Screening by composite charged particles: the case of quantum well trions

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We study the screening of an external potential produced by a two-dimensional gas of charged excitons (trions). We determine the contribution to the dielectric function induced by these composite charged particles within a random phase approximation. In mixtures of free electrons and trions, the trion response is found dominant. In the long wave-length limit, trions behave as point charges with mass equal to the sum of the three particle components. For finite wave-vectors, we show how the dielectric response is sensitive to the composite nature of trions and the internal degrees of freedom. Predictions are presented for the screening of a Coulomb potential, the scattering by charged impurities and the properties of trionic plasmons.

A major breakthrough in our understanding of the optical response of doped semiconductor quantum wells (QWs) was achieved when the existence of trions was demonstrated experimentally both in II-VI and III-V semiconductors [1, 2, 3]. These composite particles appear as the weakly bound state of an exciton (a bound electron-hole pair) to an electron or a hole depending on the n-type or p-type doping of the QW. For the sake of clarity, in the following we will deal with negatively charged excitons (X^-). The physics of these composite charged particles in the low excitation regime has attracted a considerable interest, especially concerning the metal-insulator transition [2, 4], the intrinsic radiative recombination efficiency [5, 6], the singlet-triplet crossover under strong magnetic field[7], the role of phonons in the diffusion properties [8], and the remarkable drift transport induced by an applied electric bias[9, 10].

More recently, the first investigations of a dense trion gas have been performed by resonant optical pump-probe[11]. In principle, by optical pumping of the trion resonance, it should be possible to "convert" partially or even completely the background electron gas into a trion gas. Hence, it can be possible to study the many-body properties of a gas of composite charged particles. One interesting issue to be addressed is the screening response of such a peculiar gas to an external potential, such as the Coulomb field generated by a charged impurity. In this respect, the description of a trion gas as point particles (like electrons) should be only a long wave-length approach, while at shorter wave-length the trion granularity should show up. Since the impurity-induced scattering, the dc mobility and the collective excitations are sensitive to different wave-lengths, it is desirable to determine the complete response of a dense trion gas to external disturbances and to check whether and when the internal structure can be important. In this letter, we will present a Random Phase Approximation (RPA) treatment of the trion gas response. We will show the role of the composite nature of trions in the determination of the dielectric response.

To simplify our description, we shall present results for a purely two-dimensional system, omitting the form factors due to the finite extension of the quantum well wave-functions of electrons and holes along the QW growth direction. An X^- trion state is represented by a three-body wave-function, in which the center of mass, the relative motion and the spin part can be factorized, namely

$$\Psi_{\boldsymbol{k},\eta,S_e,S_{hz}} = \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{R}}}{\sqrt{A}} \;\; \phi_{\eta,S_e}(\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2)\; \chi_{S_e,S_{hz}}(s_{1z},s_{2z},s_{hz}), \tag{1}$$

where $\boldsymbol{k}$ and $\boldsymbol{R}$ are the center of mass wave-vector and position respectively, A is the sample area, η is the relative motion quantum number, while S_e and S_{hz} denote the spin state. The relative motion part depends on the variables $\boldsymbol{\lambda}_1 = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_h$ and $\boldsymbol{\lambda}_2 = \boldsymbol{\rho}_2 - \boldsymbol{\rho}_h$, where $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_2$, $\boldsymbol{\rho}_h$ are the in-plane positions of the first, second electron and the hole respectively, while s_{1z}, s_{2z}, s_{hz} are the spin components along the perpendicular direction z. In semiconductor quantum wells, heavy-hole trions have eight spin states, where $S_e \in \{0,1\}$ represents the total spin of the two electrons and $S_{hz} \in \{\pm 3/2\}$ is the heavy-hole band angular momentum projection. The two states with $S_e = 0$ are called singlet trions, while the six states with $S_e = 1$ are triplet trions. To fulfil the Pauli exclusion principle, for a singlet (triplet) spin state, the relative motion wave-function ϕ_{η,S_e} is symmetric (anti-symmetric) under exchange of the positions of the two electrons. The X^- charge density operator reads

$$\hat{n}(\boldsymbol{\rho}) = -e\left(\delta^{(2)}(\boldsymbol{\rho}-\boldsymbol{\rho}_1) + \delta^{(2)}(\boldsymbol{\rho}-\boldsymbol{\rho}_2) - \delta^{(2)}(\boldsymbol{\rho}-\boldsymbol{\rho}_h)\right). \tag{2}$$

The matrix elements of $\hat{n}(\boldsymbol{\rho})$ on the basis of the trion states are diagonal with respect to the spin indexes. They can be written as

$$\langle \mathbf{k}',\eta',S_e,S_{hz}|\hat{n}|\mathbf{k},\eta,S_e,S_{hz}\rangle = -\frac{e}{A}\; e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{\rho}}\; \mathcal{T}^{\eta',\eta,S_e}_{\boldsymbol{k}-\boldsymbol{k}'}, \tag{3}$$

where the trion "granularity" factor $\mathcal{T}^{\eta',\eta,S_e}_{\boldsymbol{q}}$ accounting for the composite nature is

$$\mathcal{T}^{\eta',\eta,S_e}_{\boldsymbol{q}} = \alpha^{\eta',\eta,S_e}_{e,\boldsymbol{q}} + \alpha^{\eta',\eta,S_e}_{e,-\boldsymbol{q}} - \alpha^{\eta',\eta,S_e}_{h,\boldsymbol{q}}, \tag{4}$$

where the electron contribution reads

$$\alpha_{e,\boldsymbol{q}}^{\eta',\eta,S_e} = \int d^2\boldsymbol{\lambda}_1 d^2\boldsymbol{\lambda}_2 \, \phi^\star_{\eta',S_e} \, \phi_{\eta,S_e} \, e^{i\boldsymbol{q}\cdot\left(\frac{m_e}{M}\boldsymbol{\lambda}_2 - \frac{m_e+m_h}{M}\boldsymbol{\lambda}_1\right)} \, , \tag{5}$$

and the hole part is

$$\alpha_{h,\boldsymbol{q}}^{\eta',\eta,S_e} = \int d^2\boldsymbol{\lambda}_1 d^2\boldsymbol{\lambda}_2 \, \phi^\star_{\eta',S_e} \, \phi_{\eta,S_e} \, e^{i\boldsymbol{q}\cdot\left(\frac{m_e}{M}(\boldsymbol{\lambda}_1+\boldsymbol{\lambda}_2)\right)} \, , \tag{6}$$

with m_e, m_h, $M = 2m_e + m_h$ the electron, hole and trion masses respectively. If an external potential $V_{\text{ext}}(\boldsymbol{\rho},t) = \left(\sum_{\boldsymbol{q}} \frac{1}{2} V_{\text{ext}}(\boldsymbol{q},\omega) e^{i(\boldsymbol{q}\cdot\boldsymbol{\rho}-\omega t)} + h.c.\right)$ acts on the system, then the trionic wave-functions are perturbed. The perturbation of the trion wave-function generates a non-homogeneous charge density, which creates a local Hartree potential and an exchange-correlation correction (we neglect it here). Hence, the screened potential is given by the self-consistent equation $V_{\text{s}} = V_{\text{ext}} + V_{\text{loc}}$, where $V_{\text{loc}}(\boldsymbol{\rho},t) = \int d^2\boldsymbol{\rho}' \frac{e}{\kappa|\boldsymbol{\rho}-\boldsymbol{\rho}'|} \, \delta n(\boldsymbol{\rho}',t)$, being $\delta n(\boldsymbol{\rho},t)$ the induced charge density and κ is the static dielectric constant of the semiconductor quantum well. The self-consistent potential energy felt by a trion is

$$U_{\text{s}}^{\text{tr}}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\boldsymbol{\rho}_3,t) = -e\left(V_{\text{s}}(\boldsymbol{\rho}_1,t) + V_{\text{s}}(\boldsymbol{\rho}_2,t) - V_{\text{s}}(\boldsymbol{\rho}_h,t)\right) \, . \tag{7}$$

The matrix elements of the perturbation energy on the trion states at $t = 0$ are

$$\langle \mathbf{k}',\eta',S_e,S_{hz}|U_{\text{s}}^{\text{tr}}|\mathbf{k},\eta,S_e,S_{hz}\rangle = -e\,V_{\text{s}}(\boldsymbol{k}-\boldsymbol{k}',\omega)\,\mathcal{T}_{\boldsymbol{k}-\boldsymbol{k}'}^{\eta',\eta,S_e} \, . \tag{8}$$

By calculating the lowest-order perturbation theory for the trion wave-functions and summing incoherently the contribution from all the populated trion states (RPA), we get the the screened potential $V_{\text{s}}(\boldsymbol{q},\omega) = V_{\text{ext}}(\boldsymbol{q},\omega)/\epsilon_{X^-}(\boldsymbol{q},\omega)$, where the trion-induced dielectric function reads

$$\epsilon_{X^-}(\boldsymbol{q},\omega) = 1 - \frac{2\pi e^2}{\kappa q A}\Pi_{X^-}(\boldsymbol{q},\omega) \, . \tag{9}$$

The trion RPA-polarization contribution is

$$\Pi_{X^-}(\boldsymbol{q},\omega) = \sum \frac{(f_{\boldsymbol{k}+\boldsymbol{q},\eta',S_e,S_{hz}} - f_{\boldsymbol{k},\eta,S_e,S_{hz}})|\mathcal{T}_{\boldsymbol{q}}^{\eta,\eta',S_e}|^2}{E_{\boldsymbol{k}+\boldsymbol{q},\eta',S_e} - E_{\boldsymbol{k},\eta,S_e} - \hbar\omega - i0^+} \, , \tag{10}$$

where the sum is meant over $\boldsymbol{k}$, η, η', S_e, S_{hz}. The quantity $f_{\boldsymbol{k},\eta,S_e,S_{hz}}$ is the trion occupation number (not necessarily at equilibrium) and $E_{\boldsymbol{k},\eta,S_e}$ the orbital energy of the unperturbed trion state. Note that for a (spin-unpolarized) electron gas

$$\Pi_{e^-}(\boldsymbol{q},\omega) = 2\sum_{\boldsymbol{k}} \frac{f^{(e)}_{\boldsymbol{k}+\boldsymbol{q}} - f^{(e)}_{\boldsymbol{k}}}{\frac{\hbar^2|\boldsymbol{k}+\boldsymbol{q}|^2}{2m_e} - \frac{\hbar^2 k^2}{2m_e} - \hbar\omega - i0^+} \tag{11}$$

and $\epsilon_{e^-}(\boldsymbol{q},\omega) = 1 - \frac{2\pi e^2}{\kappa q A}\Pi_{e^-}(\boldsymbol{q},\omega)$. In presence of a mixed gas of trions and electrons, the linear susceptibilities of the two components add up, i.e. the polarization of the mixture is $\Pi_{mix}(\boldsymbol{q},\omega) = \Pi_{X^-}(\boldsymbol{q},\omega) + \Pi_{e^-}(\boldsymbol{q},\omega)$. Considering only the contribution of the ground trion state, the general expression in Eq. (10) can be considerably simplified. This approximation holds when only the ground state is populated and in the limit of small $\hbar\omega$. In the case of zero magnetic field, the ground state is a singlet 1s-like state, which is twice degenerate (due to the hole spin degree of freedom). By considering only the contribution of the 1s-state, we get

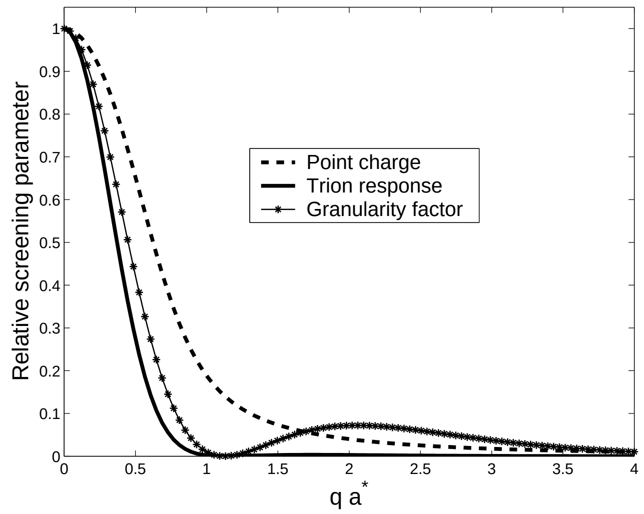

FIG. 1: Solid line: normalized static screening wave-vector $q_{\text{s}}(q,0)/q_{\text{s}}(0,0)$ versus dimensionless wave-vector qa^* for a pure gas of trions. Dashed line: same quantity, but without accounting for the trion granularity. Stars: the granularity factor $|\mathcal{T}(q)|^2 = |2\alpha_e(q) - \alpha_h(q)|^2$. Parameters: $T = 10$ K, $a^* = 8$ nm, $m_e = 0.1\ m_0$, $m_h = 0.2\ m_0$, $n_{\text{trions}} = 1 \times 10^{10}\text{cm}^{-2}$, $\kappa = 9$.

$$\Pi_{X^-}(\boldsymbol{q},\omega) \simeq 2\sum_{\boldsymbol{k}} \frac{(f_{\boldsymbol{k}+\boldsymbol{q},1s} - f_{\boldsymbol{k},1s})|\mathcal{T}(q)|^2}{\frac{\hbar^2|\boldsymbol{k}+\boldsymbol{q}|^2}{2M} - \frac{\hbar^2 k^2}{2M} - \hbar\omega - i0^+} \, . \tag{12}$$

In the following, we will consider the following trial function

$$\phi_{1s}(\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2) = \frac{1}{2\pi(a^\star)^2}\exp\left(-\frac{\lambda_1+\lambda_2}{2a^\star}\right) , \tag{13}$$

where $a^\star$ is the effective trion radius. Accurate numerical solutions for the internal motion of quantum well trions are reported in the literature (see e.g. Refs. 5, 13). With our model wave-function, the granularity factor is $\mathcal{T}(q) = 2\alpha_e(q) - \alpha_h(q)$ with

$$\alpha_e(q) = \frac{1}{\left\{\left[1+\left(\frac{m_e+m_h}{M}\ qa^\star\right)^2\right]\left[1+\left(\frac{m_e}{M}\ qa^\star\right)^2\right]\right\}^{3/2}} \tag{14}$$

and

$$\alpha_h(q) = \frac{1}{\left\{\left[1+\left(\frac{m_e}{M}\ qa^\star\right)^2\right]\right\}^3} \, . \tag{15}$$

In the long wave-length limit ($q \to 0$), $|\mathcal{T}(q)|^2 \to 1$, i.e. the granularity factor does not play any role. Hence, for small wave-vectors q, X^- trions behave as point particles with charge $-e$ and mass $M = 2m_e + m_h$, as expected. As shown in Fig. 1, the granularity factor decreases monotonically with increasing q and then has a node for the wave-vector $\bar{q}$ such that $2\alpha_e(\bar{q}) = \alpha_h(\bar{q})$. This node of the granularity factor is due to the compensation between the contributions of two electrons and the hole within the bound X^-. This kind of cancellation effect is absent in a plasma of uncorrelated electrons and holes, where the screening contributions of the two components add up. Note that the value of $\bar{q}a^\star$ depends on the mass ratio m_e/m_h and on the shape of the internal motion wave-function (for our model wave-function $\bar{q}a^\star \approx 1.1$ when $m_e/m_h = 0.5$). Finally, in the short wave-length limit ($q >> 1/a^\star$), the granularity asymptotically vanishes, because the perturbing potential oscillates too quickly in the length scale of the trion internal motion wave-function.

As a first illustrative example of our theory, we consider the trion response to a Coulomb potential induced by a charged impurity, that is $V_{\rm ext}(q) = 2\pi e \exp(-qd)/(\kappa A q)$, where d is the distance between the remote impurity donor and the quantum well plane ($d \neq 0$ for a modulation-doped sample). The screened potential can be written as

$$V_{\rm s}(q,0) = \frac{2\pi e \exp(-qd)}{\kappa A (q + q_{\rm s}(q,0))} \ , \qquad (16)$$

being $q_{\rm s}(q,0) = (\epsilon_{\rm mix}(q,0) - 1)q$ and $\epsilon_{\rm mix}$ the dielectric function for the mixed gas of trions and electrons. In Fig. 1, we show the relative screening parameter $q_{\rm s}(q,0)/q_{\rm s}(0,0)$ versus q (in units of the trion effective radius $a^\star$) for a pure trion gas, which can be obtained by resonant optical pumping. For simplicity, we have taken an equilibrium Fermi-Dirac distribution function with temperature $T = 10K$ and with density $n_{\rm tr} = 10^{10}{\rm cm}^{-2}$. We have used typical parameters for a CdTe-based semiconductor[1], namely $m_e = 0.1\ m_0$, $m_h = 0.2\ m_0$, $\kappa = 9$ (working in the CGS system) and a trion radius $a^\star = 8$ nm. The thick solid line represents $q_{\rm s}(q,0)/q_{\rm s}(0,0)$ due to the trion gas, while the dashed line shows the same quantity without including the granularity factor $|\mathcal{T}(q)|^2$. Indeed, the composite nature of trions has a major impact in their screening response, dramatically quenching the response at finite wave-vectors.

One important issue to verify is the behavior of the static screening in presence of a mixed population of trions and electrons. In Fig. 2, we show a surface plot of $q_{\rm s}(q,0)a^\star$ as a function of the trion density fraction f, which is defined by the relation $n_{\rm tr} = f(n_e + n_{\rm tr}) = f n_{\rm tot}$. Passing from a pure electron gas ($f = 0$) to a pure trion gas ($f = 1$), the static screening changes considerably. For $q \to 0$, the screening wave-vector increases for increasing trion fraction due to the heavier

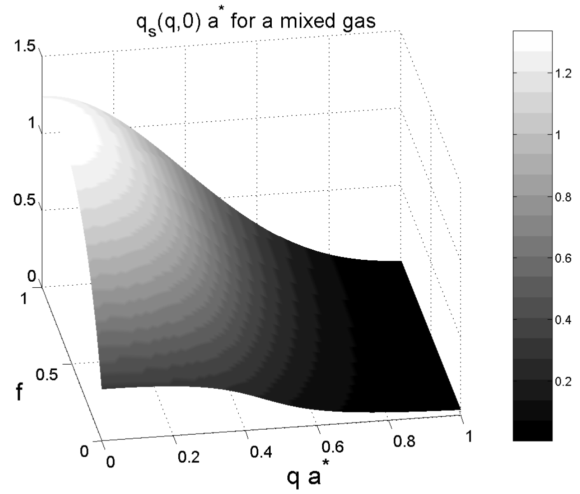

FIG. 2: Contours of the screening wave-vector $q_{\rm s}(q,0)$ (units of $1/a^\star$) as a function of qa^* and the trion density fraction f ($f = 1$ corresponds to a pure trion gas, $f = 0$ is for a pure electron gas). Parameters: total density (electrons + trions) $n_{tot} = 10^{10}{\rm cm}^{-2}$. Other parameters as in Fig. 1.

mass of the trion. In fact, in the low temperature limit $q_s(0,0) \propto m_e$ for a pure 2D electron gas[12] and therefore $q_s(0,0) \propto (2m_e + m_h)$ for a pure 2D trion gas. Fig. 2 shows that even for small trion fractions, the screening of the mixture is dominated by the trion component. Hence, the trion granularity effects at finite wave-vectors shown in Fig. 1 are important also in the mixture case.

The scattering induced by charged impurities is the main interaction process affecting the transport properties of charge carriers at low temperatures. Within the Fermi's golden rule, the wave-vector dependent lifetime of trions $\tau_{\rm tr}(k)$ in presence of the mixed gas of trions and electrons is given by the expression

$$\frac{1}{\tau_{\rm tr}(k)} = \frac{N_{\rm imp}}{A}\left(\frac{2\pi M e^4}{\kappa^2 \hbar^3}\right)\int_0^{2\pi} d\theta \frac{\exp(-2dq)}{(q\epsilon_{\rm mix}(q,0))^2} \ , \qquad (17)$$

with $q = 2k\ |\sin(\theta/2)|$ and $N_{\rm imp}/A$ is the density of impurities per unit area. Notice that the electron lifetime is $\tau_{\rm e}(k) = (M/m_e)\tau_{\rm tr}(k)$. The velocity lifetime $\tau^{\rm v}_{\rm tr}(k)$ is given by same expression in Eq. (17), but with an additional factor $(1 - \cos\theta)$ within the integral. In Fig. 3, we plot the scattering integral $I_{\rm scatt}(k) = \int_0^{2\pi} d\theta \exp(-2dq)/(q\epsilon_{\rm mix}(q,0))^2$ (in units of a^{*2}). In the case of a pure trion gas ($f = 1$, thick solid line), the k-dependence of the scattering integral has a resonant structure. The peak is due to the node of the granularity factor $|\mathcal{T}(q)|^2$ (see Fig. 1). Indeed, around the nodal wave-vector, the trion screening is dramatically quenched and the scattering efficiency enhanced. This effect is absent if the granularity of the trion is not taken into account (thick dashed line) and in the case of a pure electron gas (thin solid line). Note that in the case of a

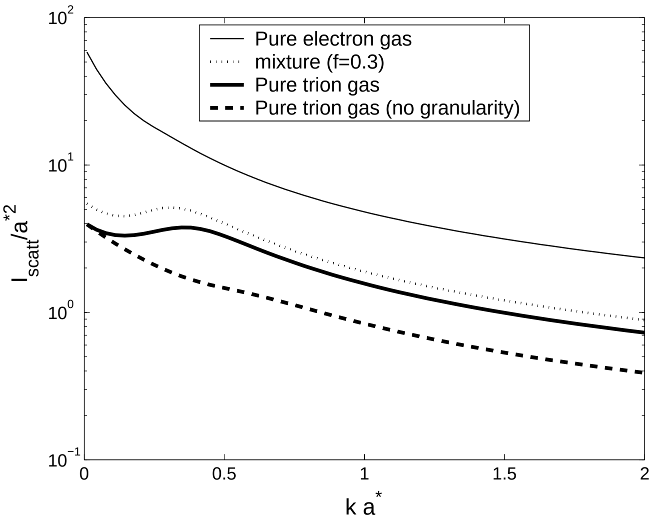

FIG. 3: Impurity scattering rate integral $I_{\rm int}$ (units of $a^{\star\ 2}$) as a function of qa^*. Thick solid line: pure trion gas ($f = 1$). Thick dashed line: the same, without granularity. Dotted line: mixture of trions and electrons ($f = 0.3$) . Thin solid line: pure electron gas ($f = 0$). Distance between impurity and carrier planes: $d = 4$nm. Other parameters as in Fig. 2.

mixture of trions and electrons, the contribution of trions is dominant, as shown by the dotted line, corresponding to a mixture where the trion fraction is only 30%. The results for the velocity lifetimes (not shown) are qualitatively analogous.

Similarly to the case of electrons, a pure gas of trions has longitudinal collective excitations (trionic plasmons), which are given by the solution of the equation $\epsilon_{X^-}(q, \omega(q)) = 0$. In the long wave-length limit, the trionic plasmons behave exactly as the electronic plasmons [12], once the electron mass m_e is replaced by the trion mass $M = 2m_e + m_h$. For finite wave-vectors, as a result of the trion granularity, the plasmon frequency is decreased, due to the quenching of the screening efficiency. For a finite wave-vector q, when only the trion ground state is involved, the behavior is that of an electron gas, once the electron charge e is replaced by the effective charge $e'(q) = e|\mathcal{T}(q)| < e$, as it can be deduced from Eqs. (9) and (12). For excitation energies of the order of the trion binding energy, the excited states will give extra plasmon branches corresponding to transition between internal states. For a mixed gas of trions and electrons, the plasmon branches are the solutions of $\epsilon_{mix}(q, \omega(q)) = \epsilon_{X^-}(q, \omega(q)) + \epsilon_{e^-}(q, \omega(q)) - 1 = 0$. Hence, a coupling is present between the trionic and the electronic branches of plasmons. The complex phenomenology of the collective excitations in presence of a mixture of electrons and trions will be addressed in detail in a future publication.

In conclusion, we have investigated the screening response of trions, taking into account their composite nature. We have obtained a RPA-dielectric function for a mixed gas of trions and electrons. This interesting physical regime is experimentally achievable by resonant optical pumping of trions in doped QWs. In the static regime, we have shown the major impact of the trion granularity in determining the dielectric response at finite wavevectors. We have calculated the scattering rates of electrons and trions due to the interaction with the charged impurities. The internal motion of trions is responsible for a quenching of the screening response for wavelengths comparable or smaller than the trion effective radius. Moreover, the granularity produces resonant features in the wave-vector dependence, due to the compensation which can occur between the contribution of the two electrons and the hole within the X^-. We hope that our study will stimulate the research in the fundamental properties of trions in the dense regime. We expect that the effect of trion granularity in the screening response will have an impact on the transport properties of these charged particles, whose investigation is underway [9, 10]. In particular, the impurity-induced localization and the onset of the metal-insulator transition[4] should be considerably modified by the conversion of electrons into trions through resonant optical pumping.

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