SINR evaluation of OFDM-CDMA systems with constant timing offset: asymptotic analysis

Youssef Nasser, Mathieu Des Noes, Laurent Ros, Geneviève Jourdain

SINR Estimation of OFDM-CDMA Systems with Constant Timing Offset : a Large System Analysis

Youssef Nasser*, Mathieu des Noes*, Laurent Ros, Geneviève Jourdain****
* CEA-LETI, 17 Avenue des Martyrs, 38054 Grenoble, France
** Laboratoire des Images et des Signaux, INPG, Grenoble, France
*youssef.Nasser@cea.fr / Mathieu.desnoes@cea.fr **laurent.ros@lis.inpg.fr / genevieve.jourdain@lis.inpg.fr

***Abstract*— This article presents the impact of a constant timing error on the performance of a downlink 2 dimensional spreading OFDM-CDMA system. This impact is measured by the Signal to Interference plus Noise Ratio (SINR) degradation after equalization and despreading. Using random matrix theory, an asymptotic evaluation of the SINR is obtained. It is independent of the value of users' spreading code while taking into account their orthogonality. Simulation results are provided to evaluate and discuss the validity of this model.**

Keywords- MC-CDMA, MC-DS-CDMA, OFDM, MMSE, Asymptotic performance, SINR.

I. INTRODUCTION

This article presents the degradation introduced by a constant timing error on the performance of a downlink 2 dimensional spreading OFDM-CDMA system [1]. By timing error we mean a constant offset of the FFT window with respect to the perfect synchronization reference. This phenomenon creates inter-carrier and inter symbol interferences (ICI and ISI) which degrade the Signal to Interference plus Noise Ratio (SINR) at the output of the equalizer.

This paper extends to an OFDM-CDMA context the work of Steendam and Moeneclay which was carried out for a pure OFDM system [3]. Moreover, exploiting some results from the random matrix theory [7], an asymptotic analytical expression of the SINR for the single user MMSE receiver is derived. This SINR formula is independent from the actual values of the spreading codes while taking into account their orthogonality. To confirm the validity of the proposed model, the theoretical SINR is compared to the SINR measured via Monte Carlo simulations. Since the inter symbol interferences are essentially non Gaussian, the SINR cannot be used to evaluate the Bit Error Rate (BER) at the output of the equalizer. Hence, BER measurements are not presented in this article.

The paper is organized as follows: section 2 describes the system model for OFDM-CDMA system with constant timing offset and gives a vector-matrix representation of the received signal, section 3 gives an asymptotic expression of the estimated SINR using some properties of random matrix theory. Eventually, section 4 gives a comparison between the asymptotic SINR model and Monte Carlo simulation results.

II. SYSTEM MODEL

A. System Description

In this section, a combination of OFDM-CDMA system is described. Figure 1 shows a 2D spreading OFDM-CDMA transmitter in a downlink communication with N_u users.

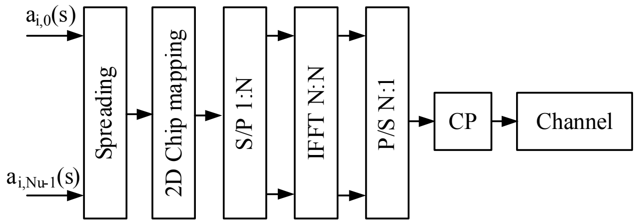

Figure 1-OFDM-CDMA transmitter.

Each symbol is first spread by a pseudo-random sequence of N_c chips. Then the resulting samples are allocated on the time/frequency grid as shown with the arrows of Figure 2.

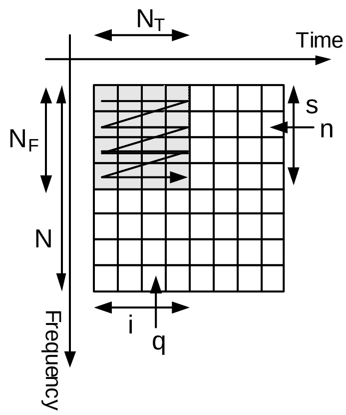

Figure 2: 2D spreading scheme.

The first N_T samples are allocated in the time direction. The next blocks of N_T chips are allocated the same way on adjacent sub-carriers. Each bin with coordinate (n,q) represents the signal transmitted on the n^{th} sub-carrier of the q^{th} OFDM symbol. The spreading factor is thus $N_c = N_F \times N_T$ where N_F is the frequency domain spreading and N_T is the time domain spreading. With a FFT size equal to N, we transmit $S=N/N_F$ data symbols $a_{i,m}$ of the m^{th} user on the i^{th} 2D OFDM-CDMA block. There is a total of S sub bands per

OFDM symbol. Particularly, if N_T=1, the system is equivalent to the MC-CDMA [4] and for N_F=1, the system is equivalent to the MC-DS-CDMA [5]. Eventually, the signal at the IFFT input is :

$$
\begin{aligned}
&b_{i,q}(sN_F+n) = \sum_{m=0}^{N_u-1} \sqrt{P_m}\, a_{i,m}[s] C_m[nN_T+q] \\
&q = 0,\dots,N_T-1; s = 0,\dots,S-1, n = 0,\dots,N_F-1
\end{aligned}
\quad \mathbf{(1)}
$$

s is the index referring to the sub-band of N_F adjacent sub-carriers used for the transmission of the symbol $a_{i,m}[s]$. P_m is the power of the m[th] user which is supposed constant in every sub-band and C_m represents his spreading sequence. At the output of the IFFT and after the cyclic prefix (CP) insertion (ν samples), the signal is written as follows :

$$
\begin{aligned}
&x_{i,q}(k) = \sum_{s=0}^{S-1}\sum_{n=0}^{N_F-1} b_{i,q}(sN_F+n) e^{j2\pi \frac{k(sN_F+n)}{N}} \\
&k = -\nu, \dots N-1
\end{aligned}
\quad \mathbf{(2)}
$$

The signal is then passed through a multipath channel with delay spread W and corrupted by a complex value Additive White Gaussian Noise (AWGN) $n_{j,q}(u)$ with variance σ^2. Notice that we use a discrete representation of the transmission with sampling-frequency *Fs* equal to the bandwidth of the real band pass transmit signal. Thus, the coefficients $g_{i,q}(k)$, of the discrete channel can be expressed as a function of the parameters of the analog channel :

$$
g_{i,q}(k) = \sum_{l=0}^{L-1} \alpha_{i,q}^{(l)} . \sin c\left\{ \pi Fs \left(\frac{k}{Fs} - \tau_{i,q}^{(l)}\right) \right\} \quad \mathbf{(3)}
$$

where $\alpha_{i,q}^{(l)}$ and $\tau_{i,q}^{(l)}$ are respectively the complex amplitudes and the delays of the l[th] paths, and sinc(x) = sin(x)/x.

At the receiver side, the guard interval is first removed. This operation assumes an implicit synchronization. In the sequel, we will assume a constant synchronization error of k_0 samples. As depicted in Figure 3 , when k_0 >0 the initial samples is selected within the cyclic prefix, while when k_0<0 it is in FFT part. Therefore, for ν > W and a constant synchronization error k_0, three cases have to be considered :

-if $-\nu + W < -k_0 \le 0$, there is no ISI and ICI.

-if $-\nu \le -k_0 \le -\nu + W$, there is an interference caused by the preceding OFDM symbol.

-if $k_0 < 0$, there is an interference caused by the succeeding OFDM symbol and also ICI.

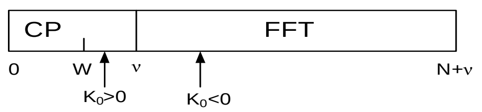

Figure 3- Synchronization error.

After the guard interval suppression, the receiver selects N consecutive samples and transposes them into the frequency domain thanks to the FFT. This is illustrated in Figure 4.

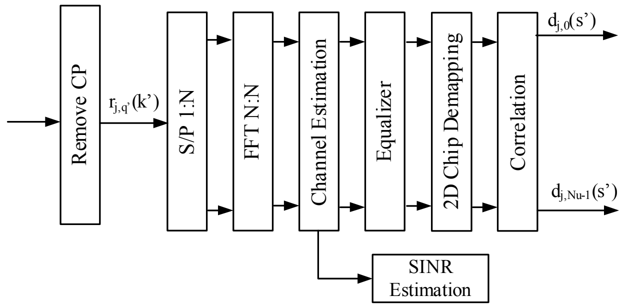

Figure 4-OFDM-CDMA receiver .

The signal at the input of the FFT can be written as follows :

$$
\begin{aligned}
&r_{j,q}(u) = \sum_{i=j-1}^{j+1}\sum_{s=0}^{S-1}\sum_{n=0}^{N_F-1}\sum_{q=0}^{N_T-1}\left[b_{i,q}(sN_F+n) e^{-j2\pi\frac{(sN_F+n)}{N}\beta} e^{j2\pi\frac{(sN_F+n)}{N}u} h_{i,q}(u,s,n) \right] + n_{j,q}(u) \\
&with\ \ \beta = k_0 + (i-j)N_T(N+v) + (q-q')(N+v)\ ; u = 0,..N-1 \\
&and\ \ h_{i,q}(u,s,n) = \sum_{k=-(N-1)+u-\beta}^{v+u-\beta} g_{i,q}(k) e^{-j2\pi\frac{(sN_F+n)}{N}k}
\end{aligned}
\quad \mathbf{(4)}
$$

Without loss of generality, we assume that one is interested in the symbols of user 0. Let us define U, $\widetilde{a}$ and Q as the matrix containing the interfering sequences, the vector of the interfering symbols and their corresponding power diagonal matrix :

$$
\begin{aligned}
&U = \left(C_1, C_2, \dots\dots, C_{N_U-1}\right)_{N_C \times (N_U-1)} \\
&\widetilde{a}_j[s] = \left(a_{j,1}(s), a_{j,2}(s), \dots, a_{j,N_U-1}(s)\right)^T \\
&Q = \begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & P_{N_U-1} \end{pmatrix}_{(N_U-1)\times(N_U-1)}
\end{aligned}
\quad \mathbf{(5)}
$$

After equalization of each carrier by the coefficient $z_{j,q'}$ and despreading by code sequence C_0, the j[th] estimated received symbol on the s' [th] sub band can be written as follows (see appendix) :

$$
\begin{aligned}
&d_{j,0}(s') = I_0 + I_1 + I_2 + I_3 + I_4 + I_5 \quad with \\
&I_0 = \sqrt{P_0} C_0^H Z_j(s') B_{j,j}(s',s') C_0 a_{j,0}(s') \\
&I_1 = \sqrt{P_0} \sum_{\substack{s=0 \\ s\neq s'}}^{S-1} C_0^H Z_j(s') B_{j,j}(s',s) C_0 a_{j,0}(s) \\
&I_2 = \sum_{s=0}^{S-1} C_0^H Z_j(s') B_{j,j}(s',s) U \sqrt{Q}\, \widetilde{a}_j(s) \\
&I_3 = \sqrt{P_0} \sum_{\substack{i=j-1 \\ i\neq j}}^{j+1} \sum_{s=0}^{S-1} C_0^H Z_j(s') B_{i,j}(s',s) C_0 a_{i,0}(s) \\
&I_4 = \sum_{\substack{i=j-1 \\ i\neq j}}^{j+1} \sum_{s=0}^{S-1} C_0^H Z_j(s') B_{i,j}(s',s) U \sqrt{Q}\, \widetilde{a}_i(s) \\
&I_5 = C_0^H Z_j(s') N_j(s')
\end{aligned}
\quad \mathbf{(6)}
$$

I_0 represents the useful signal, I_1 the interference generated by code C_0 in other sub-bands of the same j[th] block (Inter Band Interference IBI), I_2 the Multiple Access Interference (MAI) of the same j[th] block, I_3 interference generated by code C_0 in blocks j-1 or j+1 (depending on k_0- Inter Symbol Interference

ISI), I_4 the MAI generated by blocks j-1 or j+1 (ISI-MAI), and I_5 the noise. $B_{i,j}(s',s)$ is a $N_c \times N_c$ matrix which components represent the frequency channel coefficients depending on the timing offset and $Z_j(s')$ is a $N_c \times N_c$ diagonal matrix which equalizes the diagonal elements of $B_{j,j}(s',s')$ (see appendix).

B. SINR evaluation

The data symbols are assumed independent and identically distributed and having a zero mean and unit variance. The SINR for every sub band is deduced from **(6)** by calculating the expectation values of I_0, I_1, I_2, I_3, I_4 and I_5. It is given by

$$SINR = \frac{E|I_0|^2}{E|I_1|^2 + E|I_2|^2 + E|I_3|^2 + E|I_4|^2 + E|I_5|^2} \quad \textbf{(7)}$$

The expectations' values in **(7)** are given by

$$\begin{aligned}
E|I_0|^2 &= P_0 \left| C_0^H Z_j(s') B_{j,j}(s',s') C_0 \right|^2 \\
E|I_1|^2 &= P_0 \sum_{\substack{s=0 \\ s \neq s'}}^{S-1} \left| C_0^H Z_j(s') B_{j,j}(s',s) C_0 \right|^2 \\
E|I_2|^2 &= \sum_{s=0}^{S-1} C_0^H Z_j(s') B_{j,j}(s',s) U Q U^H B_{j,j}(s',s)^H Z_j(s')^H C_0 \\
E|I_3|^2 &= P_0 \sum_{\substack{i=j-1 \\ i \neq j}}^{j+1} \sum_{s=0}^{S-1} \left| C_0^H Z_j(s') B_{i,j}(s',s) C_0 \right|^2 \\
E|I_4|^2 &= \sum_{\substack{i=j-1 \\ i \neq j}}^{j+1} \sum_{s=0}^{S-1} C_0^H Z_j(s') B_{j,j}(s',s) U Q U^H B_{j,j}(s',s)^H Z_j(s')^H C_0 \\
E|I_5|^2 &= \frac{\sigma^2}{N_C} tr\left(Z_j(s') Z_j(s')^H \right)
\end{aligned} \quad \textbf{(8)}$$

These expressions show that the SINR depends on a complex way of the spreading codes i.e. **(8)** can not be used practically. In the sequel, we propose a new SINR estimation based on the results of the free probability theory.

III. Asymptotic Performance

Random matrix theory has been initially applied to CDMA systems analysis by Tse and Hanly [6], and Verdu and Shamai [11]. They studied the asymptotic performance of the multi-user MMSE receiver for a CDMA system, with random spreading and synchronous reception. They found that the dependence of the SINR on the spreading codes was vanishing in the asymptotic regime (N_c and $N_u \to \infty$ and $\alpha = N_u / N_c$). The performances only depend on the system load α, noise variance and the power distribution. This work was then extended to a multipath fading channel in [8].

Unfortunately, the model with random spreading is not accurate for the downlink of actual CDMA or OFDM-CDMA systems, since it does not take into account the orthogonality between codes.

To solve this issue [7] assumes that the spreading codes are extracted from a Haar distributed unitary matrix. Such a matrix is isometric and random. This assumption allows to apply very powerful results from the free probability theory. In [7] and [9], it was found that the dependence of the SINR on the spreading codes was also vanishing in the asymptotic regime, but this time the orthogonality between codes was accounted for. Eventually, the Haar distributed assumption is only technical. The simulation results obtained in [2][7][9] with Walsh-Hadamard spreading sequences match very well with the theoretical model.

In order to evaluate the different terms of **(8)**, we will apply a first property already used in [8] and a second which exploits the Haar distributed assumption [9].

Property 1: If A is a $N_c \times N_c$ uniformly bounded deterministic matrix and $C_m = \frac{1}{\sqrt{N_c}}(c_m(0),\ldots, c_m(N_c - 1))$ where $c_m(l)$'s are iid complex random variables with zero mean, unit variance and finite eighth order moment, then :

$$C_m^H A C_m \xrightarrow[N_c \to \infty]{} \frac{1}{N_c} tr(A) \quad \textbf{(9)}$$

In this study, C_0 is obtained by the multiplication of a Walsh-Hadamard sequence with a scrambling code of N_c chips. Hence, the assumptions needed for **(9)** are easily verified. This property is used to evaluate $E|I_0|^2$, $E|I_1|^2$ and $E|I_3|^2$.

Property 2: Let C be a Haar distributed unitary matrix of size $N_c \times N_u$ [7]. $C = (C_0, U)$ can be decomposed into a vector C_0 of size N_c and another matrix U of size $N_c \times (N_u - 1)$ given by **(5)**. Given these assumptions, [9] shows that :

$$UQU^H \xrightarrow[N_c \to \infty]{} \alpha \overline{P} \left(I - C_0 C_0^H \right) \quad \textbf{(10)}$$

$\alpha = N_u / N_c$ is the system load and $\overline{P} = \frac{1}{N_u - 1} \sum_{m=1}^{N_u - 1} P_m$ is the average power of the interfering users. This property is used to evaluate $E|I_2|^2$ and $E|I_4|^2$. Applying **(9)** and **(10)** to the computations of the different terms in **(8)**, we obtain :

$$\begin{aligned}
E|I_0|^2 &= P_0 \left| \frac{1}{N_c} tr\left(Z_j(s') B_{j,j}(s',s') \right) \right|^2 \\
E|I_1|^2 &= P_0 \sum_{\substack{s=0 \\ s \neq s'}}^{S-1} \left| \frac{1}{N_c} tr\left(Z_j(s') B_{j,j}(s',s) \right) \right|^2 \\
E|I_2|^2 &= \alpha \overline{P} \sum_{s=0}^{S-1} \left(\frac{1}{N_c} tr\left(Z_j(s') B_{j,j}(s',s) B_{j,j}(s',s)^H Z_j(s')^H \right) - \left| \frac{1}{N_c} tr\left(Z_j(s') B_{j,j}(s',s) \right) \right|^2 \right) \\
E|I_3|^2 &= P_0 \sum_{\substack{i=j-1 \\ i \neq j}}^{j+1} \sum_{s=0}^{S-1} \left| \frac{1}{N_c} tr\left(Z_j(s') B_{i,j}(s',s) \right) \right|^2 \\
E|I_4|^2 &= \alpha \overline{P} \sum_{\substack{i=j-1 \\ i \neq j}}^{j+1} \sum_{s=0}^{S-1} \left(\frac{1}{N_c} tr\left(Z_j(s') B_{i,j}(s',s) B_{i,j}(s',s)^H Z_j(s')^H \right) - \left| \frac{1}{N_c} tr\left(Z_j(s') B_{i,j}(s',s) \right) \right|^2 \right) \\
E|I_5|^2 &= \frac{\sigma^2}{N_c} tr\left(Z_j(s') Z_j(s')^H \right)
\end{aligned} \quad \textbf{(11)}$$

From these calculations, one observes that the SINR is independent of the spreading codes **(11)**. It only needs the estimation of the channel, the equalizer coefficients, the noise variance, the power allocated to each spreading code and the system load.

IV. SIMULATION RESULTS

In order to validate the asymptotic SINR model given by **(11)**, we compare its results with Monte Carlo (MC) simulations. The simulations assumptions are the following :

- FFT size: N=64, Guard Interval: ν=25 samples, Spreading factor: N_c= 32 chips, Number of users: N_u= 32 (α=1, Full load),
- Equalizer type: MMSE, Constellation: QPSK,
- OFDM-CDMA configuration : MC-CDMA (N_F = 32, N_T = 1), MC-DS-CDMA (N_F = 1, N_T = 32), OFDM-CDMA: (N_F = 8, N_T = 4).

A. Gaussian Channel

It can be easily checked that for a Gaussian channel the different components of (11) are independent of the (N_F , N_T) configuration. All the SINRs evaluated with the asymptotic model are thus equal. Figure 5 gives a comparison between the SINR measured by Monte Carlo simulations and the asymptotic model for the three OFDM-CDMA configurations. It shows that our model is consistent with simulation results even for relatively small spreading factors (N_c=32). Also, one deduces that a late synchronization ($k_0 < 0$) will introduce a large amount of interferences.

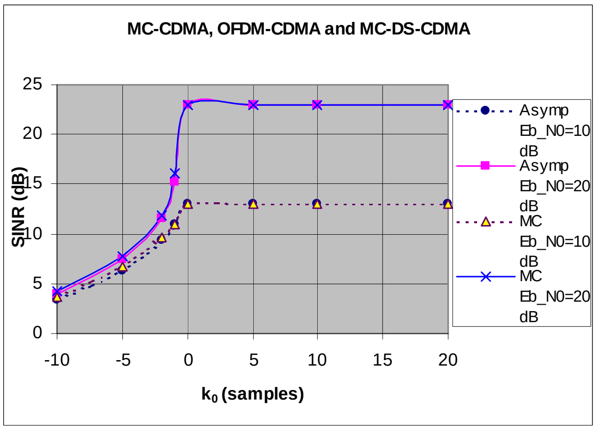

Figure 5 - Comparison between asymptotic model and simulations (Gaussian channel).

Figure 6 shows the sensitivity of the asymptotic model to the overall spreading factor for a gaussian channel.

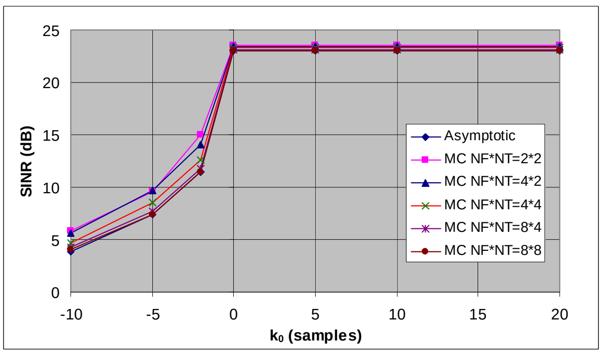

Figure 6-Influence of the (N_F,N_T) configuration for an AWGN channel.

We observe that for N_c=N_FxN_T < 32 the large system assumption is no longer valid. There is a discrepancy between the model and the simulations results.

B. Bran A channel

In this section , we give some results for a Bran A channel model [10]. It consists of W = 13 samples, obtained from L= 18 Rayleigh uncorrelated paths. Due to the variations of the channel during transmission we compare the mean SINRs, measured by simulations with the asymptotic model. We suppose that both gains and time delays of channel paths remain static during a symbol but change from an OFDM-CDMA symbol to another. The mean Energy per bit E_b of an OFDM-CDMA symbol is thus a random variable. The simulations performances are parameterized by a mean ratio $\overline{E}_b / N_0$, evaluated by $\frac{\overline{E}_b}{N_0} = \frac{\lambda}{2}.\frac{\sum_{l=0}^{L-1} E(|\alpha^{(l)}|^2)}{\sigma^2}.P_0$ where $\lambda = (N+\nu)N_T / S$. Figure 7, Figure 8 and Figure 9 present comparisons between simulation and the asymptotic model for MC-CDMA, OFDM-CDMA and MC-DS-CDMA configurations. They show that the results obtained with the asymptotic model match very well with Monte Carlo simulation. Also, we observe that for a late synchronization ($k_0 < 0$), performances rapidly degrade due to ISI. However, when the FFT window is positioned too early ($k_0 > 0$), a degradation appears at k_0 = 15 ≈ ν-W. This degradation increases with k_0 due to the ISI caused by the preceding OFDM-CDMA symbol.

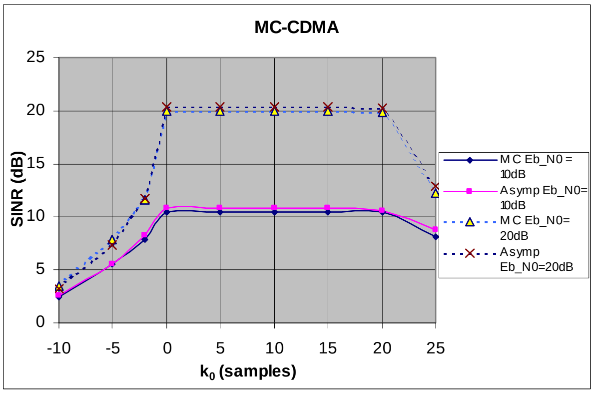

Figure 7- Comparison between asymptotic and Simulation (MC-CDMA).

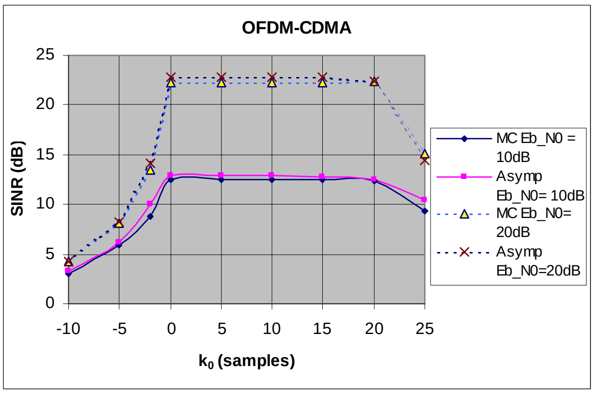

Figure 8- Comparison between asymptotic and simulation (OFDM-CDMA).

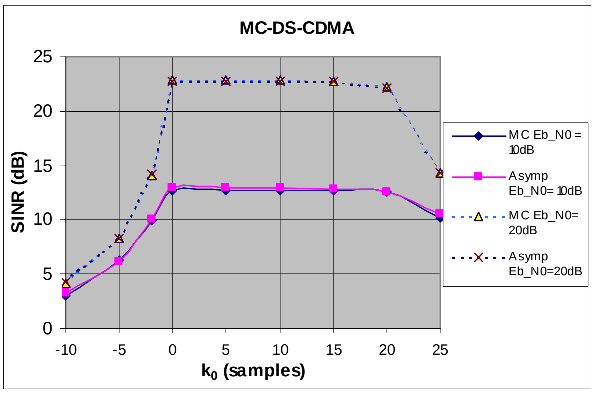

Figure 9- Comparison between asymptotic and simulation (MC-DS-CDMA).

We also observe that the average SINR is larger for MC-DS-CDMA and OFDM-CDMA configurations than for MC-CDMA. One shall not conclude that MC-DS-CDMA and OFDM-CDMA schemes offer better performances for a practical system. A fair comparison between different spreading configurations must take into account the coding rate [1].

V. CONCLUSION

In this paper, the effect of a constant timing offset at the receiving side is investigated. It is characterized by the SINR estimation when the spreading factor and the number of users are large while the system load $\alpha = \frac{N_u}{N_c}$ remains constant. Using results from the free probability theory, we obtained a SINR estimation independent of the actual spreading codes. Simulation results show the accuracy of the theoretical model for MC-CDMA, MC-DS-CDMA and 2 dimensional spreading OFDM-CDMA schemes.

APPENDIX

At the input of the receiver, after removing the guard interval, the remaining samples can be written as in **(4)**.

After equalizing each carrier frequency in each symbol by the channel coefficient $z_{j,q'}$ and correlating the equalized signal by the corresponding code sequence C_0, the j^{th} estimated received symbol on the $s'^{\,th}$ sub band can be written as

$$d_{j,0}(s') = \frac{1}{N}\sum_{n'=0}^{N_F-1}\sum_{q=0}^{N_T-1} z_{j,q}(s'N_F+n')C_0^*(n'N_T+q)\sum_{u=0}^{N-1} r_{j,q}(u)e^{-j2\pi\frac{(s'N_F+n')}{N}u}$$

$$= \sum_{m=0}^{N_U-1}\sqrt{P_m}\sum_{n'=0}^{N_F-1}\sum_{q=0}^{N_T-1} z_{j,q}(s'N_F+n')C_0^*(n'N_T+q)\sum_{i=j-1}^{j+1}\sum_{s=0}^{S-1} a_{i,m}(s)\sum_{n=0}^{N_F-1}\sum_{q=0}^{N_T-1} C_m(nN_T+q)\phi_{i,j}(..)+$$

$$\sum_{n'=0}^{N_F-1}\sum_{q=0}^{N_T-1} z_{j,q}(s'N_F+n')C_0^*(n'N_T+q)N_{j,q}(s'N_F+n')$$

where

$$\phi_{i,j}(s',s,n',n,q',q) = e^{-j2\pi\frac{(sN_F+n)}{N}\beta}\frac{1}{N}\sum_{u=0}^{N-1} h(u,s,n)e^{-j2\pi\left[\frac{(s'-s)N_F+(n'-n)}{N}\right]u}$$

Let us define the following channel coefficients matrices :

$$A_{i,j}[s',s,n',n] = \begin{pmatrix} \phi_{i,j}(s',s,n',n,0,0) & \phi_{i,j}(s',s,n',n,0,1) & \cdots & \phi_{i,j}(s',s,n',n,0,N_T-1) \\ \phi_{i,j}(s',s,n',n,1,0) & \cdots & \cdots & \phi_{i,j}(s',s,n',n,1,N_T-1) \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{i,j}(s',s,n',n,N_T-1,0) & \phi_{i,j}(s',s,n',n,N_T-1,1) & \cdots & \phi_{i,j}(s',s,n',n,N_T-1,N_T-1) \end{pmatrix}$$

$$B_{i,j}[s',s] = \begin{pmatrix} A_{i,j}[s',s,0,0] & A_{i,j}[s',s,0,1] & \cdots & A_{i,j}[s',s,0,N_F-1] \\ A_{i,j}[s',s,1,0] & \cdots & \cdots & A_{i,j}[s',s,0,N_F-1] \\ \vdots & \vdots & \vdots & \vdots \\ A_{i,j}[s',s,N_F-1,n] & A_{i,j}[s',s,N_F-1,1] & \cdots & A_{i,j}[s',s,N_F-1,N_F-1] \end{pmatrix}$$

The equalization process is modeled as a $N_c x N_c$ diagonal matrix. It compensates the diagonal elements of $B_{j,j}[s',s']$

$$Z_j[s'] = \begin{pmatrix} V_j[s',0,0] & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & V_j[s',N_F-1,N_F-1] \end{pmatrix}$$

$$V_j[s',n',n]_{q',q} = \begin{cases} z_{j,q'}(s'N_F+n') = \dfrac{\phi_{j,j}(s',s',n',n',q',q')^*}{\left|\phi_{j,j}(s',s',n',n',q',q')\right|^2 + \dfrac{\sigma^2}{\alpha\overline{P}}} & if \quad q=q' \quad for \quad q,q'=0,..N_T-1 \\ 0 & elsewhere \end{cases}$$

Combining these matrices with those already defined in **(5)**, we obtain equation **(6)**.

REFERENCES

[1] N. Maeda, Y. Kishiyama, H. Atarashi and M. Sawahashi, “Variable Spreading Factor OFCDM with Two Dimensional Spreading that Prioritizes Time domain Spreading for Forward Link Broadband Wireless Access”, Proc. of VTC spring 2003, April 2003, Jeju, Korea

[2] P. Jallon, M. des Noes, D. Ktenas, J.-M. Brossier, “Asymptotic analysis of the multiuser MMSE receiver for the downlink of a MC-CDMA system”, Proc. of VTC spring 2003, April 2003, Jeju, Korea

[3] H. Steendam and M. Moeneclay, “Optimisation of OFDM on Frequency-selective Time selective Fading Channels”, Signals, Systems, and Electronics, 1998. ISSSE 98. 1998 URSI International Symposium on, pp. 398-403, Sept-Oct. 1998.

[4] N.Yee, J.P. Linnartz and G. Fetteweis, "Multi-Carrier CDMA in Indoors wireless radio networks", proc. of PIMRC' 93, Yokohama Japan, pp:109-113

[5] A. Chouly, A. Brajal and S. Jourdan, "Orthogonal multicarrier techniques applied to direct sequence spread spectrum CDMA systems", Global Telecommunications Conference, 1993, GLOBECOM '93.,IEEE Vol. 3 pp:1723 - 1728 Nov.-Dec. 1993

[6] D.N.C. Tse and S.V. Hanly, "Linear multiuser receivers: effective interference, effective bandwidth and user capacity", IEEE Trans. on Information Theory, March 1999

[7] M. Debbah, W. Hachem, P. Loubaton, and M. de Courville, “MMSE analysis of Certain Large Isometric Random Precoded Systems”, IEEE Trans. on Information theory, May 2003.

[8] J. Evans and D.N.C. Tse, "Large system performance of linear multiuser receivers in multipath fading channels", IEEE Trans. on Information Theory, Sept. 2000

[9] J.M. Chaufray, W. Hachem and Ph. Loubaton, “Asymptotical Analysis of Optimum and Sub-Optimum CDMA Downlink MMSE Receivers”, IEEE Trans. on Information Theory, Nov. 2004.

[10] J. Medbo, H. Andersson, P. Schramm and H. Asplund, “ Channel models for HIPERLAN/2 in different indoor scenarios“, COST 259, TD98, Bradford, April 1998.

[11] S. Verdu and S. Shamai., “Spectral Efficiency of CDMA with Random Spreading”, IEEE Trans. on Information Theory, March 1999.