Granular Gas in Weightlessness: The Limit Case of Very Low Densities of Non Interacting Spheres

Experiments on non interacting balls in a vibrated box are reported. In a first experiment with an electromagnetic vibrator on earth or in board of Airbus A300 of CNES, the 1-ball dynamics exhibit little transverse motion and an intermittent quasi periodic motion along the direction parallel to the vibration. This behaviour proves a significant reduction of the phase space dimension of this billiard-like system from 11-d to 3-d or 1-d. It is caused by dissipation, which generates non ergodic dynamics. This experiment exemplifies the coupling between translation and rotation degrees of freedom during the collisions with the walls, due to solid friction at contacts. This eliminates ball rotation and freezes transverse velocity fluctuations. This trend is confirmed by 3d simulations with JJ Moreau discrete element code. A two-ball experiment performed under zero-g conditions in the Maxis 5 flight confirms the trend; the quasi-periodicity is found much greater; which is probably due to an improvement of experimental conditions. The two balls are not in perfect synchronisation showing the effect of small random noise; but the particles has never collided. This is then the normal dynamics of a gas of non-interacting dilute spherical grains in a vibrated container.

INTRODUCTION

A tremendous number of works has been devoted to the study of granular gas within these last 15 years, see [1] for a review. But a series of basic questions remain to be asked; among them one can ask what is the true role of the boundaries? does the vibrated container play the role of a thermostat or of a velostat [2], i.e. does it imposes a speed rather than a kinetic energy? Is the limit case of non interacting particles the same as the one of interacting particles, as one should expects from an analogy with normal gas, from fluid theory. In order to study the limit of non interacting particles, one is forced to diminish the density of the system, till the particles never meet. An other way is to reduce the dimension of the cell and to work with a very little number of particles. This is what is reported here. In this case one expects to recover the problem of chaotic billiard [3]. In 1d with no dissipation and in 0g, it becomes the Fermi problem [4-6]; or it is the case of a bouncing ball in 1d under gravity and no top wall [7-9]. We find in this paper (i) that the behaviour of such a gas confined in a closed box can be extremely non ergodic, (ii) that its dynamics is dominated by the coupling with the boundaries and (iii) that it is controlled by the effect of solid friction which couples translation and rotation during collisions. The paper reports first the case of a series of 1-ball experiment in the Airbus A-300-zero-g of CNES, then it reports the case of the dynamics of a 2-ball experiment in Maxis 5 rocket.

THE CASE OF A SINGLE BALL IN A CONTAINER WITH A VIBRATING PISTON

At first, this experiment has been performed in order to calibrate a gauge, to measure accurately the speed of the particles from a series of impacts and the restitution coefficient $v_{out}/v_{in} = -\varepsilon$ as a function of these speeds. It turns out that its main result is not this one, which has been achieved however, but it demonstrates also that rotations generate strong dissipation during collision that affects the mechanics of granular gas.

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Experimental technique

As shown in Fig. 1, a cylindrical cell length $L=10\text{mm}$, diameter $2R=13\text{ mm}$, is fix in the lab frame; it contains a sphere (radius $r=1.0\text{mm}$); its bottom is a vibrating piston ($z=b \cos\omega(t)$, $30\text{Hz} < f=\omega/(2\pi) < 120\text{Hz}$, $0.3\text{mm} < b < 0.7\text{mm}$ with $3g < \Gamma = b\omega^2 < 40g$, where g is the Earth's gravitation acceleration) driven by a computer-controlled electromagnetic vibrator (from Ling Dynamics Systems, Model V450) and it is closed on top by the ICP force sensor (PCB Piezotronics, Model 200B02) to be calibrated. A triaxial accelerometer (PCB Piezotronics, Model M356A08) is fixed to the piston, that measures $b\omega^2/g$ and the frequency $\omega/(2\pi)$ of the movement. Its signal is perturbed by the impact of the ball with the piston, so that it allows measuring these times of impact t_{2n} and the position z_{2n} of the piston. Data are acquired on 12bits at 2MS/s for the sensor. The impact sensor is fixed in the lab frame; this limits the perturbation due to inertial effect. It allows then determining the times t_{2n+1} of impacts and the amplitude $I_{2n+1}=I_n$ of impacts. Examples of these signal are given in right part of Fig. 1. One can then compute v_{n+} and v_{n-} before and after each impact n , and determine $\epsilon_n = -v_{n+}/v_{n-}$ as a function of v_{n-} , for the gauge and for the piston. A typical example is given in Fig. 2 in the case of the sensor. Similar results are obtained with the piston.

The noise observed on (Fig. 2.b) can be due to some uncertainty on the measurement of the impact times, as demonstrated by some simulations reported in [8]. However, similar dispersion can be also generated by some random fluctuation of the mean position and of the mean amplitude of the piston, or of g-jitter in microgravity condition. The lack of sphericity of the ball, the roughness of its surface and the roughness of the piston can also generate some dispersion by introducing some momentums during collisions. Similar normal dispersion curves have been obtained from 1d simulation when adding some Gaussian noise δt_1 and δt_2 on the measured time ($\delta t_1=10^{-4}\text{ s}$ for the piston and $\delta t_2=10^{-5}\text{ s}$ for the sensor which are approximately the reso-

Fig. 1: The experimental set-up: a sensor is placed on top of a cylindrical fix cell ($H=10\text{mm}$, $D=13\text{mm}$); this cell contains a stainless steel sphere ($d=2.0 \pm 0.002\text{mm}$); it is closed on top by a force sensor and on bottom by a vibrating piston on which an accelerometer is fixed. This allows measuring the impact times of the ball with the piston t_{2n} and with the lid t_{2n+1} . Right: A typical signal.

lution from the technical data-sheet of the manufacturers). One observes an intermittent periodic motion. Indeed a resonance shall occur when b/L is large enough, i.e. $b/L > \alpha_{\text{thres}}$ as can be obtained from 1d simulations [7], or from stability analysis of the periodic condition [8]. It can correspond to a roundtrip every n periods. Here we have observed only the case $n=1$.

Effect of collisions on rotation and transverse motion:

But the main result is not there: one observes also the freezing of the transverse motion of the ball as soon as the ball hits the lid. Indeed, this can not be demonstrated from the electronic signals of the sensors, but has to be observed optically either with the eyes or with a video camera: performing the same experiment on earth at small enough amplitude, or without the lid, one sees the ball jumping from left to right erratically; but as soon as the ball starts hitting the wall the vertical trajectory is stabilised and it evolves no more transversally (one still observes a very slowly lateral jittering, i.e. with a random transverse speed of $v_x < 1\text{mm/s}$ for 100Hz excitation frequency about). This demonstrates the freezing of the transverse degrees of freedom. We will show now that it demonstrates also the freezing of the rotation degrees of freedom. The stabilisation of the vertical trajectory comes from a coupling between transverse motion

Figure 2 Top: Variation of the restitution coefficient ϵ with the ball speed, for a 2mm diameter stainless steel sphere ($d=2.0 \pm 0.002\text{mm}$) hitting the sensor.

Bottom: Statistical distributions of the input velocity (a) of the ball with the sensor and of the restitution coefficient ϵ (b). The dotted curve corresponds to the Gaussian profile of same halfwidth at half-height. On top of (a) and (b). Similar normal dispersion curves on ϵ have been obtained from 1d simulation when adding some Gaussian noise δt_1 and δt_2 on the measured time ($\delta t_1=10^{-4}\text{ s}$ for the piston and $\delta t_2=10^{-5}\text{ s}$ for the sensor [8]).

and rotation during the collisions [8]. This coupling is due to the solid friction which exists between the surface and the ball and that acts on the ball to tend to impose non sliding condition at the contact point during collision. Let us for instance consider the case without a lid, sketched on the right part of Fig. 3. A transverse motion will impose some rotation after the first bounce; but the next bounce will preserve the same ratio of lateral speed and rotation, so that the ball can keep on moving in the same direction. With a lid things are different: any transverse component of the ball speed will force the ball to rotate in one direction, say spin +, at one wall due to the collision rules, and in the other one, say spin -, at the opposite wall. This improves much the dissipation; hence it blocks both rotation and transverse motion.

Figure 3: Sketch of collisions with rotation with bottom and top walls (left), with only top wall (right).

Figure 4: (a) Resonance rate (in %); (b) Relative piston position; and (c) Relative cavity length, as a function of frequency, where $L_{\text{max}} = L - d + A_p$, $A_p = b$ is the amplitude of vibration. In part b and c, the up and down trajectories of the ball are hardly distinguished at this scale.

Implication for the correct numerical modelling of granular gas

This effect can be generalised to ball-ball collisions in 2d or 3d: due to solid friction, and due to the fact that collision are not frontal, collision rules fix the ball spins at the end of each collision from the impact parameter at the beginning of the collision (the impact parameter describes the orientation of the relative translation speed of the balls and the orientation of the contact surface). Next collisions occur from other directions with different impact parameters so that an important part of spinning energy is lost during each collision. This effect is stronger in 3d than in 2d because the probability of frontal collision decreases with increasing the space dimension; and it does not exist in 1d. Hence one understands why dissipation looks quite large in 3d experiment on granular gas, much larger than in a 1d case. For instance, assuming a random distribution of kinetic energy, one expects that collision freezes d-2 degrees of translation-and-rotation over the 2d-1 degrees of freedom at the beginning. Hence one expects approximately a 40% energy-dissipation rate/collision.

It is rather strange that no simulation on granular gas or granular fluids has ever demonstrated and studied this effect, although this is essential to model correctly the physics of this system. In practice, until now (2004), most published simulations have just imposed some collision rules, nearly at random from paper to paper; the rules seem to be chosen in order to get a good fit with experimental data, but without coherence in between the papers; very few papers only introduce rotations, some introduce a tangent restitution coefficient instead. Some others introduce a single speed-dependent normal restitution coefficient, just to fit the experimental results; an example of such a law is given in Fig. 2, which is far to agree with the measured experimental points; however it has been used to explained our own data [9] from MiniTexus 5 results. So, simulations on granular gas simply do not take correctly care of this phenomenon actually.

The present effect is then a direct result of space research. This experiment points out also the effect of boundary conditions: indeed, the behaviour obtained in normal 1g condition without a lid (and in 3d) is different from the micro-g condition that imposes to use a lid. The effect of the lid is then to freeze a lot of degrees of freedom since the dimension of the phase space of this experiment passes from 11-d, (which is the number of freedom degrees for a 3d sphere with time dependent excitation, i.e. 3 positions, 2 angles, 3 speeds, 2 rotation speeds and the time), to 3-d, which correspond to z , v_z and v_{piston} . Comparatively, the resonance, which is also observed in our experiments, reduces the phase-space dimension much less: indeed, the synchronization links together the ball position z to the time and to the piston speed v_{piston} ; it relates also v_z and the impact times t_n to the length L of the box and to the period $T = 1/f = 2\pi/\omega$ of the forcing; this forces the attractor to be 1d instead of 3d. It diminishes then the space dimension from 3 to 1, i.e. 2 units, while the lid diminishes from 11d to 3d, i.e. 8 units!! Reduction of dimensionality is classic in dissipative sys-

tems with dissipation; this explains in particular chaotic strange attractors; however, it turns out to be peculiarly efficient in the present case. As it is told a little after, 3d simulations have been performed also with a Discrete Element code from Moreau and confirmed the experimental finding when friction is taken into account.

Intermittency

It is worth noting that 1d modelling of the 1g bouncing ball problem has concluded to the stability of the periodic bouncing [12] whatever the periodic excitation and restitution coefficient. But it is not found in real experiment [13], for which random Maxwellian- like distribution of speed is obtained]. This is likely induced by the existence of the small fluctuations.

Nevertheless, intermittent resonance has been also observed experimentally with a lid, either in 1g or in mini-g conditions i.e. during parabolic flights in board of the Airbus A-300 of CNES, cf. Fig. 4. Intermittency is most likely due to the imperfections of the experimental set- up and/or to g-jitter. Fig. 4 reports the variations with the frequency of (a) the rate of resonance in Fig. 4.a, (b) of the mean piston position when collision occurs in Fig. 4.b and (c) the mean length of the cell at that time in Fig. 4.c, where $L_{\max} = L - d + z_{\text{piston}}$. This shows that the larger the frequency (hence the larger the ball speed), the larger the periodicity rate, the less the intermittency, the smaller $\langle z \rangle / b$ and the larger the piston speed $b \sin(\omega t + \phi)$ at impact.

Comparison with numerical simulations

Such an intermittency of the synchronous motion has been observed probably in 1d numerical simulations [7] using an event-driven technique, where one sees rapid variations of the width of the speed distribution with the ratio b/L . So the stability of the periodic motion has to be studied more carefully.

3d simulations have been performed with the code of JJ. Moreau, which uses a discrete element method and collision rules which take account of restitution coefficient and friction during collision; they have confirmed the experimental finding. In particular the stabilisation of the linear path is obtained when friction is introduced; furthermore some transient trajectories which are linear and quasi periodic and which are oriented perpendicular to the direction of vibration have also been found, for peculiar initial conditions; this reveals the existence of different cyclic trajectories with their own distinct basin of attraction, with a hierarchy of Lyapounov exponents.

In the same way, some experimental test of the stability of the regular experimental trajectory of the ball has been performed forcing some change of boundary: for instance, in order to tempt destabilizing the impact position or the trajectory, i) different shapes of the surface of the piston basin have been used, ii) the cap surface of the sensor has been tilted up to 10° from the perpendicular direction of the vibration, iii) a second ball of lower diameter has been added; iv) up to 5 balls of 1 mm diameter have been introduced in the cell at the same times. Nevertheless for all these situations, the cyclic orbit parallel to

the vibration direction has remained often stable, which confirms the loss of the ergodicity of this 3d experimental system.

THE CASE OF A 2- BALL SYSTEM IN A VIBRATING CONTAINER

In the micro-g experiment of Maxus 5, these experiments have been repeated in one of the cells; the frequency and the box size were smaller ($6\text{Hz} < f < 60\text{Hz}$, $L=5\text{mm}$, cross section $S=10\text{mm} \times 10\text{mm}$) and the whole box was moving. The cell was containing 2 spheres of diameter $d'=1\text{mm}$. Intermittency has never been observed, during the whole flight; and synchronisation with the period of vibration has been obtained all the time. This proves better resonance conditions, (or more stable conditions of resonance); this is perhaps due to improved technical characteristics, but it corresponds also to a relative increase of losses. Indeed, this experiment benefits from (i) a better micro-g environment and (ii) it uses a different vibrator, working with a crank and a connecting rod. Still in this case, one observes some discrepancy with a periodic perfectly synchronous motion: as shown in Fig. 5.a, the bottom right cell of the Maxus 5 experiment contains 2 balls which moves in phase and whose impacts with a sensor have been recorded, cf. Fig. 5.b. Indeed, these impacts are not strictly concomitant, that proves non per-

Figure 5: Maxus 5 experiment (top: 10 Hz, 3mm, bottom: 15 Hz, 0.3 mm): The bottom right cell of (a) contains 2 balls ($d=1\text{mm}$) which move in phase with the cell, and whose impacts with a sensor has been recorded (b). Vibration is induced by a crank and a rod connected to a motor.

fect periodicity. In classic billiard theory applied to statistical mechanics, the real shape of the cavity plays a significant role on the ergodicity/non-ergodicity of the problem. In particular, billiards having the shape of a sphere cap should not present stable orbits. It would be interesting to demonstrate that cavity with peculiar shape would result also in the destabilisation of the resonant trajectories, hence improving the ergodicity of the dynamics. In the same way, using non spherical particles, adding some important surface roughness or adding some hard convex fixed obstacles into the cavity should also improve the "quality of the ergodicity". This is why perhaps the existence of a few balls, instead of a single one, may change the qualitative nature of the dynamics and force the problem of granular gas to be rather ergodic. However, this remains to be demonstrated.

CONCLUSION

So, the dynamics of very loose granular gas has been investigated when particles do not anymore interact with one another. We have shown in this paper (i) that the behaviour of such a gas confined in a closed box can be extremely non ergodic, (ii) that its dynamics is dominated by the coupling with the boundaries and (iii) that it is controlled by the effect of solid friction which couples translation and rotation during collisions. This study shows also that the behaviour of a dissipative billiard is quite different from a non dissipative one.

It is remarkable that the coupling between rotation and translation during collisions is often neglected in many simulations. This makes the domain of *ab initio* numerical technique little predictive at the moment and limited to interpolation in between experimentally tested situations. It is rather strange that no numerical studies have tend to define the limit of the domains of applicability, because it does not require so much computer time. This means an urgent need of testing the codes and of changing the protocols.

An other remarkable experimental result of the 1-ball experiment is that the speed v_z of this ball is always larger than the piston speed $b\omega$. In our terminology, it means a "subsonic" kind of excitation. Furthermore, the ratio $v_z/(b\omega)$ is much larger in the case of resonance than within erratic motion. This is in agreement with 1d simulations and theoretical description [11]. This shows also that the "supersonic" excitation observed when a small amount of beads fills the cell is linked to the dissipative nature of the granular gas, which is induced by ball-ball collision.

As already recounted [11], granular gas exists only in quite dilute conditions, i.e. in the so-called Knudsen regime, in the limit of fixed L and $\rho \rightarrow 0$. But we see here that when it is too much dilute "the gas loses its ergodicity". This is indeed a new phase in the dynamical diagram. Does the transition between the random "ergodic" motion and the present non ergodic one occur with hysteresis. Indeed, this makes the **granular-gas system a very peculiar one and a toy model**. The effect of the lid which renders the dynamics non ergodic may play also an

important role in other problems, such as the one of pseudo Maxwell demon [12], since it may avoid the system to empty completely. We conclude that our understanding of the low density limit of non-interacting granular gas should be revisited, in the absence of gravity, in order to investigate i) the "low-energy" regime corresponding to small aspect ratio $V_{\text{piston}}/V_{\text{ball}} \approx A_p/(L-d)$, which can permit one to check the relevance of a threshold value $\pi b/L = (1+\epsilon)/(1-\epsilon)$ needed to observe some more irregular motion in a vibrated box.

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