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1 Introduction

We present a solution of the "plasma equation" (i.e., the quasi-neutrality condition) valid in a time-independent plasma presheath in front of a solid material wall, which can be at any negative electric potential with respect to the unperturbed plasma. The plasma is composed of collisionless electrons and singly charged positive ions. The velocity distribution of the electrons is assumed to be a truncated Maxwellian due to the fact that they are partly absorbed by the wall. This assumption apparently yields a more realistic dependence of the electron density on the electric potential than the Boltzmann equilibrium factor, which is often used in various kinetic and fluid models of boundary plasmas. The ion velocity distribution is calculated as a function of the potential drop in the plasma presheath. Models of the plasma-wall transition are customarily based on the approximate two-scale analysis in which the perturbed boundary plasma is divided into a non-neutral plasma sheath at the wall and an adjacent quasi-neutral plasma presheath, where Poisson's equation is replaced with the plasma equation. In our model we follow the methodology first applied by Tonks and Langmuir [1] and further elaborated for various cases of boundary plasmas during the previous century (see, e.g., [2] and references therein). While the plasma sheath has already been treated with analyses similar to ours (e.g., by Andrews and Varey [3]), our present work represents the first treatment of this kind for the quasi-neutral plasma presheath. The results for the moments of the ion velocity distribution obtained in our study can be applied, e.g., in the expressions for boundary conditions of the above-mentioned fluid models and related computer codes.

2 Origin of truncated Maxwellian velocity distributions

The Tonks-Langmuir model is based on the approximation that the electron density n_e in the quasineutral region follows the Boltzmann profile $n_e(\Phi) = n_{e,0}\exp(e\Phi/kT^e)$, where

$n_{e,0}$ is the density at the plane of symetry of the discharge, $\Phi(x)$ is the plasma potential (satisfying the boundary condition $\Phi(0) = 0$ and monotonically decreasing towards the wall), T^e is the electron temperature, e is elementary charge, and k is the Boltzmann constant. In this model, all parameters in the quasineutral region are independent of the details of the physical boundary, with the only qualitative requirement that the boundary potential is approximately negative infinite with respect to the plasma.

In the present work, however, we suppose that the electron density follows a modified Boltzmann profile which can be obtained from the truncated Maxwellian velocity distribution reflecting the fact that the electrons are partly absorbed by an electrically biased conductive surface, generally called "electrode". Our theoretical model is based on the assumptin of *small* electrodes i.e., *plasma probes*, such that all effects of their presence in the plasma on the plasma parameters (plasma potential and velocity distributions) are strictly limited to a region of influence which is much smaller than the total region occupied by the plasma. Outside this region of influence any effect to the plasma parameters should be considered as *unmeasurable* for practical purposes. Under this condition it is possible to extract from the probe current-voltage characteristics the "exact" unperturbed plasma potential and electron velocity distribution in the unperturbed region. If, however, the region of electrode influence is so large that it cannot be distinguished from the overall unperturbed plasma region, one is faced with a complex situation where the whole system should be understood, modelled and solved self-consistently as has been done e.g., in [4] in 0D geometry approximation for an arbitrary three-dimensional plasma-wall-electrode system.

The generic formula in probe theory defining the plasma density inside the whole region of the probe influence is (e.g., Swifts and Schwar [5])

$$n_e(x) = \frac{n_{e,0}}{2} \exp\left(-\frac{e|\Phi(x)|}{kT^e}\right) \left[1 + \mathrm{erf}\sqrt{e|V_0 - \Phi(x)|/kT^e}\right] \tag{1}$$

where the reference potential $\Phi(0)$ is chosen at the "infinitely distant" point, i.e., in the unperturbed plasma and V_0 is the probe voltage (defined with respect to the reference potential).

The region of influence of an electrode consists of the quasineutral and non-neutral regions i.e., the presheath and sheath, respectively. While, due to complex physical processes going on, the detailed profiles for the unperturbed plasma cannot be calculated on the basis of formula (1), it can be done for the whole collisionless region of influence. Analytical treatment of this region is usually performed in the two-scale approximation (see e.g., [6]), i.e., separately for the presheath and sheath regions. For calculating the presheath it is convenient to put boundary conditions at the plasma-presheath boundary. Thus applying formula (1) some simple algebra leads to the density profile

$$n_e(\Phi) = n_{e,ps} \exp\left(-\frac{e|\Phi(x)|}{kT^e}\right) \frac{1 + \text{ erf } \sqrt{e|V_{ps} - \Phi(x)|/kT^e}}{1 + \text{ erf } \sqrt{e|V_{ps}|/kT_f^e}}, \tag{2}$$

where the potential $\Phi(x)$ and electrode potential V_{ps} are measured with respect to the plasma-presheath boundary (index "ps"), where the density is $n_{e,ps}$.

It can be shown (e.g. [7]) that for for collisionless regimes the ion velocity distribution $f_i(v)$ is a function of the total energy $\mathcal{E}_i = m_i v^2/2 + e\Phi$ only. Therefore the quasineutrality condition $n_i = n_e$ takes the form

$$\int_{e\Phi}^{0} \frac{f_i(\mathcal{E}_i) d\mathcal{E}_i}{\sqrt{2m_i(\mathcal{E}_i - e\Phi)}} = n_e(\Phi) \tag{3}$$

where $n_e(\Phi)$ is given by (2). In normalized variables

$$\mathcal{E}_i^* = \frac{\mathcal{E}_i}{kT^e}, \quad \Phi^* = \frac{e\Phi(x)}{kT^e}, \quad V_{ps}^* = \frac{eV_{ps}}{kT^e}, \quad n_e^*(\Phi^*) = \frac{n_e(\Phi)}{n_{e,ps}}, \quad f_i^*(\mathcal{E}_i^*) = \sqrt{\frac{2kT^e}{m_i}} \frac{f_i(\mathcal{E}_i)}{2n_{e,ps}} \tag{4}$$

and after apppropriate rearrangement on the left-hand side Eq (3) reads

$$\int_{0}^{\Phi^*} \frac{f_i^*(\mathcal{E}_i^*) d\mathcal{E}_i^*}{\sqrt{\Phi^* - \mathcal{E}_i^*}} = -i n_e^*(\Phi^*) \tag{5}$$

where $i = \sqrt{-1}$. This equation can be readily solved by using Abel inversion (eg, [8]), yielding

$$f_i^*(\Phi^*) = \frac{1}{i\pi} \frac{d}{d\Phi^*} \int_0^{\Phi^*} \frac{n_e^*(\Phi^* - \mathcal{E}_i^*) d\mathcal{E}_i^*}{\sqrt{\mathcal{E}_i^*}}. \tag{6}$$

After straightforward mathematics we obtain the analytical solution

$$\begin{aligned} f_i^*(\mathcal{E}_i^*) &= \frac{1}{\pi\sqrt{-\mathcal{E}_i^*}} - \frac{2\left[D(\sqrt{-\mathcal{E}_i^*}) + D_{\text{erf}}(\sqrt{-\mathcal{E}_i^*}, \sqrt{-V_{ps}^*})\right]}{\pi(1 + \text{ erf } \sqrt{-V_{ps}^*})} \\ &+ \frac{e^{V_{ps}^*}}{\pi^{3/2}(1 + \text{ erf } \sqrt{-V_{ps}^*})} \ln\left|\frac{2\sqrt{\mathcal{E}_i^* V_{ps}^*} + V_{ps}^* + \mathcal{E}_i^*}{V_{ps}^* - \mathcal{E}_i^*}\right| \end{aligned} \tag{7}$$

where we have introduced a new special function called here "Dawson-erf"

$$D_{\text{erf}}(y, a) = \exp(-y^2) \int_o^y e^{y'^2} \text{erf}\sqrt{y'^2 + a^2 - y^2} dy' \tag{8}$$

which for $a^2 \gg y'^2 - y^2$ reduces to "standard" (e.g. [8]) Dawson function $D_{\text{erf}}(y, \infty) = D(y) = \exp(-y^2) \int_o^y e^{y'^2} dy'$. Finally, it turns out that the logarithmic term may be neglected for any experimental bias potential.

3 Discussion and conclusion

A simple consequence of Eq (7) is that the sheath in our model dissapears for sufficiently high electrode (or probe) biases i.e., more precisely, for $V_{ps} > -1.09226... \times kT_f^e$. This means that in such cases we do not need any two-scale approach, i.e., a full solution is available up to the electrode surface. However, our model is an idelisation of the nature since in practice truncated Maxwellian distributions are difficult to obtain even at very low discharge pressures. This behaviour has for a long time been well known as the "Langmuir paradox" (e.g., [9]). We also confirmed experimentally that the electron velocity distribution near the electrodes (at least in laboratory devices) under low pressures is rather a shifted Maxwellian than a truncated one [10]. In fusion devices it might be different but to our knowledge complete experimental data (with a rotating or double plane probe) are not available up to date. This seems to be an imprtant task to be tackled for, e.g., tokamak devices.

Anyway, it should be pointed out that other velocity distributions than truncated Maxwellian could possibly lead to similar density profiles as the truncated Maxwellian, and that the exact shape of the corresponding real ion velocity distribution might not be crucial for the basic features characterising the presheath. In this sense, the present approach is intended as a first step towards shedding more light on the physics of the Scrape-of layer (SOL) in the region between the so called X-point and a divertor plate in tokamak devices.

Finally we note two facts i.e., (i) that the original Tonks-Langmuir model cannott account for the absorption effects and so it is a pure plasma model - without presheath. Our model, although more general, is a presheath model - without plasma. The plasma itself is assumed to be generated independently being a main source of full Maxwellian electrons for the presheath. In addition, neither Tonks-Langmuir model nor our model takes into account that every ion creationthe in the presheath is accompanied by the creation of a non-Maxwellian electron which is superimposed upon the main truncated population. For this reason the physics of the presheath is very difficult to resolve with arbitrary accuracy. (ii) Secondly and finally, solid surfaces are far from being ideal - they have some surface potential dispersion that could be much greater than the dispersion of the measured quantities. Therefore it is sometimes meaningless to refine too much the theories about the exact velocity distribution shapes.

4 Acknowledgements

This work was supported by the European Commission under (i) the Contract of Association between EURATOM and theAustrian Academy of Sciences and (ii) the EURATOM-Slovenia cost-sharing action contract FU06-CT-2003-00321, and by the Austrian Research Fund (FWF) under project Nos. 15013-N08 and P16807-N08. It was carried out within the framework of the European Fusion Development Agreement. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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