Convergence of utility indifference prices to superreplication price

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Abstract

A discrete-time financial market model is considered with a sequence of investors whose preferences are described by concave strictly increasing functions defined on the positive axis. Under suitable conditions we show that, whenever their absolute risk-aversion tends to infinity, the respective utility indifference prices of a given bounded contingent claim converge to the superreplication price.

Keywords: derivative pricing, utility indifference price, superreplication, utility maximization.

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1 Introduction

The utility indifference price (also called Hodges-Neuberger price or reservation price) for the seller of a contingent claim has been introduced by Hodges and Neuberger (1989). It is the minimal amount a seller should add to his or her initial wealth so as to reach an expected utility when delivering the claim which is greater than or equal to the one he or she would have obtained trading in the basic assets only. This kind of price has been studied, among others, by El Karoui and Rouge (2000), Bouchard (2000), Collin-Dufresne and Hugonnier (2004) and Delbaen *et al.* (2002) in various contexts.

The last mentioned article concentrated on the case of exponential utility with risk-aversion parameter $\alpha > 0$, *i.e.* where the investor's preferences are given by the utility function $U_\alpha(x) = -e^{-\alpha x}$, $x \in \mathbb{R}$. It has been proved that the utility indifference price of a (sufficiently integrable) contingent claim converges to its superreplication price as $\alpha \to \infty$. The superreplication price is the minimal initial wealth needed for hedging the claim without risk; this is thus a utility-free pricing concept. A related result under proportional transaction costs is presented in Bouchard *et al.* (2001).

In the present paper a sequence of investors is considered. Preferences of investor n are expressed via the choice of his or her concave strictly increasing utility function U_n. We treat the case $\mathrm{dom}(U_n) = (0, \infty)$. It is shown in discrete-time market models that (under appropriate technical conditions) the convergence of utility indifference prices to the superreplication price takes place for bounded contingent claims when the absolute risk-aversion $-U_n''/U_n'$ of the respective agents tends to infinity. The convergence of the respective optimal strategies in this context was studied in Summer (2002).

In Carassus and Rásonyi (2005) the convergence of utility prices (reservation price and Davis price) was shown when U_n tend to some limiting utility function U_∞. We remark here that the superreplication price can be considered as the utility indifference price for the function

$$U_\infty(y) := -\infty,\ y < x, \quad U_\infty(y) := 0,\ y \geq x,$$

where x is the agent's initial capital, see section 4 for details. So we will use the ideas of that paper, though those techniques are not directly applicable since they are based on smoothness of U_∞.

In section 2 we set up our model and give formal definitions of the concepts involved. Section 3 sums up a few facts about utility maximization which we will need in the sequel, based on Rásonyi and Stettner (2005b). Section 4 proves the main result.

2 Definitions, assumptions and results

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0\leq t\leq T}, P)$ be a discrete-time filtered probability space with time horizon $T \in \mathbb{N}$. We assume that $\mathcal{F}_0$ coincides with the family of P-zero sets. Let $\{S_t,\ 0 \leq t \leq T\}$ be a d-dimensional adapted process representing the discounted (by some numéraire) price of d securities in a given economy. The notation $\Delta S_t := S_t - S_{t-1}$ will often be used. Denote by $D_t(\omega)$ the smallest affine hyperplane containing the support of the (regular) conditional distribution of ΔS_t with respect to $\mathcal{F}_{t-1}$, see Proposition 8.1 of Rásonyi and Stettner (2005a) for more information about the random set D_t.

Trading strategies are given by d-dimensional processes $\{\phi_t,\ 1 \leq t \leq T\}$ which are supposed to be predictable (*i.e.* ϕ_t is $\mathcal{F}_{t-1}$-measurable). The class of all such strategies is denoted by Φ. Denote by L^∞, L^∞_+ the sets of bounded, nonnegative bounded random variables, respectively, equipped with the supremum norm $\|\cdot\|_\infty$. Trading is assumed to be self-financing, so the value process of a portfolio $\phi \in \Phi$ is

$$V_t^{z,\phi} := z + \sum_{j=1}^{t} \langle \phi_t, \Delta S_j \rangle,$$

where z is the initial capital of the agent in consideration and $\langle \cdot, \cdot \rangle$ denotes scalar product in $\mathbb{R}^d$.

The following absence of arbitrage condition is standard:

$$(NA): \ \forall \phi \in \Phi \ (V_T^{0,\phi} \geq 0 \text{ a.s.} \Rightarrow V_T^{0,\phi} = 0 \text{ a.s.}).$$

However, we need to assume a certain strengthening of the above concept hence an alternative characterization is provided in the Proposition below. Let Ξ_t denote the set of $\mathcal{F}_t$-measurable d-dimensional random variables,

$$\tilde{\Xi}_t := \{\xi \in \Xi_t : \ \xi \in D_{t+1} \text{ a.s.}, \ |\xi| = 1 \text{ on } \{D_{t+1} \neq \{0\}\}\}.$$

Proposition 2.1 *(NA) holds iff there exist $\mathcal{F}_t$-measurable random variables $\beta_t, \ 0 \leq t \leq T-1$ such that*

$$\text{(1)} \qquad \operatorname*{ess.\,inf}_{\xi \in \tilde{\Xi}_t} P(\langle \xi, \Delta S_{t+1} \rangle < -\beta_t | \mathcal{F}_t) > 0 \text{ a.s. on } \{D_{t+1} \neq \{0\}\}.$$

Proof. The direction $(NA) \Rightarrow (1)$ is Proposition 3.3 of Rásonyi and Stettner (2005a). The other direction is clear from the implication $(g) \Rightarrow (a)$ in Theorem 3 of Jacod and Shiryaev (1998). □

The following condition is called "uniform no-arbitrage" and was introduced by Schäl (2000).

Assumption 2.2 There exists a constant $\beta > 0$ such that for $0 \leq t \leq T-1$

$$\operatorname*{ess.\,inf}_{\xi \in \tilde{\Xi}_t} P(\langle \xi, \Delta S_{t+1} \rangle < -\beta | \mathcal{F}_t) > 0 \text{ a.s. on } \{D_{t+1} \neq \{0\}\}.$$

Let $G \in L_+^\infty$ be a random variable which will be interpreted as the payoff of some derivative security at time T. Now the concept of superreplication price is formally introduced as the minimal initial wealth needed for hedging without risk the given contingent claim:

$$\pi(G) := \inf\{z \in \mathbb{R} : \ V_T^{z,\phi} \geq G \text{ for some } \phi \in \Phi\}.$$

We refer to Karatzas and Cvitanić (1993), El Karoui and Quenez (1995), Kramkov (1996) and Föllmer and Kabanov (1998) for more information about this notion.

We go on incorporating a sequence of agents in our model with concave utility functions U_n. The functions r_n below express the absolute risk-aversion of the respective agents.

Assumption 2.3 Suppose that $U_n : \mathbb{R}_+ \to \mathbb{R},\ n \in \mathbb{N}$ is a sequence of concave strictly increasing twice continuously differentiable functions such that

$$\forall x \in (0,\infty) \quad r_n(x) := -\frac{U_n''(x)}{U_n'(x)} \to \infty,\ n \to \infty.$$

Example 2.4 Typical examples are the sequences $U_n(x) = -e^{-\gamma_n x},\ x > 0$ where $0 < \gamma_n$ and $\gamma_n \to \infty$ or the utility functions with derivatives $U_n'(x) = e^{-\gamma_n x^2},\ x > 0$ where $0 < \gamma_n$ and $\gamma_n \to \infty$.

Define for each $x \geq \pi(G)$:

$$\mathcal{A}(G,x) := \{\phi \in \Phi :\ V_T^{x,\phi} \geq G \text{ a.s.}\}.$$

In this case the set $\mathcal{A}(G,x)$ admits an alternative characterization, see Proposition 2.7 below.

Define the supremum of expected utility at the terminal date when delivering claim G, starting from initial wealth $x \in (0,\infty)$:

$$u_n(G,x) := \sup_{\phi \in \mathcal{A}(G,x)} EU_n(V_T^{x,\phi} - G), \tag{2}$$

where we assumed that the expectations exist (it will be shown that under the hypotheses of our main result this is indeed the case).

Definition 2.5 The utility indifference price $p_n(G,x)$ is defined as

$$p_n(G,x) = \inf\{z \in \mathbb{R} : u_n(G,x+z) \geq u_n(0,x)\}.$$

We wish to find conditions on S and U_n which guarantee that $p_n(G,x)$ tends to $\pi(G)$ whenever Assumption 2.3 holds. Our main result is the following Theorem, see also Remark 4.4 for possible generalizations.

Theorem 2.6 *Suppose that $x \in (0,\infty)$, S is bounded, Assumptions 2.2 and 2.3 hold. Then the utility indifference prices $p_n(G,x)$ are well-defined and converge to $\pi(G)$ as $n \to \infty$.*

Before closing this section, an alternative characterization of the superreplication price and $\mathcal{A}(G,x)$ is provided. Take any $G \in L^\infty_+$. Define

$$\begin{aligned}
\pi_T(G) &:= G, \\
\pi_t(G) &= \operatorname{ess.inf}\{X : \sigma(X) \subset \mathcal{F}_t, \exists \phi \in \Xi_t \text{ such that} \\
&\qquad X + \langle \phi, \Delta S_{t+1}\rangle \geq \pi_{t+1}(G) \text{ a.s.}\},
\end{aligned}$$

for $0 \leq t \leq T-1$. Note that $\pi_0(G)$ can be chosen constant, by the triviality of $\mathcal{F}_0$.

Proposition 2.7 *We have*

$$\pi_0(G) = \pi(G).$$

Furthermore, $\mathcal{A}(G,x)$ can be characterized as

$$\{\phi \in \Phi : \; V_t^{x,\phi} \geq \pi_t(G) \text{ a.s.}, \; 0 \leq t \leq T\}. \tag{3}$$

Proof. Take x and ϕ such that $V_T^{x,\phi} \geq G \, a.s.$ Let us prove by induction that for all t, $V_t^{x,\phi} \geq \pi_t(G) \, a.s.$ This holds true trivially for $t = T$. Assume it is true for $t+1$, then

$$V_t^{x,\phi} + \langle \phi_{t+1}, \Delta S_{t+1}\rangle \geq \pi_{t+1}(G) \quad a.s,$$

and thus $V_t^{x,\phi} \geq \pi_t(G) \, a.s.$ This proves (3) (the other inclusion being trivial). Applying the preceding argument for $t = 0$ and taking the infimum, we get $\pi(G) \geq \pi_0(G)$ by definition of $\pi(G)$. In order to show the other inequality, fix $\varepsilon > 0$. There exist $X_{t-1}, \; \{\phi_t, \; 1 \leq t \leq T\}$ such that X_{t-1} is $\mathcal{F}_{t-1}$-measurable, $\phi_t \in \Xi_{t-1}$ and

$$\begin{aligned}
X_{t-1} + \langle \phi_t, \Delta S_t\rangle &\geq \pi_t(G) \text{ a.s.} \\
\pi_{t-1}(G) + \varepsilon/T &> X_{t-1} \text{ a.s.}
\end{aligned}$$

which implies that

$$\pi_{t-1}(G) + \varepsilon/T + \langle \phi_t, \Delta S_t\rangle \geq \pi_t(G) \text{ a.s.}$$

Summing over all $t = 1, \ldots, T$,

$$\pi_0(G) + \varepsilon + \sum_{t=1}^{T} \langle \phi_t, \Delta S_t\rangle \geq G \text{ a.s.}$$

follows and therefore

$$\pi_0(G) + \varepsilon \geq \pi(G),$$

so letting $\varepsilon \to 0$ proves the first statement of the Proposition. $\square$

3 Utility maximization

We evoke a few facts about utility maximization, based on the paper Rásonyi and Stettner (2005b). Fix a concave nondecrasing function $U : (0, \infty) \to \mathbb{R}$. Define

$$\Xi_t^x := \{\xi \in \Xi_t : x + \langle \xi, \Delta S_{t+1} \rangle \geq 0 \text{ a.s.}\}.$$

Theorem 3.1 *Let $S_t \in L^\infty$, $0 \leq t \leq T$ and supppose that Assumption 2.2 holds. Then the functions U_t below are well-defined for all $x \geq 0$,*

$$U_T(x) := U(x), \quad U_t(x) = \operatorname{ess.}\sup_{\xi \in \Xi_t^x} E(U_{t+1}(x + \langle \xi, \Delta S_t \rangle)|\mathcal{F}_t),$$

and there exist (finite-valued) random variables J_t such that

$$U_t(x) \leq J_t x, \ x \in (0, \infty), \ 0 \leq t \leq T.$$

Consequently, $u(G, x)$ is well-defined and finite for all $G \in L^\infty$ and $x > \pi(G)$. Furthermore, there exists $\phi^*(x) = \phi^*(G, x) \in \mathcal{A}(G, x)$ such that

$$u(G, x) = EU(V_T^{x, \phi^*(x)} - G).$$

In fact, $\phi^*(x)$ can be constructed in such a manner that it satisfies

$$\phi_t^*(x) \in D_t, \text{ a.s.}, \ 1 \leq t \leq T.$$

Proof. The estimations for U_t can be found in the proof of Proposition 13 of Rásonyi and Stettner (2005b); Theorem 3 and Theorem 1 of the same paper implies the rest. $\square$

Lemma 3.2 *Take any strategy $\phi \in \mathcal{A}(G, x)$ satisfying $\phi_t \in D_t$, $1 \leq t \leq T - 1$. There exist increasing functions $M_t(x) \geq 0$ such that*

$$V_t^{x, \phi} \leq M_t(x).$$

Proof. For $t = 0$ take $M_0(x) := x$. Suppose that the statement has been shown up to $t - 1$. We claim that

$$|\phi_t| \le \frac{V_{t-1}^{x,\phi}}{\beta}. \tag{4}$$

Indeed, define

$$A := \left\{ |\phi_t| > \frac{V_{t-1}^{x,\phi}}{\beta} \right\} \in \mathcal{F}_{t-1}, \;\; B := \left\{ \langle \frac{\phi_t}{|\phi_t|}, \Delta S_t \rangle < -\beta \right\}.$$

Clearly, $\{V_t^{x,\phi} < 0\} \supset A \cap B$ and

$$P(A \cap B) = E[E[I_{A\cap B}|\mathcal{F}_{t-1}]] = E[I_A[E(I_B|\mathcal{F}_{t-1})]].$$

By Assumption 2.2, $P(B|\mathcal{F}_{t-1}) > 0$, thus $P(A) > 0$ would contradict $\phi \in \mathcal{A}(G, x)$ (note that $\pi_t(G) \ge 0$ and see Proposition 2.7), we get that (4) holds.

Thus by the induction hypothesis

$$V_t^{x,\phi} \le M_{t-1}(x) + \|\Delta S_t\|_\infty M_{t-1}(x)/\beta =: M_t(x),$$

which defines a suitable $M_t(x)$. □

4 Proof of the main result

Denote by L^0 the set of all real-valued random variables on $(\Omega, \mathcal{F}, P)$ equipped with the topology of convergence in probability. The notation L^0_+ stands for the set of nonnegative random variables. Define for $z \in \mathbb{R}$

$$K_z := \{V_T^{z,\phi} : \; \phi \in \Phi\}.$$

We recall the following fundamental fact, see Kabanov and Stricker (2001) or Schachermayer (1992) for a proof.

Theorem 4.1 *Under (NA), the set $K_z - L^0_+$ is closed in probability.* □

Lemma 4.2 *Let $B \in L^0$ such that $B \notin K_z - L^0_+$. Then there exists $\varepsilon > 0$ such that*

$$\inf_{\theta \in K_z} P(\theta \le B - \varepsilon) \ge \varepsilon.$$

Proof. Suppose that the statement is false. Then for all n there is $\theta_n \in K_z$ such that

$$P(\theta_n \leq B - 1/n) \leq 1/n,$$

hence for $\kappa_n := [\theta_n - (B - 1/n)] I_{\{\theta_n > B - 1/n\}} \in L^0_+$:

$$P(\theta_n - \kappa_n = B - 1/n) \geq 1 - 1/n.$$

This implies $\theta_n - \kappa_n \to B$ in probability, hence $B \in \overline{K_z - L^0_+} = K_z - L^0_+$, a contradiction. □

Lemma 4.3 *Suppose that $U_n,\ n \in \mathbb{N}$ satisfy Assumption 2.3 as well as*

$$\forall n \in \mathbb{N} \quad U_n(x) = 0, \quad U_n'(x) = 1,$$

for some fixed $x \in (0, \infty)$. Then

$$\forall y < x \quad U_n(y) \to -\infty,\ n \to \infty, \quad \forall y \geq x \quad U_n(y) \to 0,\ n \to \infty.$$

Proof. First take $y < x$. As U_n' is decreasing, $U_n'(u) \geq U_n'(x) = 1$, for $u \leq x$, hence $r_n(u) \leq -U_n''(u)$. Necessarily

$$U_n'(y) = U_n'(x) - \int_y^x U_n''(u) du \geq 1 + \int_y^x r_n(u) du \to \infty,$$

as $n \to \infty$, by the Fatou-lemma. Also

$$U_n(y) = U_n(x) - \int_y^x U_n'(u) du \to -\infty,$$

by the same reasoning, using the previous convergence observation.

Now for any $y \geq x$ we claim that $U_n'(y) \to 0$. If this were not the case, along a subsequence n_k, for all k

$$U_{n_k}'(y) \geq \alpha > 0.$$

Then by monotonicity $U_{n_k}'(u) \geq \alpha$, for all $u \leq y$, so $r_n(u) \to \infty$ implies that $-U_{n_k}''(u) \to \infty,\ k \to \infty,\ u \leq y$. Then necessarily

$$0 \leq U_{n_k}'(y) = U_{n_k}'(x) + \int_x^y U_{n_k}''(u) du = 1 + \int_x^y U_{n_k}''(u) du \to -\infty,$$

a contradiction proving the second assertion of this Lemma. □

Proof of Theorem 2.6. Fix $x > 0$. As we have already pointed out in Theorem 3.1, $u_n(G, x)$ is well-defined and finite. It is also easy to see that $\mathrm{ran}(u(G, \cdot)) = \mathrm{ran}(u(0, \cdot))$, so the $p_n(G, x)$ are well-defined (in the sense that the infimum is taken over a nonempty set).

Notice that Assumption 2.3 remains true if we replace each U_n by $\alpha_n U_n + \beta_n$ for some $\alpha_n > 0,\ \beta_n \in \mathbb{R}$. Also, the utility indifference prices defined by these new functions are the same as the ones defined by the original U_n. Hence by choosing $\alpha_n := 1/U_n'(x)$ and $\beta_n := -U_n(x)/U_n'(x)$, we may and will suppose that for all $n \in \mathbb{N}$

$$U_n(x) = 0, \quad U_n'(x) = 1. \tag{5}$$

Fix $\pi(G) < y < x + \pi(G)$. Then

$$x + G \notin K_y - L^0_+,$$

by the definition of the superreplication price. Take $0 < \varepsilon$ given by Lemma 4.2 appplied with $B := x + G$ and $z = y$. Notice that the function $M_T(x)$ figuring in Lemma 3.2 does not depend on the particular choice of the strategy ϕ and hence can be chosen uniformly for all $\phi^*_n(y)$, $n \in \mathbb{N}$, where $\phi^*_n(y)$ is the optimal strategy for the problem (2) with initial capital y (see Theorem 3.1). Define the sets

$$A_n := \{\omega \in \Omega : \ V_T^{y,\phi^*_n(y)}(\omega) \le x + G(\omega) - \varepsilon\}.$$

As for all $n,\ 1 \le t \le T-1$, $\phi^*_{n,t}(y) \in D_t$, Lemma 4.2 says that $P(A_n) \ge \varepsilon$. We get

$$\begin{aligned} u_n(G, y) &= EU_n(V_T^{y,\phi^*_n(y)} - G) \\ &\le EI_{A_n} U_n(x - \varepsilon) + EI_{A_n^C} U_n(M_T(y)) \\ &\le P(A_n) U_n(x - \varepsilon) + U_n(M_T(y) + x) P(A_n^C) \\ &\le \varepsilon U_n(x - \varepsilon) + U_n(M_T(y) + x) \to -\infty, \end{aligned} \tag{6}$$

by Lemma 4.3. For the last inequality we used the fact that $U_n(x-\varepsilon) \le U_n(x) = 0$ and that $U_n(z) \ge 0$ for all $z \ge x$.

We also see from (5) and the definition of $u_n(0,x)$ that

$$\liminf_{n\to\infty} u_n(0,x) \geq \liminf_{n\to\infty} U_n(x) = 0. \tag{7}$$

One may easily check that

$$p_n(G,x) \leq \pi(G). \tag{8}$$

Indeed, for any $\delta > 0$ we may take a strategy $\hat{\phi}(\delta) \in \mathcal{A}(G, \pi(G) + \delta)$ such that

$$V_T^{\pi(G)+\delta,\hat{\phi}(\delta)} \geq G.$$

Then, as U_n is non decreasing,

$$\begin{aligned} u_n(0,x) &\leq \sup_{\phi\in\mathcal{A}(0,x)} EU_n(V_T^{x+\pi(G)+\delta,\phi+\hat{\phi}(\delta)} - G) \\ &\leq \sup_{\phi\in\mathcal{A}(G,x+\pi(G)+\delta)} EU_n(V_T^{x+\pi(G)+\delta,\phi} - G) = u_n(G, x+\pi(G)+\delta), \end{aligned}$$

so by the definition of the utility indifference price $p_n(G,x) \leq \pi(G) + \delta$ and (8) follows by letting $\delta \to 0$.

Now it remains to prove

$$\liminf_{n\to\infty} p_n(G,x) \geq \pi(G). \tag{9}$$

Suppose that this fails, *i.e.* for some $x > \eta > 0$ and a subsequence n_k

$$p_{n_k}(G,x) \leq \pi(G) - \eta$$

holds, for all $k \in \mathbb{N}$. Again, by Definition 2.5,

$$u_{n_k}(G, x + \pi(G) - \eta) \geq u_{n_k}(0,x),$$

the left-hand side tends to $-\infty$ by (6) applied to $y = x + \pi(G) - \eta$ and the liminf of the right-hand side is nonnegative by (7), a contradiction proving (9) and hence the Theorem. □

Remark 4.4 It is possible to extend Theorem 2.6 to certain unbounded price processes and relax Assumption 2.2, too. Define $\mathcal{W}$ as the set of random variables with finite moments of all orders. Suppose $\Delta S_t \in \mathcal{W}$, $1/\beta_{t-1} \in \mathcal{W}$, $1 \leq t \leq T$

and Assumption 2.3. Then $p_n(G,x)$ tends to $\pi(G)$. Indeed, Theorem 3.1 follows again from Theorem 3 and Proposition 13 of Rásonyi and Stettner (2005b), and Lemma 3.2 can be shown with random variables $M_t(x) \in \mathcal{W}$ instead of constants, in the same way. Then the same argument works, just like in (6) we get

$$u_n(G,y) \leq \varepsilon U_n(x-\varepsilon) + EI_{A_n^C} U_n(M_T(y)+x).$$

Here

$$I_{A_n^C} U_n(x + M_T(y)) \leq I_{A_n^C}[U_n(x) + U_n'(x)(M_T(y))] \leq M_T(y),$$

and this is integrable (in fact, lies in $\mathcal{W}$), hence an application of the Fatou-lemma shows $u_n(G,y) \to -\infty$ for $\pi(G) < y < \pi(G) + x$. The rest of the proof is identical.

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