

Entanglement storage in atomic ensembles

Aurelien Dantan, Alberto Bramati, Michel Pinard

▶ **To cite this version:**

Aurelien Dantan, Alberto Bramati, Michel Pinard. Entanglement storage in atomic ensembles. EPL - Europhysics Letters, 2004, 67, pp.881. hal-00001522

HAL Id: hal-00001522
https://hal.science/hal-00001522

Submitted on 4 May 2004

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Entanglement storage in atomic ensembles

A. Dantan, A. Bramati and M. Pinard

Laboratoire Kastler Brossel, Université Pierre et Marie Curie, 4 place Jussieu, 75252 Paris Cedex 05, France

PACS. 03.67.-a – Quantum information.
PACS. 03.67.Mn – Entanglement production, characterization and manipulation.
PACS. 42.50.Lc – Quantum fluctuations, quantum noise and quantum jumps.

Abstract. – We propose to entangle macroscopic atomic ensembles in cavity using EPR-correlated beams. We show how the field entanglement can be almost perfectly mapped onto the long-lived atomic spins associated with the ground states of the ensembles, and how it can be retrieved in the fields exiting the cavities after a variable storage time. Such a continuous variable quantum memory is of interest for manipulating entanglement in quantum networks.

ccsd-00001522, version 1 - 4 May 2004

Entanglement is one of the most intriguing feature of quantum mechanics and, since the enunciation of the famous Einstein-Podolsky-Rosen paradox [1], has always attracted a lot of attention. In particular, it is at the heart of quantum communication and quantum information protocols such as quantum cryptography, teleportation, dense coding, quantum computing [2]. The past few years have seen many realizations of entangled beams in the continuous variable regime, using $\chi^{(2)}$ process in optical parametric amplifiers (OPAs) [3, 4, 5, 6], Kerr effect in optical fibers [7, 8] or in cold atoms [9]. Efficient sources of entangled beams now exist and strong correlations have been achieved over rather broad bandwidth [6]. In order to build quantum communication networks in which light beams connect atomic ensembles, a major issue is to be able to store entanglement into the atoms [10, 11]. Entanglement between two atomic ensembles has been successfully demonstrated by Julsgaard *et al.* by sending pulses of coherent light through two atomic vapor cells [12] and measuring the outgoing field. However, the possibility to store entanglement between quantum-correlated beams into atoms remains to be demonstrated. In this Letter we propose a cw scheme to achieve entanglement between two cold atom ensembles placed in cavities by using EPR-entangled beams, as produced by OPAs for instance, and coherent control fields. The entanglement between the beams is mapped onto the ground state spins of the atoms and no measurement of the field is required. Given the long lifetime of the cold atoms spin the entanglement can thus be stored for a rather long time when the control field is switched off. It can then be retrieved in the vacuum modes exiting the cavities by switching on the control field again after a variable storage time. We then give a method to directly measure the entanglement of the outgoing beams - and, consequently, the atomic entanglement - in one simultaneous measurement of the EPR variances with two homodyne detections and a single local oscillator.

© EDP Sciences

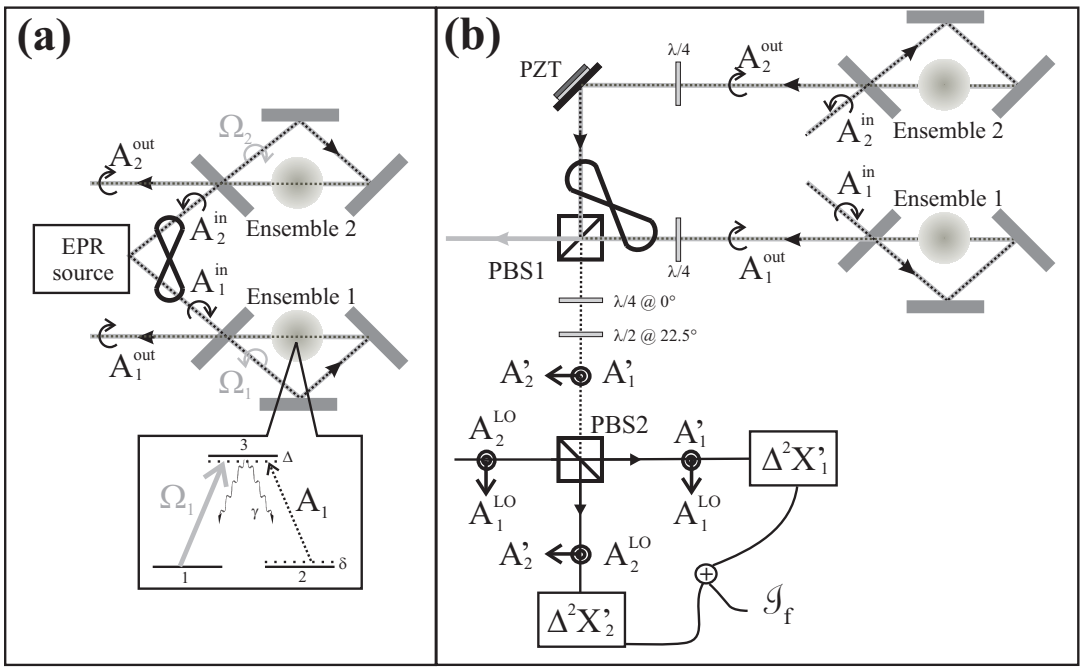


Fig. 1 – (a) Entanglement mapping onto the atoms: the incoming vacuum fields $A^{in}_{1,2}$ are entangled, $\Omega_{1,2}$ are intracavity coherent control fields. The insert shows the Λ structure of the atomic levels considered. (b) Readout: the incident vacuum fields are in a coherent vacuum state, the outgoing vacuum fields are entangled when the control fields are switched on again. PBS: polarizing beamsplitter, LO: local oscillator, PZT: piezo-electric ceramic.

As shown in fig. 1, we consider two identical sets of N Λ-type 3-level atoms, each set interacting with a control field Ω_i, $(i = 1, 2)$ and with one of the EPR-entangled beams A_i. Without loss of generality we assume the control fields to be σ_+-polarized and the entangled vacuum fields σ_--polarized. The entangled vacuum fields can be obtained for instance from OPAs [5] or from an OPO below threshold [6], although other schemes can be equivalently envisaged. To simplify, the entanglement bandwidth is assumed to be larger than the cavity bandwidth κ. First, we examine how to create entanglement between the atoms and start by studying the interaction of light with one ensemble. In previous works [13, 14, 15], we showed that "Raman"- or "EIT"-type interaction of light with Λ-type atoms could lead to squeeze the atomic spin, either in a non linear regime, when the incoming field is in a coherent state ("self spin squeezing"), or by transfer, when the incoming field is a broadband squeezed vacuum. In the latter, we examined how squeezing could be transferred from the field to the atoms in different configurations. A quasiperfect transfer is predicted either in an "EIT" (on one- and two-photon resonance) or a "Raman" (on two-photon resonance, but large one-photon detunings) configurations. Basing ourselves on these results we consider for instance an EIT situation in which the fields are both one- and two-photon resonant ($\Delta = \delta = 0$) and cancel the cavity detuning. Since $\langle A^{in}_1 \rangle = 0$, the atoms are pumped by control field Ω_1 into level $|2\rangle$ and the spin is aligned along z in steady state: $\langle J_{z1} \rangle = N/2$. The collective atomic spin can then be treated as a harmonic oscillator, the non-commutating spin components J_{x1} and J_{y1} in the plane orthogonal to the mean spin playing the same role as the field quadrature

operators $X_1 = A_1 + A_1^\dagger$ and $Y_1 = i(A_1^\dagger - A_1)$. We thus seek to map the EPR correlations existing between the quadratures X_i, Y_i onto the two ensembles spin components J_{xi}, J_{yi}. It is possible to choose the control field pumping rate $\Gamma_E = \Omega_1^2/\gamma$ so that the ground state observables evolve slowly with respect to the fields or the optical dipoles and get simplified equations for the atomic spin fluctuations in the Fourier domain [15]

$$(\tilde{\gamma}_0 - i\omega)\delta J_{x1}(\omega) = -\beta\delta X_1^{in}(\omega) + \tilde{f}_{x1} \quad (1)$$
$$(\tilde{\gamma}_0 - i\omega)\delta J_{y1}(\omega) = -\beta\delta Y_1^{in}(\omega) + \tilde{f}_{y1} \quad (2)$$

where $\tilde{\gamma}_0 = \gamma_0 + \Gamma_E/(1+2C)$ is the effective atomic decay constant, satisfying $\gamma_0 \ll \tilde{\gamma}_0 \ll \gamma, \kappa$, $C = g^2N/T\gamma$ is the standard cooperativity parameter, T the coupling mirror transmission, g the atom-field coupling constant and $\tilde{f}_{x1}, \tilde{f}_{y1}$ are Langevin operators, the correlation functions of which can be calculated via the quantum regression theorem. These operators account for the noise due to loss of coherence in the ground state ($\propto \gamma_0$) and for the noise contribution of the optical dipole via spontaneous emission ($\propto \Gamma_E$). $\beta = gN\Omega_1/\gamma\sqrt{T}(1+2C)$ represents the effective coupling with the incoming EPR field A_1^{in}. To simplify we assume a symmetrical configuration for ensemble 2 ($\Omega_2 = \Omega_1$, same number of atoms, $\langle J_{z2}\rangle = N/2$, etc), so that the equations for the ground state spin fluctuations are similar to eqs. (1-2) by substituting subscript 1 by 2. We then obtain very simple equations for the fluctuations of the ground state spins operators

$$(\tilde{\gamma}_0 - i\omega)(\delta J_{x1} - \delta J_{x2}) = -\beta(\delta X_1^{in} - \delta X_2^{in}) + \tilde{f}_{x1} - \tilde{f}_{x2} \quad (3)$$
$$(\tilde{\gamma}_0 - i\omega)(\delta J_{y1} + \delta J_{y2}) = -\beta(\delta Y_1^{in} + \delta Y_2^{in}) + \tilde{f}_{y1} + \tilde{f}_{y2} \quad (4)$$

which are valid if the effective optical pumping rate satisfy $\gamma_0 \ll \tilde{\gamma}_0 \ll \kappa, \gamma$. Since the fluctuations of both spins are related to those of the incident beams quadratures, one expects that the field correlations will reflect on the atoms.

To quantify the entanglement we make use of the inseparability criterion derived by Duan *et al.* and Simon [16], which rely on the sum of the variances of EPR-type Gaussian operators, such as $X_1 - X_2$ and $Y_1 + Y_2$ for the field.

$$\mathcal{I}_f = \frac{1}{2}[\Delta^2(X_1 - X_2) + \Delta^2(Y_1 + Y_2)] < 2 \quad (5)$$

is a necessary condition for modes 1 and 2 to be entangled. Moreover, when modes 1 and 2 are symmetric it has been shown by Giedke *et al.* that this condition is also sufficient and directly related to the Entanglement of Formation (EoF), thus providing a good measure of entanglement [17]. For OPAs pumped below threshold strong amplitude correlations and phase anti-correlations exist, and both $\Delta^2(X_1 - X_2)$ and $\Delta^2(Y_1 + Y_2)$ can be strongly reduced below the separable beams value of 2 on a broad bandwidth [6]. In a similar fashion the EPR-type operators for the atomic ensembles are $J_{x1} - J_{x2}$ and $J_{y1} + J_{y2}$, which satisfy $\langle[J_{x1} - J_{x2}, J_{y1} + J_{y2}]\rangle = i\langle J_{z1} - J_{z2}\rangle = 0$. The atomic ensembles are then entangled if the following inseparability criterion is satisfied [12]

$$\Delta^2(J_{x1} - J_{x2}) + \Delta^2(J_{y1} + J_{y2}) < |\langle J_{z1}\rangle| + |\langle J_{z2}\rangle| = N \quad (6)$$

It is then obvious from eqs. (3-4) that any entanglement between modes 1 and 2 will be transferred to the atomic spin, provided the coupling β is large enough with respect to the noises of the process $\tilde{f}_{\alpha i}$. This is indeed possible in EIT or Raman situations for which the noises are substantially reduced by the cooperative behavior of the atoms [15], while the

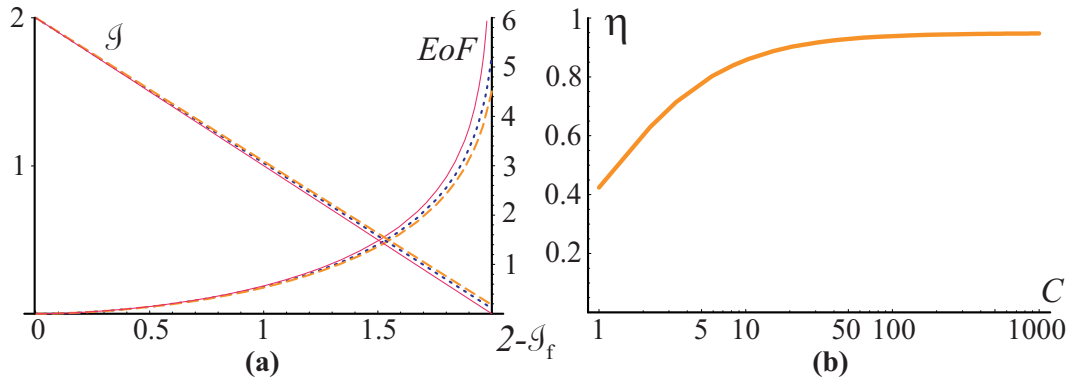


Fig. 2 – (a) Left axis: Inseparability criterion for the incident fields $\mathcal{I}_f$ [plain] and the atoms $\mathcal{I}_{at}$ [dotted: simple model approximation, dashed: full calculations] vs EPR correlations $2-\mathcal{I}_f$. Right axis: entanglement of formation for incident fields and atoms vs EPR correlations [parameters: $C = 100$, $\kappa = 2\gamma$, $\gamma = 1000\gamma_0$, $\Gamma_E = 15\gamma$]. (b) Optimized mapping fidelity $\eta = f(\mathcal{I}_{at})/f(\mathcal{I}_f)$ vs cooperativity. For each value of C the pumping rate is optimized [$\gamma = 1000\gamma_0, \kappa = 2\gamma, \mathcal{I}_f = 1$].

coupling is enhanced by the number of atoms, as we will show further. In other words, the signal-to-noise ratio of the transfer process is increased by the atoms cooperative behavior, even though the strong coupling regime is not reached [18]. Assuming broad bandwidth amplitude correlations and phase anti-correlations and in the regime $\gamma_0 \ll \tilde{\gamma}_0 \ll \gamma, \kappa$, the sum of the atomic EPR variances, $\mathcal{I}_{at} = \frac{2}{N}[\Delta^2(J_{x1} - J_{x2}) + \Delta^2(J_{y1} + J_{y2})]$ (normalized to 2) is directly related to the amount of EPR-type correlations of the incident beams $\mathcal{I}_f$

$$\mathcal{I}_{at} = \frac{2C}{1+2C}\frac{\Gamma_E}{(1+2C)\tilde{\gamma}_0}\mathcal{I}_f + 2\left[\frac{\gamma_0}{\tilde{\gamma}_0} + \frac{\Gamma_E}{(1+2C)^2\tilde{\gamma}_0}\right] \tag{7}$$

The first term in (7), proportional to $\mathcal{I}_f$, can be understood as the atom-field coupling factor and can be very close to 1 for C and Γ_E large enough. The second and third terms represent the noises of the transfer process due to the loss of coherence in the ground state ($\propto \gamma_0$) and spontaneous emission ($\propto \Gamma_E$), respectively. Both can be made small in the regime chosen for the pumping and for large values of C. Typical experimental values of C of 100-1000 ensure a quasiperfect entanglement mapping from fields to atoms. Note also that, if the incident fields are not entangled ($\mathcal{I}_f = 2$), so are the atoms ($\mathcal{I}_{at} = 2$). The accuracy of our simple picture has been checked by full calculations involving the three-level atomic structure and the exact covariance matrix of the whole atom-field system. The results are represented in fig. 2(a) in which we plot $\mathcal{I}_{at}$ as a function of the EPR correlations $2 - \mathcal{I}_f$. Very good agreement is found between the simple model [Eq. (7), dotted] and the numerical simulations [dashed]. As a comparison the inseparability criterion for the incident fields $\mathcal{I}_f$ is also represented [plain]. In a symmetrical configuration the inseparability criterion (5) is a measure of entanglement via the entanglement of formation [17]. The incident EPR fields and the atomic EoFs (see [19] for details) are also both plotted in fig. 2(a). To quantify the efficiency of the mapping we plot in fig. 2(b) the ratio of the atomic EoF to that of the EPR fields, $\eta = f(\mathcal{I}_{at})/f(\mathcal{I}_f)$, as a function of the cooperativity. An excellent mapping ($\eta \sim 100\%$) is obtained for easily accessible values of C.

Once the field entanglement has been mapped onto the atoms the fields can be switched off and the atoms return to the coherent spin state very slowly, on a time scale given by $1/\gamma_0$.

We now address the readout problem and show that it is possible to retrieve the entanglement in the vacuum fields exiting the cavities by switching on again *only* the control fields. Indeed, starting now with correlated atomic ensembles and incoming coherent vacuum fields we expect the fluctuations of $J_{x1}-J_{x2}$ and $J_{y1}+J_{y2}$ to imprint on the fluctuations of, respectively, $X_1^{out}-X_2^{out}$ and $Y_1^{out}+Y_2^{out}$. As represented in fig. 1 the outgoing vacuum fields are combined using quarter-wave plates on the first PBS while the outgoing control fields are discarded. After PBS1 one disposes of two entangled vacuum modes with orthogonal polarization. The path dephasing can easily be cancelled using the control fields interference signal to lock the piezo-electric ceramic. Using the method developed in ref. [20] we then rotate the polarization basis so as to retrieve two squeezed modes for the same quadrature. This can be easily done with a quarter-wave plate at 0° in order to rotate the noise ellipsoid of one mode by $\pi/2$ with respect to the other. One then uses a half-wave plate at 22.5° to get the ±45° polarization modes, which are now squeezed for the same quadratures. To simultaneously measure the squeezing of both modes we use the technique of ref. [9] and combine on PBS2 the beam to be measured with a local oscillator polarized at 45° to the cube axes. We then perform two balanced homodyne detections, the first of which measures the noise of $X_1'=(X_1^{out}-X_2^{out})/\sqrt{2}$, the second, $X_2'=(Y_1^{out}+Y_2^{out})/\sqrt{2}$. The sum of the signals therefore gives directly the value of the inseparability criterion (5).

More quantitatively one may express the correlation function of X_1' in the regime $\gamma_0 \ll \tilde{\gamma}_0 \ll \kappa,\gamma$ as functions of the atomic EPR variances at the switching time

$$\mathcal{C}_1'(t,t') \equiv \langle \delta X_1'(t)\delta X_1'(t')\rangle = \delta(t-t') - \frac{4C\Gamma_E}{(1+2C)^2}\left[1-\frac{\Delta^2(J_{x1}-J_{x2})}{N/2}\right]e^{-\tilde{\gamma}_0(t+t')}$$

A similar expression holds for the correlation function of X_2', replacing $J_{x1}-J_{x2}$ by $J_{y1}+J_{y2}$. When the atoms are in coherent spin states ($\Delta^2(J_{x1}-J_{x2})=N/2$) one retrieves the standard δ-function of a free field. We must now specify how the homodyne detections are performed. In order to correctly measure the squeezing of modes A_1' and A_2', and, therefore, the field entanglement at the output of the cavity, we choose for the LO a temporal profile in $e^{-\tilde{\gamma}_0 t}$ which matches that of the vacuum modes (as can be seen from the correlation functions). In this case the normalized power measured by a Fourier-limited spectrum analyzer integrating over a time large with respect to $1/\tilde{\gamma}_0$ is given by

$$\begin{aligned} P_1(t) &\equiv \int_{-\pi/T_0}^{\pi/T_0}\frac{d\omega}{2\pi}\int_t^{t+T_0}d\tau\int_t^{t+T_0}d\tau' e^{-i\omega(\tau-\tau')}E_{LO}(\tau)E_{LO}(\tau')\mathcal{C}_1'(\tau,\tau') \\ &= \mathcal{N}-\mathcal{S}\left[1-\frac{\Delta^2(J_{x1}-J_{x2})}{N/2}\right]e^{-2\tilde{\gamma}_0 t} \end{aligned}$$

again with a symmetrical expression for the second homodyne detection. $\mathcal{N}$ and $\mathcal{S}$ are integrals depending on T_0, $\tilde{\gamma}_0$ and C. $\mathcal{N}$ represents the shot noise (i.e. the noise level for uncorrelated atoms, and, therefore, uncorrelated fields), so that $\mathcal{S}/\mathcal{N}$ is the signal-to-noise ratio of the measurement. $\mathcal{N}$ can be easily measured in a preliminary experiment in which the atoms are prepared in a coherent spin state. The signal-to-noise ratio can be shown to be close to 1 when $\tilde{\gamma}_0 T_0 \gg 1$ and $C \gg 1$ [15]. Note that this is possible because we have chosen the right matching profile for the LO. For short times the atomic entanglement is given by

$$\frac{1}{\mathcal{N}}[P_1(0)+P_2(0)] \simeq \frac{2}{N}[\Delta^2(J_{x1}-J_{x2})+\Delta^2(J_{y1}+J_{y2})] = \mathcal{I}_{at}$$

We indeed retrieve the value of the inseparability criterion (6) for the atomic ensembles, i.e. measuring the atomic entanglement is equivalent to measuring the entanglement between the outgoing modes. It is then possible with this technique to detect the atomic entanglement with nearly 100% efficiency.

In conclusion we have proposed a scheme to achieve continuous entanglement between atomic ensembles in cavities using a pair of EPR-correlated beams. The entanglement can be stored for a long time in the ground-state atomic spins and retrieved at will in the fields exiting the cavities by switching on the control fields. We also propose a technique to perform the atomic entanglement readout in a single shot measurement with one local oscillator. It is worth noticing that all the results obtained in an EIT configuration can be readily transposed in a Raman configuration, in which a perfect entanglement storage is also predicted in the regime $\gamma_0 \ll (1+2C)\Gamma_R \ll \kappa, \gamma$ (where $\Gamma_R = \gamma\Omega^2/\Delta^2$ is the Raman optical pumping rate). We would like to point out that, although the storage time is given by the inverse of the natural decay rate of the ground state $1/\gamma_0$, the memory bandwidth, i.e. the frequency bandwidth over which the entanglement is stored is much broader ($\propto \tilde{\gamma}_0$), because of the cavity interaction with the field. Such a quantum memory could allow to store and manipulate entanglement which is a key challenge for quantum communication and information.

* * *

REFERENCES

[1] Einstein A., Podolsky B. and Rosen R., *Phys. Rev.*, **47** (1935) 777.

[2] DiVincenzo D.P., *Science*, **270** (1995) 255; Furusawa A., Sorensen J., Braustein S. Fuchs C., Kimble H.J. and Polzik E.S., *Science*, **282** (1998) 706; Braustein S.L. and Kimble H.J., *Phys. Rev. A*, **61** (2000) 042302; Li X., Pan Q., Jing J., Zhang J., Xie C. and Peng K., *Phys. Rev. Lett.*, **88** (2002) 047904.

[3] Ou Z.Y., Pereira S.F., Kimble H.J. and Peng K., *Phys. Rev. Lett.*, **68** (1992) 3663.

[4] Zhang Y., Wang H., Li X., Jing J., Xie C. and Peng K., *Phys. Rev. A*, **62** (2000) 023813.

[5] Bowen W.P., Treps N., Schnabel R. and Lam P.K., *Phys. Rev. Lett.*, **89** (2002) 253601; Bowen W.P., Schnable R. Lam P.K. and Ralph T.C., *Phys. Rev. Lett.*, **90** (2003) 043601.

[6] Laurat J., Coudreau T., Keller G., Treps N. and Fabre C., (2004), quant-ph/0403224.

[7] Silberhorn C., Lam P.K., WeissO., König F., Korolkova N. and Leuchs G., *Phys. Rev. Lett.*, **86** (2001) 4267.

[8] Glöckl O., Lorenz S., Marquardt C., Heersink J., Brownnutt M., Silberhorn C., Pan Q., van Loock P., Korolkova N. and Leuchs G., *Phys. Rev. A*, **68** (2003) 012319.

[9] Josse V., Dantan A., Bramati A., Pinard M. and Giacobino E., *Phys. Rev. Lett.*, **92** (2004) 123601.

[10] Duan L.M., Cirac J.I., Zoller P. and Polzik E.S., *Phys. Rev. Lett.*, **85** (2000) 5643.

[11] Lukin M.D., *Rev. Mod. Phys.*, **75** (2003) 457.

[12] Julsgaard B., Kozhekin A. and Polzik E.S., *Nature*, **413** (2001) 400.

[13] Dantan A., Pinard M., Josse V., Nayak S. and Berman P.R., *Phys. Rev. A*, **67** (2003) 045801.

[14] Dantan A., Pinard M. and Berman P.R., *Eur. Phys. J. D*, **27** (2003) 193.

[15] Dantan A. and Pinard M., *Phys. Rev. A*, **69** (2004) 043810.

[16] Duan L.M., Giedke G., Cirac. J.I. and Zoller P., *Phys. Rev. Lett.*, **84** (2000) 2722; Simon R., *Phys. Rev. Lett.*, **84** (2000) 2726.

[17] Giedke G., Wolf M.M., Krüger O., Werner R.F. and Cirac J.I., *Phys. Rev. Lett.*, **91** (2003) 107901.

[18] Duan L.M., Lukin M.D., Cirac J.I. and Zoller P., *Nature (London)*, **414** (2001) 413.
[19] Given $\mathcal{I}_\alpha$ for symmetric states the EoF is calculated using $f(\mathcal{I}_\alpha) = c_+(\mathcal{I}_\alpha/2)\log_2[c_+(\mathcal{I}_\alpha/2)] - c_-(\mathcal{I}_\alpha/2)\log_2[c_-(\mathcal{I}_\alpha/2)]$, where $c_\pm(x) = (x^{1/2} \pm x^{-1/2})^2/4$ [17].
[20] Josse V., Dantan A., Bramati A. and Giacobino E., *J. Opt. B: Quant. Semiclass.*, (2004) to be published, quant-ph/0310139.