Leader modeling of gigantic jets connecting thunderclouds to the ionosphere
Lizhu Tong, Kenichi Nanbu, Hiroshi Fukunishi

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A randomly stepped leader propagation model is developed to study gigantic jets, a new type of lightning, connecting thunderclouds to the ionosphere. The thundercloud is considered as an electrode igniting gigantic jets and the ionosphere is assumed as the other. The propagation of stepped leader is considered as a field controlled random growth process. The electric field is considered due to the thundercloud charges and the self-consistently propagating leader. A leader propagation probability is proposed to determine if the leader grows at next step and to choose the step direction of the leader in case of growth. The results show that leader propagation spans ~72 km from igniting position to the ionosphere. The simulation of leader propagation appears to be in agreement with the structure of observed gigantic jets.

Keywords: Lightning, gigantic jet, leader modeling, thundercloud, ionosphere

1. Introduction

The breakdown in air long gaps, e.g., several tens of meters, occurs via a growth of a leader from one electrode to the other with a high electrical conductivity. One of the most important features of lightning discharge consists of the random behavior of its trajectory, which is the behavior of leader discharge. It is known that complicated discharge patterns can be described by an object with fractional dimensionality. This approach allows us to analyze quantitative characteristics of lightning discharge.

Gigantic jet is a new type of lightning between thundercloud and the ionosphere. It is a rarely discharge phenomenon to be detected because the condition igniting such a giant discharge is not easy to be satisfied, but it is of interest to theoretically study discharge characteristics of gigantic jets. This will help us to understand their contribution to the global electrical circuit.

In this paper we develop a randomly stepped leader propagation model for studying gigantic jets. The thundercloud is considered as an electrode igniting gigantic jets and the ionosphere is assumed as the other. Lightning formation is described by a random growth of leader discharge channels, which is determined by the electrostatic field produced due to thundercloud charges and leader discharge channels. A leader propagation probability is proposed to determine if the leader grows at next step and to choose the step direction of the leader in case of growth. The results yield a three-dimensional overall picture of leader propagation, which appear to be in agreement with the structure of observed gigantic jets.

2. Critical fields for leader propagation

Mechanisms usually considered responsible for the upper lightning discharge include the propagation of streamers in the quasi-electrostatic field of thunderclouds and breakdown of runaway electrons. In the first case,
the lightning is ignited and developed without any external factors being involved, whereas in the second case the ignition of the lightning is influenced by external factors such as cosmic rays which generate high energy electrons. The first mechanism is consistent with laboratory experiments. Extensive experimental data have been acquired in laboratory situations to support the electrical nature of the formation and development of a leader discharge.\(^9\) The physical processes underlying leader breakdown consist of igniting electrons, leading an avalanche to streamer corona criterion, and evolving streamer corona to leader channel. It is known that the electric field for ionization threshold can be estimated by 
\[ E_k = E_0 \cdot \left( \frac{N}{N_0} \right), \]
where 
\[ E_0 = 3.14 \times 10^4 \text{ V} \cdot \text{m}^{-1}, \]
\[ N_0 = 2.688 \times 10^{21} \text{ m}^{-3}, \]
and \( N \) is the neutral atmospheric density, taken from US Standard Atmosphere (1976).\(^10\)
It should be noted that the field \( E_k \) is required to ignite a streamer discharge or an avalanche-streamer transition but not for streamer propagation.\(^11\)

Observations of gigantic jets showed an upward transport of negative charges\(^4\) and it was suggested\(^5\) that gigantic jets might be negative cloud-to-ionosphere (NCI) discharges. Therefore, here we pay our attention on negative streamer propagation. A remarkable feature of streamer discharges is that in spite of their internal structural complexity, involving multiple highly branched streamer channels, its macroscopic characteristics remain relatively stable under a variety of external conditions. The minimum field \( -E_c \) required for propagation of negative streamers in air at atmospheric pressure is \( \sim -12.5 \text{ kV/cm} \), which value has been used to study blue jets.\(^7\) The condition for a streamer-leader transition can be satisfied if the electric field over the length of the streamer zone in a leader exceeds the critical value.\(^9\) Since the electric field decreasing with upper distance from the thundercloud is lower than the exponential decrease of the pressure with height, the condition for a streamer-leader transition can be substantially satisfied to ignite an upper lightning discharge. In the present work, the critical field \( E^* \) that governs the leader propagation in the air is assumed to be equal to the field in the streamer zone, i.e., \( E^* = E_c \).\(^13\) Also, \( E^* \) and \( E_c \) are simply scaled with height proportionally to the neutral atmospheric density.\(^7\)

3. Leader model

The propagation of the lightning can be considered as a field controlled random growth process, which is a discrete process. We schematize the leader propagation as a sequence of connections between the points of a spatial Cartesian lattice. The solution of electric field is divided into two stages: one is for the accumulation of the thundercloud charge and the other is for the propagation of the leader. The former is calculated until the arrival of ionization threshold. The point which field value reaches ionization threshold is considered as the igniting point of a leader. The potential of this igniting point is then fixed and the discharge is propagated by adding additional links. The continuity equation on the basis of charge conservation law is\(^14\)

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot E + \rho \sigma / \varepsilon_0 = 0, \quad (1) \]

where \( \rho \) is the charge density, \( \sigma \) is the conductivity, and \( t \) is the time. \( E \) is electrostatic field governed by

\[ \nabla \cdot E = (\rho + \rho_s) / \varepsilon_0, \quad (2) \]

where \( \rho_s \) is the thundercloud source charge density, i.e., \( \rho_s = \rho \) in this work. We consider the electrostatic field at the first stage to be axisymmetrical. In the cylindrical coordinate system \((r,z)\), the charge density \( \rho (r,z,t) \) is assumed to be a Gaussian spatial distribution given by

\[ \rho (r,z,t) = \rho(t)e^{-[(r-r_c)^2+(z-z_c)^2]/a^2}, \]

where \( z_c \) is the mean height of negative thundercloud charges and \( \rho(t) \) is the charge density corresponding to \( Q(t) = Q_0 \).
\[ \text{tanh}\left(\frac{t}{\tau}\right)/\text{tanh}(1) \], where \( Q_0 \) is the magnitude of thundercloud charge and \( \tau \) is the duration for the accumulation of thundercloud charge. The detailed discussion related to the charge distribution of thundercloud has been reported in our previous research.\(^{11} \) In this work, we set \( z = 16 \text{ km}, a = 2 \text{ km}.\)\(^{5} \) Before the ionization threshold is reached, electron conductivity below 60 km height is low. Therefore the total conductivity \( \sigma \) is dominated by ion conductivity, taken by \( \sigma = 5 \times 10^{14} e^{-z/(60 \text{ km})} \text{ S/m}.\)^{15} The ordinary finite difference method\(^{16} \) is used to solve eq. (1). The Fourier transform method and Thomas algorithm\(^{17} \) are used to solve eq.(2).

From the start of the ignition of the leader we solve three-dimensional electrostatic field to satisfy random spatial trajectories of leader propagation. The equation governing three-dimensional electrostatic field is Laplace equation. The field consists of moving boundaries, following leader propagation, which is determined at each growth step of the leader. A three-dimensional isoparametric finite element method is used to solve the Laplace equation.\(^{18} \) At every growth step, a new mesh point is linked with the leader channel, which is regarded as an imposed boundary for the recalculation of the electrostatic field. The mesh points located in leader channels are assumed to retain the potential which they have acquired until the end of the simulation. The potential drop along leader channels is determined by the field value of leader channel, e.g., \( \sim 1 \text{ kV/cm} \) for propagation of positive and negative leaders in atmospheric pressure.\(^{1} \) Next growth point of the leader is randomly chosen from the neighboring points around the leader tip. We parameterize the probability \( p \) for the formation of a leader step as

\[
 p = \left| E_i - E^* \right|^{\alpha} \sum_{i=1}^{K} \left| E_i - E^* \right|^{\alpha} ,
\]

where \( E^* \) is the critical field required for the leader propagation, i.e., \( E^* = E_c \) in this work, \( K \) is the total number of the neighboring points around the leader tip, and \( E_i \) is the average field along the growth direction of the leader.

\[ E_i \text{ between points } i \text{ and } j \text{ is calculated by } E_i = (U_i - U_j)/d \text{, where } U_i \text{ and } U_j \text{ are potentials at the points } i \text{ and } j \text{, and } d \text{ is the distance between } i \text{ and } j. \text{ We assume } \alpha = 1, \text{ following the previous research.}\(^{3,19} \) Based on eq.(3), a uniform random number \( R \) between 0 and 1 is introduced to choose one possible growth direction. If we suppose there are four possible growth directions at the leader tip \( i \), as shown in Fig.1, i.e., points \( j, k, l, m \), and their relative bond direction probabilities are \( 0.4, 0.1, 0.3, 0.2 \), when \( R = 0.7 \), the point \( l \) will be considered as next leader growth point, as shown in Fig.2. The growth of the leader stops self-consistently when all probabilities become zero.

We note that the field value in leader channel can not be simply scaled with height proportionally to the neutral atmospheric density due to its high electrical conductivity. It is known that the change in density with height is approximately exponential. We have \( \rho(z) = \rho(0)e^{-\alpha z} \), where \( \alpha (= 1.25 \times 10^4) \) is the scaling parameter and \( z \) is the...
height above sea level in meters. The conditions generating a streamer require a certain quantity of electrons to be reached. In order to satisfy this condition, the streamer zone becomes longer as the decrease of density with height. However, the leader length is much longer than streamer zone for long leaders. Bazelyan and Raizer showed some examples of long leaders, e.g., a leader with the length of 11.4 m consists of streamer zone of 3.6 m. Thus, in this work we consider the leader length \( L \) by \( L(z) = L(0)e^{-cz} \), where \( c \) is the parameter that characterizes the variation of leader channel. We assume that the change of leader channel field with leader length is linear for long leaders with sizes exceeding several meters, i.e., linearly decreasing as the increase of leader length, such as the experimental data shown in Table 6.1 of Bazelyan and Raizer. Therefore, we have \( E(z) = E(0)[L(0) / L(z)] = E(0)e^{-cz} \). Assigning \( \beta = c\alpha \), we obtain \( E(z) = E(0)e^{-\alpha z} \). In this work, we set \( \beta = 5 \times 10^{-5} - 1.5 \times 10^{-4} \).

4. Results and discussion

We calculate the quasi-electrostatic field during the accumulation of thundercloud charge on the basis of the solution of eqs. (1) and (2). Figure 3 shows the distribution of the electric field at the time that the ionization threshold is reached. The thundercloud charge required to reach the ionization threshold is 203.57 C. The electric field generated by thundercloud charges just exceeds the critical field \( E_k \) for ionization, which is much larger than the minimum field \( E_c \) required for supporting the propagation of streamers, as shown in Fig.3(b).

(a) Electric field on \( r-z \) plane

(b) Electric field on the axis \( (r = 0) \)

Fig. 3. Electric field distribution at the time that the ionization threshold is reached.
The simulation of leader propagation starts from the time that the ionization threshold is reached. Because the field produced by thundercloud charges is much larger than the critical field $E^*$ for leader propagation as shown in Fig. 3(b), the leader discharge starts to propagate upwards from the position of ionization threshold around the top of thundercloud. The calculation shows that most of the leader discharges terminate below the height of ~40 km, as seen in Fig. 4. Only a few of them propagate over the height and arrive at the ionosphere. It is known that blue jets propagate from cloud tops to a height of ~ 40 km.\textsuperscript{21,22} The present results give a reasonable theoretical explanation for such a fact that gigantic jets are rarely observed in comparison with blue jets.

![Fig. 4. Leader discharge under the condition that the critical field is reached.](image)

Once the leader starts its propagation, the charges would be transported upwards. The accumulation of thundercloud charge can be suppressed not enough to support the leader discharge reaching at the ionosphere. In the present work we use a simple charge distribution of thundercloud in consideration of the effect of the intracloud (IC) discharge or the positive cloud to gourd (+CG) discharge.\textsuperscript{11} Under the situation the accumulation of thundercloud charge could be maintained. Figure 5 (a) gives the calculated result when the thundercloud charge is accumulated up to be 300 C. The prevailing leader discharges span the long distance (= ~72 km) from the igniting position around the top of thundercloud to the ionosphere. The leader propagation appears a three-dimensional overall picture in agreement with the observed gigantic jets, as shown in Fig.5 (b).

5. Conclusion

We develop a randomly stepped leader propagation model to study gigantic jets connecting thunderclouds to the ionosphere. The critical electric field for the leader propagation in upper atmosphere is proposed on the basis of the experimental data in laboratory situations. A leader propagation probability is introduced to determine if the leader grows at next step and to choose the step direction of the leader in case of growth. The results show that the leader propagation can span ~72 km from igniting position to the ionosphere. The simulation presents a three-dimensional overall picture of leader propagation, which appear to be in agreement with the structure of observed gigantic jets. The model developed in this paper could be also applied to study blue jets and red sprites.
Fig. 5. Leader propagation between thundercloud and the ionosphere. (a) The simulation result based on the present model. (b) Image of a typical gigantic jet.

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