$$E \dot{x} = A x + B u + F w \tag{2.a}$$

$$y = C x + D w \tag{2.b}$$

where $y = \begin{bmatrix} -B_1 u \\ y^* \end{bmatrix} \in \mathbb{R}^t$, $C = \begin{bmatrix} A_1 \\ C^* \end{bmatrix} \in \mathbb{R}^{t.n}$ and $D = \begin{bmatrix} F_1 \\ G^* \end{bmatrix} \in \mathbb{R}^{t.q}$, with t = m + p - r.

One can easily prove that assumption i) is equivalent to rank $\begin{bmatrix} F \\ D \end{bmatrix} = q$.

Let rank $D = q_1 \leq q$, then there exist two non-singular matrices U and V such that

$$U\,D\,V = \begin{bmatrix} I_{q_1} & 0 \\ 0 & 0 \end{bmatrix}.$$

System (2) can now be written as

$$E\dot{x} = \Phi x + B u + F_{11} y_1 + F_{12} w_2 \tag{3.a}$$

$$y_1 = C_{11} x + w_1 \tag{3.b}$$

$$y_2 = C_{12} x \tag{3.c}$$

where $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = Uy$, $\begin{bmatrix} C_{11} \\ C_{12} \end{bmatrix} = UC$, $w = V \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$, $F\,V = [F_{11} \quad F_{12}]$, and $\Phi = A - F_{11} C_{11}$.

One can easily obtain rank $C_{12} = p_1 = t - q_1$ from rank $[C^* \quad G^*] = p$ and the above matrix decomposition.

Our aim is to design an observer in the form

$$\dot{z} = \Pi z + L_1 y_1 + L_2 y_2 + H u \tag{4.a}$$

$$\hat{x} = M z + N y_2 \tag{4.b}$$

where $z \in \mathbb{R}^{n-p_1}$.

The problem of the observer design is reduced to finding matrices Π, L_1, L_2, H, M and N such that the estimate $\hat{x}$ converge asymptotically to the state x.

3. Reduced-order observer design

In this section we present a new method to design a reduced-order observer for singular system (1) with unknown inputs. The solution of this problem is given in the following theorem.

Theorem 3.1. Let T be an $(n- p_1).r$ matrix such that

$$T\Phi - \Pi T E = L_2 C_{12} \tag{5.a}$$

$$T F_{12} = 0 \tag{5.b}$$

where $\det \begin{bmatrix} TE \\ C_{12} \end{bmatrix} \neq 0$.

Then for

$$H = T B \tag{5.c}$$

$$L_1 = T F_{11} \tag{5.d}$$

and

$$\begin{bmatrix} TE \\ C_{12} \end{bmatrix} [\,M \quad N\,] = \begin{bmatrix} I_{n-p_1} & 0 \\ 0 & I_{p_1} \end{bmatrix} \tag{5.e}$$

we have

$$\hat{x}(t) - x(t) = M e^{\Pi t} (z(0) - TE\,x(0)).$$

The convergence of the reduced-order observer is obtained when Π is a stability matrix.

Since this theorem is easy to prove, its proof is omitted.

Equation (5.a) is the so called generalized Sylvester equation, which must be satisfied under the constraint (5.b).

Define the following non-singular matrix

$$\begin{bmatrix} R \\ C_{12} \end{bmatrix} = \begin{bmatrix} I_{n-p_1} & K \\ 0 & I_{p_1} \end{bmatrix} \begin{bmatrix} TE \\ C_{12} \end{bmatrix} \tag{6}$$

where K is an $(n-p_1).p_1$ arbitrary matrix and R is an $(n-p_1).n$ matrix of full row rank, then we have

$$T\,E = R - K\,C_{12} \tag{7}$$

Equations (5.b) and (7) can be written as

$$[\,T \quad K\,] \begin{bmatrix} E & F_{12} \\ C_{12} & 0 \end{bmatrix} = [\,R \quad 0\,] \tag{8}$$

The solution of this equation depends on the rank of matrix $\begin{bmatrix} E & F_{12} \\ C_{12} & 0 \end{bmatrix}$. The solution exists if

$$\operatorname{rank} \begin{bmatrix} E & F_{12} \\ C_{12} & 0 \end{bmatrix} = n + \dim w_2.$$

We can now establish the following result.

Lemma 3.1. For systems (1) and (3) we have the following statements:

1) $\operatorname{rank} \begin{bmatrix} E & F_{12} \\ C_{12} & 0 \end{bmatrix} = n + \dim w_2$ if and only if assumption ii) is satisfied.

2) $\operatorname{rank} \begin{bmatrix} sE - \Phi & -F_{12} \\ C_{12} & 0 \end{bmatrix} = \operatorname{rank} \begin{bmatrix} sE^* - A^* & -F^* \\ C^* & G^* \end{bmatrix} - q_1, \ \forall s \in \mathbb{C}.$

Proof. 1) Define the following non-singular matrices

$$U_1 = \begin{bmatrix} P & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & I_p & 0 \\ 0 & 0 & 0 & I_p \end{bmatrix}, V_1 = \begin{bmatrix} Q & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & I_q & 0 \\ 0 & 0 & 0 & I_q \end{bmatrix}, U_2 = \begin{bmatrix} P & 0 \\ 0 & I_p \end{bmatrix}, V_2 = \begin{bmatrix} Q & 0 \\ 0 & I_q \end{bmatrix},$$

$$U_3 = \begin{bmatrix} U & 0 & 0 \\ 0 & I_r & 0 \\ 0 & 0 & U \end{bmatrix}, V_3 = \begin{bmatrix} I_n & 0 & 0 \\ 0 & V & 0 \\ 0 & 0 & V \end{bmatrix}, U_4 = \begin{bmatrix} I_r & 0 \\ 0 & U \end{bmatrix}, V_4 = \begin{bmatrix} I_n & 0 \\ 0 & V \end{bmatrix} \text{ and}$$

$$U_5 = \begin{bmatrix} I_r \, F_{11} & 0 \\ 0 \;\; I_{q_1} & 0 \\ 0 \;\; 0 & I_{p_1} \end{bmatrix},$$

then from assumption ii) we obtain

$$\text{rank } U_1 \begin{bmatrix} E^* & A^* & F^* & 0 \\ 0 & E^* & 0 & F^* \\ 0 & C^* & G^* & 0 \\ 0 & 0 & 0 & G^* \end{bmatrix} V_1 - \text{rank } U_2 \begin{bmatrix} E^* & F^* \\ 0 & G^* \end{bmatrix} V_2 = n + q$$

or equivalently rank $\begin{bmatrix} C & D & 0 \\ E & 0 & F \\ 0 & 0 & D \end{bmatrix}$ - rank D = n + q, since E is of full row rank.

Then rank $U_3 \begin{bmatrix} C & D & 0 \\ E & 0 & F \\ 0 & 0 & D \end{bmatrix} V_3$ - rank U D V = n + q, it follows that

$$\text{rank} \begin{bmatrix} E & F_{12} \\ C_{12} & 0 \end{bmatrix} = n + \dim w_2 \tag{9}$$

2) We have the following relations

$$\begin{aligned} \text{rank} \begin{bmatrix} sE^* - A^* & -F^* \\ C^* & G^* \end{bmatrix} &= \text{rank } U_2 \begin{bmatrix} sE^* - A^* & -F^* \\ C^* & G^* \end{bmatrix} V_2 \\ &= \text{rank } U_4 \begin{bmatrix} sE - A & -F \\ C & D \end{bmatrix} V_4 \\ &= \text{rank } U_5 \begin{bmatrix} sE - \Phi & -F_{11} & -F_{12} \\ C_{11} & I_{q_1} & 0 \\ C_{12} & 0 & 0 \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} sE - \Phi & -F_{12} \\ C_{12} & 0 \end{bmatrix} + q_1 \end{aligned} \tag{10}$$ ♦

Now, equation (7) can be written as

$$[\, T \quad K \,] \begin{bmatrix} E \\ C_{12} \end{bmatrix} = R \tag{11}$$

It follows from (9) that rank $\begin{bmatrix} E \\ C_{12} \end{bmatrix}$ = n, then the general solution of (11) is therefore given by

$$[T \quad K] = R \begin{bmatrix} E \\ C_{12} \end{bmatrix}^{+} + Y \left(I_{r+p_1} - \begin{bmatrix} E \\ C_{12} \end{bmatrix} . \begin{bmatrix} E \\ C_{12} \end{bmatrix}^{+}\right) \tag{12}$$

where $\begin{bmatrix} E \\ C_{12} \end{bmatrix}^{+}$ is the generalized inverse of matrix $\begin{bmatrix} E \\ C_{12} \end{bmatrix}$, given by $\begin{bmatrix} E \\ C_{12} \end{bmatrix}^{+} = \Delta [E^T \ C_{12}^T]$, where $\Delta = (E^T E + C_{12}^T C_{12})^{-1}$ and Y is an arbitrary matrix of appropriate dimension.

From equation (12), we have

$$T = R \, \Delta \, E^T + Y\varphi \tag{13}$$

where $\varphi = \begin{bmatrix} I_r - E\Delta E^T \\ -C_{12}\Delta E^T \end{bmatrix}$. Substituting (13) into (5.b) gives

$$Y \, \varphi \, F_{12} = - R \, \Delta \, E^T \, F_{12} \tag{14}$$

The solution of this equation exists if $\varphi \, F_{12}$ is of full column rank. We now state the following result.

Lemma 3.2. The matrix $(\varphi \, F_{12})$ is of full column rank if and only if assumption ii) is satisfied.

Proof. Define the following full column rank matrix

$$S = \begin{bmatrix} \begin{bmatrix} E \\ C_{12} \end{bmatrix}^{+} \\ \left(I_{r+p_1} - \begin{bmatrix} E \\ C_{12} \end{bmatrix} . \begin{bmatrix} E \\ C_{12} \end{bmatrix}^{+}\right) \end{bmatrix},$$

then from lemma 3.1 we have

$$\text{rank} \begin{bmatrix} E & F_{12} \\ C_{12} & 0 \end{bmatrix} = n + \dim w_2$$

$$= \text{rank } S \begin{bmatrix} E & F_{12} \\ C_{12} & 0 \end{bmatrix}$$

it follows that rank $\varphi \, F_{12} = \dim w_2$ if and only if assumption ii) is verified. ♦

From lemma 3.2 the solution of (14) is given by

$$Y = -R \, \Delta \, E^T \, F_{12} \, (\varphi \, F_{12})^{+} + Z \, (I_{r+p_1} - (\varphi \, F_{12}) \, (\varphi \, F_{12})^{+}) \tag{15}$$

substituting (15) into (13) gives

$$T = R\,\Delta\,E^T - R\,\Delta\,E^T\,F_{12}(\varphi\,F_{12})^+\varphi + Z\,(I_{r+p_1} - (\varphi\,F_{12})\,(\varphi\,F_{12})^+)\,\varphi \tag{16}$$

where Z is an arbitrary matrix.

Now, from (5.a) and (5.e) we have

$$\Pi = T\,\Phi\,M \tag{17}$$
$$L_2 = T\,\Phi\,N \tag{18}$$

Substituting (16) into (17) gives

$$\Pi = \Omega + Z\,\Gamma \tag{19}$$

where $\Omega = R\,\Delta\,E^T\Phi\,M - R\,\Delta\,E^T F_{12}\,(\varphi\,F_{12})^+\varphi\,\Phi\,M$ (20)

and $\Gamma = (I_{r+p_1} - (\varphi\,F_{12})\,(\varphi\,F_{12})^+)\,\varphi\,\Phi\,M$ (21)

If the pair (Ω, Γ) is detectable, one can design a reduced-order observer from the standard methods, in the form (4).

In the sequel, we will give sufficient conditions for the existence of the reduced-order observer.

Lemma 3.3. Under the assumption rank $\begin{bmatrix} E \\ C_{12} \end{bmatrix} = n$, the reduced-order observer (4) exists if

$$\text{rank}\begin{bmatrix} sE - \Phi & -F_{12} \\ C_{12} & 0 \end{bmatrix} = n + \dim w_2,\ \forall s \in \mathbb{C},\ \text{Re}(s) \geq 0 \tag{22}$$

Proof. First note that rank $\begin{bmatrix} sE - \Phi & -F_{12} \\ C_{12} & 0 \end{bmatrix}$ = rank $\begin{bmatrix} I_r & 0 & 0 \\ 0 & I_{p_1} & -sI_{p_1} \\ 0 & 0 & I_{p_1} \end{bmatrix}\begin{bmatrix} sE - \Phi & -F_{12} \\ sC_{12} & 0 \\ C_{12} & 0 \end{bmatrix}$

$$= \text{rank}\begin{bmatrix} sE - \Phi & -F_{12} \\ sC_{12} & 0 \\ C_{12} & 0 \end{bmatrix}$$

Now, define the following matrices $S_1 = \begin{bmatrix} \begin{bmatrix} R \\ C_{12} \end{bmatrix} \cdot \begin{bmatrix} E \\ C_{12} \end{bmatrix}^+ & 0 \\ (I_{r+p_1} - \begin{bmatrix} E \\ C_{12} \end{bmatrix} \cdot \begin{bmatrix} E \\ C_{12} \end{bmatrix}^+) & 0 \\ 0 & I_{p_1} \end{bmatrix}$,

$S_2 = \begin{bmatrix} M & N & 0 \\ 0 & 0 & -I_{\dim w_2} \end{bmatrix}$, and $S_3 = \begin{bmatrix} I_{n-p_1} & 0 & -R\Delta E^T F_{12}(\varphi F_{12})^+ \\ 0 & I_{p_1} & -C_{12}\Delta E^T F_{12}(\varphi F_{12})^+ \\ 0 & 0 & (\varphi F_{12})^+ \\ 0 & 0 & I_{r+p_1} - (\varphi F_{12})\,(\varphi F_{12})^+ \end{bmatrix}$, then we have

$$\operatorname{rank}\begin{bmatrix} sE - \Phi & -F_{12} \\ sC_{12} & 0 \\ C_{12} & 0 \end{bmatrix} = \operatorname{rank} S_1 \begin{bmatrix} sE - \Phi & -F_{12} \\ sC_{12} & 0 \\ C_{12} & 0 \end{bmatrix} S_2$$

$$= \operatorname{rank}\begin{bmatrix} sI_{n-p_1} - R\Delta E^T\Phi M & (sR - R\Delta E^T\Phi)N & R\Delta E^T F_{12} \\ -C_{12}\Delta E^T\Phi M & (sC_{12} - C_{12}\Delta E^T\Phi)N & C_{12}\Delta E^T F_{12} \\ -\varphi\Phi M & -\varphi\Phi N & \varphi F_{12} \\ 0 & I_{p_1} & 0 \end{bmatrix},$$

$$= n + \dim w_2,\ \forall s \in \mathbb{C},\ \mathrm{Re}(s) \geq 0,$$

Since $R\,M = I_{n-p_1}$ and $C_{12}\,M = 0$, then it follows that

$$\operatorname{rank}\begin{bmatrix} sI_{n-p_1} - R\Delta E^T\Phi M & R\Delta E^T F_{12} \\ -C_{12}\Delta E^T\Phi M & C_{12}\Delta E^T F_{12} \\ -\varphi\Phi\, M & \varphi F_{12} \end{bmatrix} = \dim w_2 + n - p_1,\ \forall s \in \mathbb{C},\ \mathrm{Re}(s) \geq 0$$

or equivalently

$$\operatorname{rank} S_3 \begin{bmatrix} sI_{n-p_1} - R\Delta E^T\Phi\, M & R\Delta E^T F_{12} \\ -C_{12}\Delta E^T\Phi M & C_{12}\Delta E^T F_{12} \\ -\varphi\Phi M & \varphi F_{12} \end{bmatrix}$$

$$= \operatorname{rank}\begin{bmatrix} sI_{n-p_1} - \Omega & 0 \\ -C_{12}\Delta E^T\Phi M + C_{12}\Delta E^T F_{12}(\varphi F_{12})^+\varphi\Phi M & 0 \\ (\varphi F_{12})^+\varphi\Phi M & I_{\dim w_2} \\ \Gamma & 0 \end{bmatrix}$$

$$= \dim w_2 + n - p_1,\ \forall s \in \mathbb{C},\ \mathrm{Re}(s) \geq 0$$

which gives $\operatorname{rank}\begin{bmatrix} sI_{n-p_1} - \Omega \\ [\,0 \quad I_{p_1}\,]\,\Gamma \\ \Gamma \end{bmatrix} = n - p_1$, since $C_{12}\Delta E^T = -[\,0 \quad I_{p_1}\,]\,\varphi$, and finally we have

$$\operatorname{rank}\begin{bmatrix} sI_{n-p_1} - \Omega \\ \Gamma \end{bmatrix} = n - p_1,\ \forall s \in \mathbb{C},\ \mathrm{Re}(s) \geq 0$$

which is the detectability of the pair (Ω, Γ), and Π is a stability matrix. ♦

The above results leads to the following theorem.

Theorem 3.2. Sufficient conditions for the observer (4) to exist are assumptions i), ii) and

$$\operatorname{rank}\begin{bmatrix} sE^* - A^* & -F^* \\ C^* & G^* \end{bmatrix} = n + q,\ \forall s \in \mathbb{C},\ \mathrm{Re}(s) \geq 0 \tag{23}$$

The proof can be straightforwardly deduced from lemma 3.1, 3.2 and 3.3, and is omitted.

Remarks: i) Condition (23) generalizes the notion of stable transmission zeros of the square singular systems.

ii) The minimal order ρ of the observer is given by : $\rho = n - \text{rank } C_{12}$.

iii) Matrix Z in equation (19) can be chosen to make the poles of the observer have specified locations if, in theorem 3.2, $\forall s \in \mathbb{C}$, $\text{Re}(s) \geq 0$ is replaced by $\forall s \in \mathbb{C}$.

iv) The design of the observer presented in this paper requires only straightforward matrix operations. A procedure to compute efficiently the observer matrices can be obtained from the singular values decomposition of the full column matrices (φF_{12}) and $\begin{bmatrix} E \\ C_{12} \end{bmatrix}$. This problem has not been presented in this paper, the reader can refer to [8] for more details on the computations problems. ■

Now, we can summarize the procedure for designing the observer :

1) Choose an $(n-p_1).n$ matrix R such that $\begin{bmatrix} R \\ C_{12} \end{bmatrix}$ is non-singular, this can be always done, since C_{12} is of full row rank.

2) The matrix M can be obtained from (5.e) and (11)

$$M = \begin{bmatrix} R \\ C_{12} \end{bmatrix}^{-1} \begin{bmatrix} I_{n-p_1} \\ 0 \end{bmatrix} \tag{24}$$

3) Compute

$$\alpha = I_{r+p_1} - F_{12} (\varphi F_{12})^{+}\varphi,\ \Delta = \left(\begin{bmatrix} E \\ C_{12} \end{bmatrix}^{T} \begin{bmatrix} E \\ C_{12} \end{bmatrix} \right)^{-1},\ \Omega = R\, \Delta\, E^{T}\, \alpha\, \Phi\, M,$$

and $\Gamma = \varphi\, \alpha\, \Phi\, M$.

4) If the pair (Ω, Γ) is detectable, we can find Z such that the observer is asymptotically stable, then we can compute $T = (R\, \Delta\, E^{T} + Z\, \varphi)\, \alpha$.

5) The matrix N can be obtained from (5.e)

$$N = \begin{bmatrix} TE \\ C_{12} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ I_{p_1} \end{bmatrix} \tag{25}$$

6) Matrices H, L_1 and L_2 can be obtained from (5.c), (5.d) and (18).

4. Numerical example

The following example illustrates the above design method. Consider the singular system (1) described by [19]

$$E^* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A^* = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, F^* = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, G^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and $C^* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

In this case the transformation matrix is $P = I_4$. We obtain the r.s.e singular system (4) described by

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, F = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ and } y = \begin{bmatrix} -u_1 \\ y^* \end{bmatrix}$$

Choose $R = [\,0 \quad 1 \quad 0 \quad 0\,]$, then

$$M = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \Omega = 0 \text{ and } \Gamma = \begin{bmatrix} \gamma \\ 0 \\ -1/3 \\ -1/3 \\ -\gamma \\ 1/3 \end{bmatrix} \text{ where } \gamma \in R$$

For $\Pi = -1$, we can choose $Z = [\,0 \quad 0 \quad 3 \quad 3 \quad 0 \quad 3\,]$, then

$$T = [\,0 \quad 1 \quad 1\,], N = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ -1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, L_1 = 0, L_2 = [\,1 \quad -1 \quad 0\,] \text{ and } H = [\,0 \quad 1\,].$$

Thus, the reduced-order observer is

$$\dot{z} = -z + y_1 - y_2 + u_2$$

$$\hat{x} = \begin{bmatrix} y_2 \\ z + y_1 - y_3 \\ -y_1 + y_3 \\ y_1 \end{bmatrix}$$

5. Conclusion

We have presented a systematic design method for a reduced-order observer for linear singular systems with unknown inputs. This method is based on the constrained generalized

Sylvester equation. The existence conditions of the observer are given and generalize those currently used for unknown inputs observer problem of singular systems.

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