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On non-zero space average density perturbation effects in tokamak plasma reflectometer signals

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Abstract: The effects of the non-zero average density perturbation on phase and amplitude measured by reflectometry are presented. The non-zero average density perturbation on the phase variation can be seen as an index effect as soon as the shape of the density perturbation does not introduce spectral effects. Amplitude modulation in time follows generally the properties of the cut-off layer seen as a mirror but some specific situations produce a time modulation two times higher than the input time variation of the density perturbation as observed in Tore Supra. The introduction of secondary cut-off can exhibit this effect as shown in 2D simulations.

Introduction: A detailed explanation on the shapes of the phase and amplitude variations is given for different types of oscillating density perturbations with a zero or non-zero space average. The phase variations can be seen far away from the cut-off layer when the space average of the oscillating density fluctuation is non-zero. The previous studies [1] in reflectometry devoted to the connection between the phase fluctuation spectrum and the radial wave-number spectrum of the density fluctuation use a homogeneous turbulence model. These study do not consider the fact that the density perturbation can have a non-zero average value but some spectral effects have been shown when the density perturbation has a very small width [2]. Other numerical studies have demonstrated that is possible to obtain information on the evolution of the radial wave number spectrum without considering the non-zero average density perturbations, which introduces small radial wave number component. This can be easily understood by considering Gaussian density perturbation whose has a wavenumber spectrum centred on the zero radial wavenumber. But the spectrum width depends on the width of Gaussian density perturbation, if the width is larger than the Airy wavelength the phase perturbation looks like the initial perturbation [3]. When some Bragg backscattering arises, the phase response can be modify and can be expressed in term of $K_{1/4}$ Bessel function. To understand the non-zero density perturbation effect, as mentioned before some part of the
contribution comes from the Bragg backscattering and the other one is associated to an index effect or forward scattering as reported in [4]. Using WKB approximation can do the comparison of the index effects between zero and non-zero density perturbation. The amplitude of these phase variations is given as a function of the distance between the cut-off layer and the density fluctuation position. On Tore Supra tokamak a X-mode Fluctuation Reflectometer (TSFR) covering 105-155 GHz has been recently installed to measure the amplitude and phase of density fluctuation from low field to high magnetic field side depending of the plasma scenario. To better understand the measured TSFR signal simulations are needed especially in presence of magnetic islands in the probing zone. Comparison between the TSFR signals and simulations have shown that it is possible to find a shape density variation in the vicinity of the magnetic island, which reproduces the experimental results qualitatively. 2D modelling is required to stand for the amplitude variations seen by the TSFR. The role of the non-zero space average density fluctuation on the correlation reflectometry can be seen in terms of forward scattering due to the existence of small wave number component in the spectrum associated to this kind of the density perturbation.

**Expressions of phase variations for zero and non-zero average density perturbation.**

A very simple model based on the WKB approximation shows the dependencies of the phase variations induced by a constant density perturbation over a distance $2d_f$. The position of the density perturbation is defined by the difference between the cut-off layer position corresponding to $x=L$ and the current position of the centre of the density perturbation named $x=x_0$. The amplitude of the density perturbation is $\delta n$ and the shape is assumed to be symmetric as shown on figure 1.

![Figure 1: Schematic view of the modelling with the different parameters used.](image-url)
For simplicity a linear density profile is used in the case of the ordinary mode without changing the aim of the physical results. To obtain the expression of the phase variation as a function of the distance we have first integrated the wavenumber over the plasma with and without the perturbation. Then the difference between the perturbed and the unperturbed case is made to have an expression of the phase variation. As the WKB approximation imposes to avoid the secondary cut-off the amplitude of the density perturbation should be small. In this case it is possible to use series expansion in term of \( \delta n \) at the first order and square root identities to have an expression of the phase variation as a function of \( L-x_0=d \). For the non-zero density perturbation, this expression is:

\[
\delta \varphi = -2k_o d_f \sqrt{L} \frac{\delta n}{\sqrt{L-x_0-d_f} + \sqrt{L-x_0+d_f} n_c}
\]  

(1)

where \( n_c \) is the density at the cut-off layer. This expression can be reduced if \( d \gg d_f \) to

\[
\delta \varphi = -k_o d_f \frac{\sqrt{L}}{d} \frac{\delta n}{n_c}.
\]  

(2)

This formula shows that the non-zero average density perturbation can influence the phase fluctuation far from the cut-off. With the same method, the phase variation for a zero average is written as

\[
\delta \varphi = 2k_o d_f \frac{d_f}{L} \frac{\sqrt{L}}{\sqrt{L-x_0-d_f} + \sqrt{L-x_0+d_f} \left( \sqrt{L-x_0+d_f} + \sqrt{L-x_0-d_f} \right) n_c} \delta n
\]

with the same additional assumption used to establish Eq.(2), the dependencies of the phase variation can be defined as

\[
\delta \varphi = k_o d_f^2 \frac{\sqrt{L}}{\sqrt{2d^3}} \frac{\delta n}{n_c}
\]  

(3)

As expected, a zero average density perturbation has an effect on the phase on a range shorter than a non-zero average density perturbation. However the perturbation on the phase depends on the square of the width of the density perturbation. The limits of this model are clearly defined by the assumptions that is to say, the density perturbation should be coherent and below the cut-off. These situations can correspond to the fluctuation reflectometer experiment especially when some islands oscillate between the cut-off and the edge. Other phenomenon can also induce phase modulation as the mirror effect or destructive interference [4].
**Full wave computation of the zero and non-zero average density perturbation.**

To illustrate the behaviour of these kinds of density perturbation we have computed the phase variation during the motion of the perturbation in the density with a probing wave at fixed frequency, which gives similar results when the frequency is swept. The chosen conditions to shown that a zero average density perturbation minimizes the phase variation avoid the spectral effect for a blob or a burst [5]. On the figure 2, the density perturbations with a fixed width and modulated ratio of the negative part over positive part are drawn and the phase variation associated to them. This figure 2 exhibits clearly that at the zero average density perturbation the phase variation reaches a minimal value.

![Figure 2: Density perturbation with different values of the average given by different value of the shift $x_s$ in $(x-x_s)e^{-ax^2}$ and the phase variation induced by these perturbations.](image)

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The second part of this study is to evaluate when the width becomes enough small to generate spectral effect and to know what kind of response give the zero and non-zero average density perturbation.

![Figure 3: Gaussian density perturbation with different widths and their phase variation in the case of non-zero average density perturbation (X-mode probing wave).](image)

Figure 3: Gaussian density perturbation with different widths and their phase variation in the case of non-zero average density perturbation (X-mode probing wave).
As expected when the width becomes of order of the Airy wavelength the spectral effect arises. For smaller values of the width, the phase variation correspond to the square of the Airy function as demonstrated in [3]. The important point is to notice that the phase perturbation is seen until the density perturbation leaves the plasma independently of the width. In the case of zero average density perturbation, the first part of the phase response corresponds to the crossing of the positive part of density perturbation. Then when the density perturbation is totally is the probing zone, the decrease of the phase variation becomes faster than those observed for the non-zero average density. For widths slightly higher the Airy wavelength, the phase variation becomes negligible before the density perturbation leaves the plasma. The overshoot on the phase when the perturbation leaves the plasma corresponds to a local over density at the edge.

\[ \text{Figure 4: Phase variation in the case of zero average density perturbation induced by density perturbation following the expression } (x-x_0)\exp(-w(x-x_0)^2) \text{ with different widths (X-mode).} \]

As we try to know if the two kinds of density perturbation can be superposed, some discrepancy appears due to non-linear effects. However the blobs or bursts, which can be associated to non-zero average density perturbation exist rather at the edge of the plasma. For the interaction of the probing wave with the islands the 2D effects should be considered especially when the amplitude variation are studied as it is in Tore Supra.

**Experimental results and interpretation.**

On Tore Supra tokamak a X-mode Fluctuation Reflectometer (TSFR) covering 105-155 GHz has been recently installed to measure the amplitude and phase of density fluctuation (see figure 5) from low field to high magnetic field side depending of the plasma scenario. To better understand the measured TSFR signal simulations are needed especially in presence of magnetic islands in the probing zone. Comparison between the TSFR signals and simulations have shown that it is possible to find a shape density variation in the vicinity of the magnetic island, which reproduces the experimental results qualitatively. To recover the experimental behaviour we
have introduced a secondary cut-off inside the islands that is to say an over density on the plateau at the outer edge of the island. 2D modelling with representative dimensions and parameters [6] is required to stand for the amplitude variations seen by the TSFR.

\[ \text{Figure 5: experimental temporal evolution of the phase and amplitude associated to a } m=2 \text{ mode in Tore Supra plasma.} \]

\[ \text{Figure 6: 2D O-mode reflectometry simulation of the temporal evolution of the phase and amplitude associated to a chain of islands exhibiting a double frequency for the amplitude that one for the phase.} \]

\textbf{Other explanation for interpreting the TSFR experiments.}

Another explanation should be given by the dynamical properties of the wave connected to the flux conservation. The variation can arrive when the cut-off layer moves at some velocity close to the group velocity. This can be found when the index gradient length becomes very high, for example, near the top of the density profile. To access to these phenomena a wave equation code is needed to simulate the temporal evolution of the amplitude of reflected wave. To reproduce
similar behaviour of the TSFR experiments, it is necessary to put one or two oscillating structures between the edge of the plasma and the cut-off corresponding, for example, to the situation where two rational surfaces with islands are probed simultaneously. The chosen parameters do not correspond to experimental one due to the consuming time for one run. This situation can amplify the amplitude variation. Different simulations have reproduced such behaviour observed experimentally.

Figure: The temporal evolution of the phase (left) and amplitude(right) associated to a double oscillation system in opposition exhibiting a double frequency for the amplitude

The role of the zero and non-zero average density perturbation needs further studies. As it is mentioned in [4] the forward scattering is connected to the small value of the wavenumber, which can be associated to a non-zero average density perturbation with a Gaussian shape without modulation. Due to the small space scale of the micro turbulence, the density fluctuation can be described as a zero average density perturbation except at the edge of the plasma where some phenomenon as ELM's, blobs, streamers can exist.

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