

DYNAMIC BAYESIAN NETWORKS IN SYSTEM RELIABILITY ANALYSIS

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Abstract: Today industrial systems are characterized by a set of dependencies among the components and the environment of the system. To address these difficulties, this paper presents a method for modelling and analyzing the reliability of a complex system based on Dynamic Bayesian Networks (DBN). This method allows to take into account the influence of time or exogenous variables on the failure (degradation) modes of the system. The DBN graphical structure provides an easy way to specify the dependencies and, hence, to provide a compact representation of the model. In addition, the DBN formalism is associated to simulation tools that enable an efficient processing for the models. Copyright © 2006 IFAC

Keywords: Reliability evaluation, Dynamic Bayesian Network, Markov process, Hidden Markov Model, Markov Switching Model, Input-Output HMM.

1. INTRODUCTION

The reliability assessment of complex systems is usually realized thanks to combinatorial methods such as Fault Tree (FT) or Reliability Block Diagram (RBD). However, these methods are not very useful to model firstly, the influence of time and exogenous variables on the degradation processes and secondly, the interdependencies among the components. Consequently, practical applications to industrial systems remain limited in number (Labeau, *et al.*, 2000).

To make these methods more applicable, one promising solution consists in using Bayesian Networks (BN) to obtain some additional power both at the modelling and analysis levels (Bobbio, *et al.*, 2001). BN provide a framework for handling probabilistic events and they have proven to be a powerful formalism to express complex dependencies between random variables.

Recently, their popularity started to grow among system reliability analysts. Bobbio explains, for

example, how the Fault Tree (FT) can be achieved using BN. In the same way, (Torres-Toledano, *et al.*, 2003) present the potential benefits given by Dynamic Bayesian Networks (DBN) formalism to improve the efficiency of Reliability Block Diagram (RBD). (Boudali and Dugan, 2005) describe the relation between dynamic FT and DBN. (Weber and Jouffe, 2006) propose an Object Oriented Dynamic Bayesian Network model of system reliability formalized from a dual functioning/dis-functioning modelling approach.

In our previous work we have shown that DBN is an extension of Markov Chains (MC) (Weber and Jouffe, 2003), Hidden Markov Model (HMM) and Input Output Hidden Markov Model (IOHMM) (Weber *et al.*, 2004), which decreases the effect of combinatorial explosion by a more synthetic description of a complex system. For this reason, DBN can be considered as a dynamic reliability method (Labeau, *et al.*, 2000) capable of handling the component aging, the maintenance operations and the evolution of the environment.

This paper introduces a modelling approach based on DBN to analyse reliability of complex systems. Thus, this study shows how DBN can be used for modelling reliability systems. Following this introduction, section 2 outlines the major concepts, proprieties and tools of the stochastic modelling of reliability. To overcome some stochastic modelling difficulties, section 3 presents the discrete-time DBN reliability framework proposed. Finally, a conclusion including some perspectives is developed in section 4.

2. STOCHASTIC MODEL OF SYSTEM RELIABILITY

Representing system failure in a probabilistic way is attractive because it naturally accounts for the uncertainty and ambiguity. To make such representation, the process behaviour is considered as a random variable that takes its values from a finite state space corresponding to the possible process states.

2.1 Time-independent Markov process

Deterioration can be modelled by a discrete time-dependent stochastic process $\{X_k, k \geq 0\}$ where X_k is a random quantity for all $k \geq 0$. The reliability of systems is usually assumed to be a Markov process.

Markov process is a stochastic process evolving under the Markov property. Given a value of X_k , the values of X_{k+i} where $i > 0$ are independent of values of X_{k-j} where $j > 0$. In the case of finite or countable state space, Markov processes can be represented by a graphic called Markov Chain (Ansell, *et al.*, 1994). Markov chain models a sequence of random variables $\{X_k, k = 0, 1, 2, \dots\}$ for which the Markovian property is held. Let $\{s_1, \dots, s_M\}$ be a finite set of the possible mutually-exclusive states of each X_k . The probability distribution over these states is represented by the vector $p(X_k)$:

$$p(X_k) = [p(X_k = s_1), \dots, p(X_k = s_m), \dots, p(X_k = s_M)]$$

$$\text{with } \sum_{m=1}^M p(X_k = s_m) = 1 \quad (1)$$

In this section, only systems having constant failure rates are considered. Homogenous Markov chains are used for this case because of the stationarity of transition probabilities. In such model, transition probabilities are time invariant and depend only on values states:

$$p_{ij} = p(X_{k+1} = s_i | X_k = s_j)$$

$$= p(X_{k+2} = s_i | X_{k+1} = s_j) \quad (2)$$

In a homogeneous discrete-time MC, the transition matrix \mathbf{P}_{MC} between the states is defined from failure rate parameters. For instance let us consider a system with two states $\{1, 2\}$, then:

$$\mathbf{P}_{MC} = \begin{bmatrix} 1 - p_{12} & p_{12} \\ 0 & 1 \end{bmatrix} \quad (3)$$

Where $p_{12} \equiv \lambda \Delta t : \lambda$ presents a constant failure rate and Δt is the time interval. The probability p_{12} can be interpreted as the probability that the component fails after the time Δt .

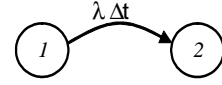


Fig. 1. Markov Chain for modelling deterioration model

Given an initial distribution over states $p(X_0)$, the probability distribution over states after k stage $p(X_k)$ is obtained from the Chapman-Kolmogorov equation:

$$p(X_k) = p(X_0) \prod_{i=0}^{k-1} \mathbf{P}_{MC} \quad (4)$$

Assuming that $i \in \{0 \dots l\}$ represents the functioning states, system reliability is defined as:

$$R(k) = \sum_{i \in \{0 \dots l\}} p(X_k = s_i) \quad (5)$$

The use of classic MC to model deterioration in systems needs to enumerate all possible states which lead sometimes to a huge transition matrix. Indeed, parameters quickly increase exponentially with respect to the number of reachable system states. Unfortunately, it is often infeasible to do this sort of exhaustive enumeration to determine the transition matrix.

2.2 Time-dependent Markov process

The basic assumption of discrete-time Markovian models is that the sojourn time in each state is geometrically distributed. Under this assumption, the failure rate of a component is considered as independent of time. Unfortunately, the geometric distribution is unsuitable for the majority of engineering problems because it does not reflect the fact that most component continuously deteriorates due to usage, wear or age.

To take it into account, the matrix defined in eq.3 becomes time-dependant and the MC is not homogeneous anymore.

$$\mathbf{P}_{MC}(X_k | X_{k-1}) = \begin{bmatrix} 1 - p_{12}(k) & p_{12}(k) \\ 0 & 1 \end{bmatrix} \quad (6)$$

One solution to model a time-dependent Markov process (e.g. Semi-Markov process (Limnios, 2001)) consists in an approximation of the initial process by an equivalent time-independent MC. This transformation is usually provided by approaches such as the supplementary variables (Cox, 1955a) or equivalent rate methods as phase type distribution approximation (also called state space expansion or method of stages). The method consists in increasing the number of system states to model the dynamics or degradations (Cox, D.R., 1955b). Discrete phase type distributions (DPH)

have been introduced and formalized by (Neuts, 1981). They are used in the numerical solution of non-markovian processes (Janssen *et al.*, 2001), or closely related to physical observations.

According to the structure of the transition matrix P_{MC} and the initial probability vector $p(X_0)$, many (discrete or continuous) distributions can be approximated by the absorbing-time distribution of the MC (Fig. 1). Nevertheless, the fitting problem for the DPH class has received very little attention and precautions must be taken in order to know the accuracy and the restrictions of the method (Bobbio *et al.*, 2004).

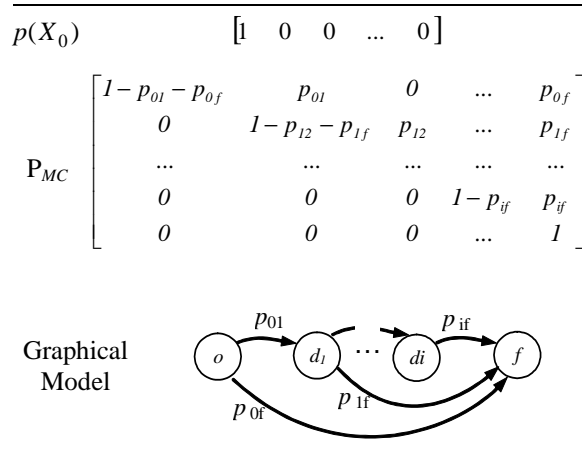


Fig. 2 - Canonical form CF3 (Bobbio *et al.*, 2004).

2.3 Non observable degradation model

In the previous sections stochastic processes X_k are supposed to be completely observable. In practice this is seldom the reality because the physical degradations of a component result in a change of its state which is observed only through a variation in the component functionality. Therefore a failure is an event impacting the component state which is modelled as a stochastic process X_k , nevertheless the only one phenomenon observed is the occurrence of failure modes in the component function which could be modelled as stochastic processes Y_k .

To model such processes with unobservable states or if the observation of this information is too expensive to be acquired, we propose to use a substitution variable X_k to characterize the deterioration level and the component states. The random variables X_k are assumed to be hidden stochastic process. In this case, model could be described as a Hidden Markov Model (HMM) where $p(Y_k|X_k)$ is the emission distribution. Usually used in speech recognition (Rabiner, 1989) or financial analysis, HMM is doubly stochastic process with an underlying stochastic process that is not observable (hidden). The process is observed through another set of stochastic processes that produces the sequence of observations. Therefore the general model of component degradation can then be formalized from HMM.

2.4 Modelling under exogenous events

Failure times are often considered as the only factor that can explain the reliability characteristics of the system. This may be insufficient (Singpurwalla, 1995). Operating and environmental conditions (e.g. humidity, temperature) may also influence the reliability of a system. All those factors are referred to as covariates or explanatory variables (Bagdonavicius, *et al.*, 2001). The reliability characteristics of a system can be then explained in a better way by considering the effects of covariates. To take into account exogenous events, several models represent each situation according to an environmental context, and models become complex when the system is composed by n components with several failure modes.

Markov Switching Models (MSM) are introduced to model this kind of stochastic process with exogenous events. These models are also viewed as conditional MC where the selection of transition distributions is dependent on exogenous variables. The MSM models the non-stationary due to abrupt changes in the functioning modes of the system (Bengio Y., 1999). A MSM represents the conditional distribution $p(X_k|U_k)$ given the input sequence $U_k = [u_0, u_1, \dots, u_k]$ where u_k represents the exogenous constraint with states $\{s_1^u, \dots, s_M^u\}$.

The simulation of MSM is based on switching and initialisation procedure of MC depending on the state of the exogenous variable. It is unsuitable to obtain the analytic solution of this kind of hybrid differential system.

Moreover if it is a distribution over the states of the exogenous variable U_k (input) which condition the solution, then we proposed the formalism of Input-Output HMM (IOHMM) to model the stochastic process. Let Y_k represent the observations (output), X_k the hidden variables and the external exogenous observable variables denoted as U_k which induces several behaviours of the hidden process X_k . Moreover, the result of the hidden process is observed through the failure modes of the component defined as outputs Y_k . To model this complex stochastic process $p(Y_k|U_k, X_k)$, the formalism of Input-Output HMM (IOHMM) is well suitable.

3. BAYESIAN NETWORK MODELLING

The MC, HMM, MSM or IOHMM are well suited models in the reliability analysis of a complex entity or system of low dimension. However, within the framework of general complex systems, the combinatorial explosion of states makes these methods unmanageable. To overcome this problem, a method based on DBN formalism is presented in the next section.

3.1 Bayesian Network notation

Bayesian networks (BN) (Pearl, 1988) are directed acyclic graphs (DAG) representing a joint

probability density function (Pdf). Nodes represent variables and directed arcs indicate causal relations between the connected nodes. According to d-connexion notion, the lack of arc between two nodes X^j and X^l (Fig. 3) means that the knowledge of X^l is irrelevant to estimate the state value of X^j when the state of X^i is known. Whereas the existence of arc between two variables indicates a direct probability dependency between them as X^i and X^j in Fig. 2. For more details see (Jensen, 1996).

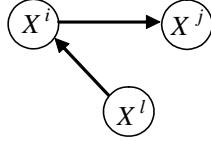


Fig. 3. Example of BN

Nodes which has not parent are described by a marginal distribution: unconditional probability. So they are characterized by a probability table (PT) composed of the class belonging probabilities $p(X^i)$ over the all possible states of X^i . Whereas, nodes having one parent at least (children nodes) are characterized by conditional distributions probabilities. So, Conditional Probabilities Tables (CPT) are associated with these type of nodes. PT and CPT together with the independence assumption defined by the graph, present a unique joint distribution over all the variables. Thus, it is possible to quantify probabilities for each variable conditioned on all the possible values the others variables in the graph.

3.2 Dynamic Bayesian Network

In this section, special case of BN named Dynamic Bayesian Networks (DBN) is proposed to model different types of time-dependent process.

DBN includes a temporal dimension managed by time-indexed random variables. The process is represented at time step k by a node X_k^i with a finite number of states $S_{X^i} : \{s_1^{X^i}, \dots, s_M^{X^i}\}$ and arcs represent dependencies across time points. The state space Ω is the cross product of the value spaces for individual states variables: $\Omega = \prod_{i=1}^N S_{X^i}$. Let $p(X_k^i)$ denotes the probability distribution over the variables states at time step k . Nodes correspond to the state variables can be partitioned into two sets: those corresponding to state of the variable at the current time step (k), and those corresponding to state system at the following time step ($k+1$). In this case, the variable is represented at successive times. In the DBN, shows in (Fig. 4), the temporal evolution is represented in two slices of time k and $k+1$, so network is called by (Boyen and Koller, 1998) a two-Time slices Bayesian Network (2-TBN). Defining the *transition-probabilities* between the states of the variable at time step k and time step

$k+1$ leads to define CPT relatively to inter-time slices. So, given any $p(X_k^i)$ at time step k , the network induces a unique distribution $p(X_{k+1}^i)$ at time $k+1$ which is calculated by inference. Starting at time $k=0$ with the distribution $p(X_0^i)$, computation of $p(X_T^i)$ can be realized by iterative inferences.

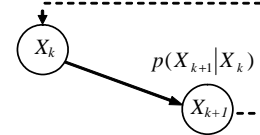


Fig. 4. A 2-TBN modelling MC with node X_k

A) *Markov Chain model*: The knowledge of the distribution probabilities and the CPT allows the calculation of the distribution probabilities at time $k+1$ by making inference. So, with this model, the future ($k+1$) is conditionally independent of the past given the present (k), which means that the CPT or $p(X_{k+1}|X_k)$ respects the Markov property (Kjaerulff, 1995). In the classical MC representation (§2.1), parameters are calculated for all the possibilities of states variables leading to the state space Ω , and the notion of independence between variables is unexploited. Contrary to this, DBN allows to take into account this consideration to represent joint Pdf in a factorized manner. In real world, the number of system states is very important, so a matrix representation is fastidious and usually unfeasible. The use of DBN modelling to analyse the deterioration system is more suitable to solve this problem, it permits to minimize considerably the number of parameters to stand for a joint Pdf (Weber and Jouffe, 2003).

B) *N-order Markov model*: For many stochastic processes, the first order MC is not sufficient to describe the deterioration model. Thus, usually we need to know more than one state in the past to predict the future. DBN offer an elegant way to model such hypothesis. The concept of 2-TBN can be generalized to higher order. Indeed, (N-1) order Markov property can be modelled by N-Time slices Bayesian Network (N-TBN) (Ben Salem, *et al.*, 2004). In this case, the past is resumed in the N last times slices. Distribution probabilities are obtained by inference using CPT containing the state evolution over N-time slices. The degree of the temporal regression is not bounded and it is possible to use any past instance information to determine probabilities distribution of the future.

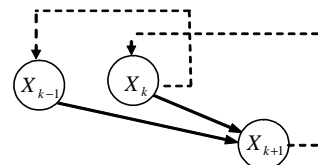


Fig. 5. A 3-TBN modelling second order MC.

Fig. 5 shows, how a second order MC can be modelled by a 3-TBN. Unfortunately, more the degree of the regression is important more complex

is the CPT of the node X_{k+1} . Nevertheless this model is interesting to take into account the dynamic comportment of the behaviour of the distribution of the probability of X_{k+1} . This model provide an alternative solution to the problem described in § 2.2.

C) semi-Markov model: with DBN the CPT models the dependence between the past and the future as the conditional probability $p(X_{k+1}|X_k)$. This CPT can be indexed by time then the CPT is not constant but involve according to k . In this way a 2-TBN is able to simulate a semi Markov process.

D) Non observable degradation model: with DBN it is also possible to model non observable system through emission distribution $p(Y_k | X_k)$ with Y_k the observed variable and X_k the hidden variable. In this case when all the temporal nodes are independent, an exact inference algorithm is still used to simulate the model. A 2-TBN model of HMM is presented as a compact DBN form in (Fig.6). This framework provide a graphical modelling solution to the problem explained in §2.3.

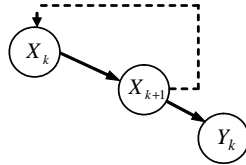


Fig. 6. A 2-TBN modelling MC with temporal node X_k and exogenous observations U_k .

E) Exogenous event model: in DBN non homogeneous stochastic model is able to be computed with taking into account the effect of the exogenous events on systems (like accidentally rupture of a component). The formalism to realize this are MSM and IOHMM as it is presented in §2.4. (Bengio and Frasconi, 1995) demonstrate how it is possible to model IOHMM with DBN. Weber *et al.* (2004), show an example of system reliability analysis formalised as MSM with DBN.

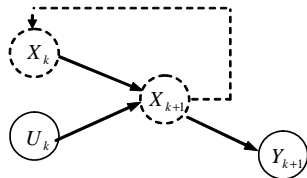


Fig. 7. A 2-TBN modelling IOHMM with temporal node X_k and exogenous observations U_k and Y_{k+1} .

Let's concenter the covariate as a (exogenous) discrete random variable U_k . This variable is modelled by a node containing information about exogenous events see (Fig. 7.). The node is realised in the current time slice k and can be introduced as (hard) evidence or probability distributions observations. So, the introduction of the context made by the means of U_k governs the behaviour of X_k . Observations at time k condition the state probabilities distributions at time $k+1$, because of

the CPT which are conditioned by the exogenous variables.

3.3 DBN to model failure dependencies

The reliability of component can be modelled as a DBN made of some nodes as presented in the §3.2. Thus independent components in the system are modelled using DBN equivalent to independent MC/HMM/IOHMM. The power of this method is that the DBN allows to merge the MC, HMM or IOHMM through a factorised form (Murphy, 2002). This aggregation is made in the time slice ($k+1$) with a method similar to the fault tree but without the limitation of independent elementary event and the variables can be described over n states (several failures or degradations), Fig. 8.

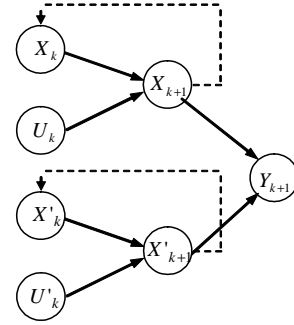


Fig. 8. DBN model of a system composed from tow independent stochastic processes.

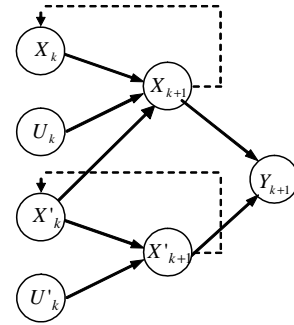


Fig. 9. DBN model of a system composed from dependent stochastic processes

One of the most difficult tasks in reliability analysis is to model a simultaneous failure processes. Two cases appear: interconnected failures processes or independent failure processes. Original solutions can be proposed based on DBN to model both cases. For the first case of system, the idea consist to develop N models of (number of interconnected failure process) DBN evolving at the same time and connected by node representing the global system state. Fig. 8 gives an example of model with two independent failure processes modelled by DBN. This formalism leads to model global system reliability in spite of the independence between some of their failure subsystems (Weber and Jouffe 2006). Algorithms as junction tree are used to make exact inference. The time increment is carried out by setting the computed marginal probabilities of the node at time step $k+1$ as observations for its corresponding node in the previous time slice. Nevertheless, if dependences exist in the stochastic

processes (Fig. 9.) then exact inference algorithms can not be employed because it leads to errors. Therefore approximate inference algorithms are proposed for instance the (Boyen and Koller, 1998) inference algorithm allows providing a bounded error or other approximate inference methods, as for example particle filtering (Koller and Lerner, 2000).

This approach represents a useful method to decrease the complexity of the model by grouping interconnected subsystems in a global deterioration model. So, through this method, it is possible to analyse in one time both of the entire system and its subsystems. For example, instead of including distinct technical components with dependencies, this structure could be represented as one entity. Under this method, the BN leads to model independent stochastic process as IOHMM then the global model of the system is simplified. If the dynamics of process are independent, then the exact inference method based on junction tree compute the simulation of the system reliability.

CONCLUSION AND FURTHER WORK

The proposed method, based on the Dynamic Bayesian Networks theory, easily allows designing DBN structures for the modelling of temporal evolution of complex systems. The correspondence between Markov Chains and DBN is presented and applied to the system reliability estimation.

The proposed method is a good solution to model the reliability of complex stochastic process. Indeed, the number of states needed to model a complex system with MC increases exponentially. As the DBNs representation is based on the modelling of process entities, the obtained model is more compact and readable than MC.

In future works, in order to achieve this modelling technique we have to define how the learning algorithms of BN can contribute to model the dynamics of the system reliability and how the parameters behaviour can be then modelled.

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