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What should an unmanaged earnings distribution look like?

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Abstract

In asserting that the number of firms reporting small profits is abnormally high, thus suggesting that earnings management has taken place, accounting researchers assume that the distribution of reported earnings should be smooth for unmanaged earnings. This has never in fact been demonstrated.

This article seeks to confirm this assumption through a laboratory experiment, and also sets out to identify the general distribution pattern to be expected for unmanaged earnings. Normal distribution does not appear to be a good fit. The study’s results also highlight the existence of downward management of earnings by firms with higher-than-average profits.

Keywords
Earnings management, earnings distribution, experiment, accounting threshold.
Introduction

Since the late 1990s, one stream of accounting literature has focused on the distribution of earnings reported by firms. It is also referred to as the literature on accounting thresholds, or earnings management to meet an objective. Researchers estimate that every year the number of firms reporting small profits is abnormally high, and the number of firms reporting small losses is abnormally low. This suggests that a significant number of firms manage their accounts every year to avoid reporting a loss (Hayn, 1995; Burgstahler and Dichev, 1997; Degeorge, Patel and Zeckhauser, 1999). All the studies carried out so far appear to confirm this.

In reaching this conclusion, this stream of research has relied on a fundamental premise concerning earnings distribution patterns: if earnings are not managed, the distribution pattern should be smooth (Burgstahler and Dichev, 1997). This basic premise has never yet been demonstrated, as observation of unmanaged earnings is in practice impossible. The only earnings that can be observed are the earnings actually reported, which may have been managed. The “real” earnings cannot be found in any database. The smooth distribution premise therefore results from deductive reasoning (Vidal, 2008). Researchers can find no explanation why distributions of earnings, if unmanaged, display discontinuities, especially around the zero profit mark.

This paper presents a study which proposes to demonstrate this assumption empirically for the first time. It uses an experiment-based approach in which business game participants simulate the management of a fictitious firm competing with other firms. They make management decisions, but can under no circumstances interfere with the accounts. Accounting manipulation is a decision made with the sole objective of modifying the earnings after they have been calculated or estimated. In the experimental protocol, earnings are calculated absolutely impartially by a software, using the same rules for all firms. One thousand six hundred and twenty annual earnings figures are analyzed in this way. The first finding of this study confirms the fundamental premise and thus supports the assumption that earnings are managed to avoid reporting losses. Secondary findings concerning the distribution pattern show that the Normal distribution does not appear to be a good fit for earnings distributions.

Part 1: the distribution pattern of unmanaged earnings

This section discusses the pattern that unmanaged earnings distributions should follow. It reviews the relatively small body of literature on the subject, leading to formulation of the study’s two research questions.

The literature on accounting thresholds examines distribution irregularities. Discussion of irregularities in an earnings distribution requires conscious or unconscious reference to a benchmark distribution considered “regular”\(^\text{20}\). Comparison of an empirical distribution with the expected distribution may reveal differences that are interpreted as irregularities. However, it is impossible to observe distributions of unmanaged earnings. The benchmark distribution pattern is therefore unknown. Researchers have adopted a prudent approach in addressing this problem, leading to non-parametric measures of the irregularities, although some authors still refer more or less explicitly to a Gaussian distribution pattern. A few authors have attempted to introduce parametric measurement approaches, making explicit reference to a specific distribution (Vidal, 2009).

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\(^{20}\) Also known as the “expected distribution” or “theoretical distribution”.

\(^{21}\) Or “real” or “observed” distribution.
1.1. Studies not based on a specific distribution law

The first approach used to examine irregularities in the reported earnings distributions is very much the dominant approach in the accounting literature (Burgstahler et Dichev, 1997; Degeorge, Patel et Zeckhauser, 1999; Brown, 2001; Burgstahler et Eames, 2003; Das et Zhang, 2003; Dechow, Richardson et Tuna, 2003; Holland et Ramsey, 2003; Leuz, Nanda et Wysocki, 2003; Brown et Caylor, 2005; Coppsens et Peek, 2005; Burgstahler, Hail et Leuz, 2006; Daske, Gebhardt et McLeay, 2006; Roychowdhury, 2006; Beaver, McNichols et Nelson, 2007). In this approach, known as “non-parametric”, the parameters of the overall distribution law are unknown. However, whatever law applies, the researchers assume that if earnings have not been managed the distribution should be “smooth”.

This approach calls for several comments:

- It is easy to implement because it uses very few mathematical tools. Irregularities are measured without reference to the distribution parameters, using local estimations based on the observed numbers in the classes surrounding the interval studied.
- It is robust, for the underlying conditions involve very few restrictions. Only discontinuities (the “peaks” or “breaks”) in the distribution are taken into consideration. A “strange-looking” distribution pattern will not be considered irregular as long as it remains “smooth”. For example, a uniform earnings distribution pattern (a totally flat line) would be considered regular. The notion of irregularity is restricted to discontinuities.

Non-parametric measures have the advantages associated with their drawbacks: they are straightforward and robust, but nonetheless imprecise (Chen, Lin, Wang et Wu, 2005; Vidal, 2008). The reason the non-parametric approach is the most commonly used is that researchers are prudent. As they are unable to observe the distributions of unmanaged earnings, they do not take the risk of inventing the mathematical parameters of those distributions.

1.2. Are the distributions Gaussian?

Although researchers generally avoid the risk of explicitly proposing a distribution law for corporate earnings, several studies are based on an implicit assumption: distribution of earnings should follow a Gaussian pattern, i.e. Normal distribution. This is particularly true of Burgstahler and Dichev (1997) who measure irregularities by symmetry. Measuring irregularities by symmetry requires a hypothesis regarding at least one of the parameters of the distribution. Mard (2004) goes further,

![Figure 3: Example of smooth irregular distribution](image-url)
adjusting the estimations by explicit reference to a Normal distribution. Yet paradoxically, in both these cases, the authors highlight the asymmetrical nature of the empirical distributions observed. Jeanjean (1999) writes “Theoretically, in a sufficiently large sample, scaled earnings distribution should be normal”. These references to Normal distribution result from the frequent use of this law to reflect economic phenomena. They assume that corporate earnings are (a) data with random distribution and/or (b) that as there are large numbers of them in databases, the central limit theorem can be applied.

(a) Corporate earnings are not random data. They depend on the firm’s actual business activity and its managers’ strategic decisions, which may vary in suitability. Business activity generates returns that are not random\(^\text{22}\). Independently of the risk factor that is omnipresent in almost all decisions, to rephrase Einstein, “businesses do not play dice”. Every year, the earnings distribution of a population of firms will therefore present characteristics that cannot be assumed in advance.

(b) The central limit theorem teaches that when a large number of drawings of a variable following the same law are aggregated, the aggregate distribution approaches Normal distribution. To attain such a normal distribution, a large number of drawings need to be aggregated - in other words, we need to examine the distribution of corporate earnings in a given country over several hundred years. This is currently impossible, as no such data are available. In fact it will always be impossible, since the components of the population studied, and the economic context, undergo changes on such a scale over such a long period that we cannot accept the idea that the distribution of earnings will remain unchanged over the very long term. In other words, the central limit theorem cannot apply here. Whether or not they are aggregated over several years, the distribution law for earnings is unknown.

### 1.3. Studies based on an explicit distribution law

Chen, Lin, Wang and Wu (2005) paper stand out for its attempt to measure irregularities in distribution by using a mathematical law for earnings distribution. This paper posit (without explanatory arguments) that earnings should follow a mixed normal distribution. The most interesting aspect of these parametric approaches deserves emphasis: if the distribution law is known, then “parametric” measures can be introduced for irregularities. In other words, irregularities are measured by calculating the surface separating the expected distribution from the observed distribution. These measures are more precise than non-parametric measures, and make it possible to assess the total number of firms in an irregular position and the amounts that have been “managed”. These advantages, however, come with associated drawbacks: they depend on the relevant distribution law, which is as yet unknown.

### 1.4. Research questions

Ultimately, a dual research question is addressed. Observing the distribution of unmanaged earnings, the study has two objectives:

1. It seeks to verify the premise that an unmanaged earnings distribution is smooth (without discontinuities). If supported, this initial point will confirm the relevance of all the research on accounting thresholds since 1995.

2. It seeks to identify the mathematical distribution law for earnings reported by firms. This second point opens up the field of research to parametric methodologies for measuring irregularities.

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\(^{22}\) Even if earnings were random, they would not necessarily follow a Normal distribution.
Before presenting the results of our experimental research, it is useful to observe the distributions of earnings as reported (and therefore potentially manipulated) by firms, and review the specificities of these distributions. To identify and discuss any irregularities (other than local discontinuities), a benchmark pattern is needed. Since the relevant theoretical distribution is unknown, it is presumably impossible to identify irregularities. To get around this problem, we work on the *reductio ad absurdum* assumption that a Normal distribution pattern should apply. The comparison between the actual distribution and the normal distribution identifies zones of potential irregularities which are discussed.

2.1. Differences between observed distributions and Normal distribution
Most studies on earnings distributions observe the earnings variable scaled by a size variable such as total assets. However, it is of some value to begin by first observing the distribution pattern for unmanaged earnings.

2.1.1. Unscaled earnings distribution
The graphs below (see Figure 2) show the earnings distribution for French listed companies from 1992 to 2004 as reported in the Compustat Global Vantage database.

![Figure 2: Distribution of earnings reported by French firms](image)

Earnings reported by French listed firms from 1992 to 1994
(as stated in the Compustat Global Vantage database)
(Central portion of the distribution)

(Tails of the distribution)
Comparison of the distribution for the earnings variable with a Normal distribution having the same mean and standard deviation shows the following differences:

- The empirical distribution is dissymmetrical, whereas the Normal distribution is symmetrical. The empirical distribution has thicker distribution tails (particularly on the left) than the Normal distribution. Finally, the mode for the empirical distribution lies not on the mean nor the median, but on the zero earnings mark.

• The empirical distribution is more highly concentrated (leptokurtic) than the Normal distribution. Rather than being bell-shaped, it is shaped like an upturned funnel. There is a upward phase then a downward phase, but no “flat” summit.

2.1.2. Scaled earnings distribution

The earnings of French listed firms from 1992 to 1994 have been scaled by the “total assets” variable. The distribution is shown below (Figure 4).

**Figure 3: Distribution of earnings scaled by Total Assets**

![Figure 3: Distribution of earnings scaled by Total Assets](image)

Except for the mode, which no longer lies around zero but around the mean (mean and median are practically the same), the differences observed in the unscaled earnings distribution are also observed in the scaled earnings distribution.

2.2. Discussion of the observed differences

2.2.1. Dissymmetry

Reported earnings follow a dissymmetrical distribution. The dissymmetry observed in France is also noted by Burgstahler and Dichev (1997) in 76,000 observations for US firm earnings. More specifically, three sources of dissymmetry are observed: large losses, a thickness in the distribution to the left, and smaller numbers of firms to the left of the mean.

(a) Large losses

There is no economic limit to a loss, but there is a limit to earnings increases. The effect of competition in a market economy means that a competitive advantage cannot generate economic rents indefinitely. Competitors will try to copy the resources that generate exceptional rents. Generating income necessarily requires a resource-consuming activity which itself generates charges. Value cannot be created out of nothing.

However, this constraint applicable to an increase does not exist for a downward trend. It is technically possible to incur expenses indefinitely without generating income, and this automatically leads to infinite losses. The market constraint will make such a firm disappear rapidly, but for a few years (at least one), the firm may report very significant losses. In general, a firm caught up in a spiral of deficit can
make extremely large losses. We cannot thus consider high losses as a distribution irregularity due solely to earnings management practices. (b) Thickness of the distribution on the left The conservatism principle may explain this dissymmetry. For Givoly and Hayn (2002), the broad tail in the distribution reflects accounting distrust (the conservatism principle) of high-risk firms, whereas the low spread to the right results from a reluctance to translate good performance into accounting terms. Even so, can the dissymmetry be attributed to accounting conservatism alone? If all firms are subject to the conservatism principle in the same way, we would expect to see a shift towards the left hand side of the earnings distribution curve, but not necessarily any dissymmetry. But firms are not necessarily faced with risk in similar ways, and firms with worse results than their peers have to cope more with unfavorable events. Therefore, these firms (whose earnings are below the median) are more likely to suffer the consequences of the conservatism principle and their earnings are weighed down more by provisions. This may explain why the left hand side of the curve shows a steeper slope and spreads out further to the left. The conservatism principle can therefore legitimately explain a dissymmetry that apparently cannot be considered solely as an expression of accounting manipulation. (c) The low numbers “just to the left” of the mean As we have just seen, the steeper slope to the left of the mean can be explained by the conservatism principle. Also, considering that firms manage a portfolio of activities, they are likely to discontinue their least profitable activities first, in order to concentrate their efforts and resources on the most profitable activities. In other words, even if accounting conservatism alone does not fully explain the dissymmetry, a process of economic adjustment leads firms to leave the left hand part of the earnings curve by managing their portfolio of activities in such a way as to approach the central part of the distribution curve. The theory of effort (Dechow, Richardson and Tuna, 2003) supports this explanation. All this suggests there is may be nothing abnormal about observing dissymmetry. 2.2.2. High concentration The earnings show a “pointed” distribution pattern typical of high concentration. This concentration is located around the median and/or the mean when the variable is scaled, but when the variable is unscaled it lies around the zero earnings point. This observation raises questions as to the economic nature of the phenomenon observed: the earnings. (d) Concentration around the peak If a Normal distribution is assumed, high concentration must be considered as an irregularity. In other words, firms manage their accounts so they can report earnings that are close to the mean (median). The annual mean (median) of reported earnings could thus be interpreted as a new threshold that has not yet been considered in the literature. Explanations for this hypothesis can be imagined: publishing close-to-average earnings enables the firm to convey a picture of relative health, and it is not impossible that reporting results that are lower than competitors’ earnings involves a cost. This could encourage managers to aim for that level. But the concentration phenomenon could also be explained by the non-random nature of earnings distribution. (1) There is a certain inertia in profit levels. A firm that is highly profitable one year is very unlikely to make a significant loss the following year. It will go through increasingly difficult years before reaching a disastrous situation. (2) Also, in a competitive economic universe, firms are under market law pressure that tends to homogenize their performances. (3) The theory of effort (Dechow, Richardson et Tuna, 2003) can also provide an explanation for the concentration.
It is thus natural to see high concentration around the mean in earnings distributions, and this phenomenon cannot be attributed solely to accounting manipulation. 

(e) Concentration around the zero earnings mark
In the unscaled earnings distribution (Figure 3), there is high concentration around zero. This is largely explained by the sample composition. The sample contains many small firms and few large firms. The small firms are achieving results commensurate with their investments, whether they are losses or profits. It may appear normal that small unscaled earnings should be concentrated around zero. However, there is a break in the distribution between slightly negative earnings and zero earnings, and this discontinuity cannot be explained by the sample composition. Small size should have the same effect on both the negative and positive side, but here the concentration is not symmetrical. In short, the size effect makes it difficult to interpret the unscaled earnings distribution - but this does not justify the high concentration of exclusively positive earnings above zero.

2.3. Summary
In the first section we saw that it is impossible to apply the central limit theorem in order to posit a Gaussian distribution pattern for earnings. Observation of published results shows that every year, earnings distributions have recurring characteristics (upturned funnel curve, dissymmetry, and concentration). These characteristics cannot apparently be attributed solely to accounting manipulations. For all these reasons, aggregate earnings distributions cannot be considered to follow a Normal distribution pattern. Until it becomes possible to observe a distribution of unmanaged earnings, accounting researchers will be obliged to advance with great caution.

Part 3: the experimental protocol

In an ideal business world, managers take their management decisions, and accountants translate the effects of those decisions on the life of the firm into the financial statements with total impartiality. In reality, managers, who oversee publication of the accounts, may have motives for orienting the information disclosed to the public (Healy and Whalen, 1999; Breton and Stolowy, 2003). In such cases there is a presumption of earnings management. More specifically, accounting manipulation is considered to have taken place when there is a deliberate intent not to report the “real” earnings calculated or estimated impartially under accounting rules, but to release a different earnings figure justified by management through the selected accounting options (provisions, useful lives etc) or management decisions that they would not have made in normal circumstances (delaying or advancing investments, granting or refusing extended payment terms, etc).

In a business game that simulates running an organization, it is possible to restrict participants to management decisions alone, in total isolation from the production of financial statements. But to arrive at a statistically relevant earnings distribution, the experimental protocol requires a very large number of simulations. The research in this paper is based on a timely opportunity: from the archives of an introduction to management course involving the use of business games, it is possible to establish the distribution of 1,641 unmanaged annual earnings figures.

3.1. Presentation of the business game
The Win-Firme business game, a teaching software developed in 1995, is used in some fifty schools and universities, mostly in France\(^2\). The way it works is typical of this kind of business game software. Participants are put into teams. Each team

\(^2\) The website [www.winfirme.com](http://www.winfirme.com) describes the software and how it works.
runs a fictitious firm that manufactures and sells products. Participants take operational decisions (product manufacturing quantities, sale price, communication and quality budgets, staff hiring and pay, and the research and development budget for marketing new products). These decisions are entered into a computer, and the software compares them with competing firms’ decisions and allocates consumers according to predefined parameters.

Two aspects of the game should be highlighted. The first is that this simulation excludes chance. There is no random event. Yet it remains impossible to predict the future, since competitors’ decisions are unknown. The second important aspect is that sales are allocated essentially on the basis of relative decisions by the different firms. There is no right or wrong decision, because the algorithm is not looking for a standard solution. Therefore, whatever the demand parameters are, firms adjust their budgets and prices to generate profits. In other words, firm profitability does not depend on the game parameters but on the coherence of the team's decisions compared to its competitors. Consequently, whatever the scenario, the earnings distribution cannot be predicted.

43 games were scrutinized to develop the database used for this study (see Table 1). A game lasts an average 5 to 6 rounds (and a round generally lasts 2 hours), corresponding to 2 to 4-day business game seminars. The number of competing teams generally ranges from 5 to 8, and seminars are attended by thirty to forty students. In all, 1,620 annual earnings were entered to form the database. The size variables (total assets and sales) were also entered. Total assets (sales) amounted to zero in six (four) cases. After eliminating these data from the base, the scaled results provide 1,614 (1,616) observations.

<table>
<thead>
<tr>
<th>Number of firms</th>
<th>Game duration</th>
<th>Number of games</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>6.5</td>
<td>5.7 rounds</td>
<td>43</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>9</td>
<td>9 rounds</td>
<td></td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>4</td>
<td>3 rounds</td>
<td></td>
</tr>
</tbody>
</table>

The 43 game seminars contributing to the database took place in 23 different schools and universities between 2004 and 2009. Participants are students, in both scientific fields (mostly first-year students at engineering schools) and economic disciplines (management and/or economics degree students). 5 seminars were attended by students with more diverse backgrounds (adults on in-service training, students from other course types) as part of their masters qualifications in research or vocational subjects. While all participants were students when they attended the seminar, the mixed range of profiles avoids the bias of an over-homogeneous population that is frequent in experimental protocols.
Table 2: Characteristics of experiment participants

<table>
<thead>
<tr>
<th></th>
<th>Science students</th>
<th>Economics and/or management students</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 years’ higher education</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>3 years’ higher education/Degree level</td>
<td>7</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>4 years’ higher education/Master 1</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5 years’ higher education/Master 2</td>
<td>7</td>
<td>12</td>
<td>19</td>
</tr>
<tr>
<td>Other</td>
<td>18</td>
<td>24</td>
<td>1</td>
</tr>
</tbody>
</table>

In all, nearly 1,200 participants contributed to data collection. Excluding the time needed for data recovery and entry, the duration of the experimental protocol can be estimated at 800 hours.

3.2. The advantage of experiments

Using laboratory-type experiments to study earnings distributions offers an enormous advantage: the “reported” earnings in the game are calculated by the software, without the participants (managers) being able to take any accounting decision. The software plays the role of a totally independent accountant. In the business game, earnings are not managed because no accounting choices are included in the game. The same rules apply for all firms. The accounts are a purely technical result of the firms’ business decisions. However, accounting options are not the only lever for earnings management. Managers may also use the timing of certain investment decisions to achieve an earnings objective. Degeorge, Patel and Zeckhauser (1999) use the term “direct managing”, Glaum, Lichtblau and Lindemann (2004) talk of “manipulation of cash flows”, Roychowdhury (2006) refers to “real activities manipulation”, and Burgstahler, Hail and Leuz (2006) refer to “Business Management”. Graham, Harvey and Rajgopal (2005) show that these practices are in fact dominant in business. In a business game, managers are arguably able to influence earnings through their R&D, communication and quality budgets. But the game is a simplified model of reality, and time is divided into rounds. Decisions are taken irrevocably at the start of the round. In other words, it is as if the decisions for year N were taken in the night of December 31, N-1 to January 1, N. This makes it impossible to adjust budgets during the year when the first earnings estimates are calculated. Manipulation of cash flows to achieve an earnings objective is thus impossible. For all these reasons, use of simulations to study earnings management is particularly judicious.

3.3. The limitations of experiments

While an experiment can isolate the accounting process from attempts at opportunistic earnings management, it diverges from reality in that it is based on economic modeling, and furthermore is implemented in a teaching context. These two factors may lead to divergences.
between the earnings distribution resulting from the game and the theoretical distribution of actual earnings. They are reviewed below to examine how far they can be controlled for.

### 3.3.1. Economic modeling

In a simulation, the participants are players whose decisions are risk-free, as they will not actually experience the effects (on their pay or their career) simulated in the game. This may lead them to adopt different behaviors from those seen in real life. This bias is limited by the fact that the software makes no assumption about performance, and the duration of each seminar obliges the “firms” to introduce long-term strategies. If firms reduce their price in the face of higher pressure from competitors, the average firm performances will be lower and firms will have to take the initiative of adjusting their strategy to improve their lot. This can come about through a downturn in competitive pressure or disappearance of the weakest competitors. In other words, if the competitors take greater risks, the effects on results should ultimately lead to compensating adjustments by participants, since a balance is reached without any intervention by the facilitator, or alteration of the software settings.

Other possible phenomena are radical optimization strategies at the end of the game, or under-assertive decisions at the start of the game when participants have not yet assimilated the way the game works. To control for this bias, the earnings distribution is also traced after eliminating the first and last year of the game from the database.

### 3.3.2. Teaching context

The simulations used to construct this study were originally developed for an educational purpose. It is interesting to examine the possible biases associated with this purpose. First of all, the scenarios are generally constructed such that firms are identical at the start of the game. Their markets generally grow in the first few rounds, so that students are not under too much pressure in the learning phase of the game. Generally, the industries simulate a maturity phase from the third or fourth round, which increases the competitive pressure and has a damaging effect on firm performance. While crisis phases happen in real life, their effects on earnings distributions are smoothed by the fact that a large number of sectors exist, and firms manage more diversified product portfolios than in the game. However, this bias is offset by the large number of games studied, and partly controlled for by eliminating the first year of the game.

The game facilitator’s role can also introduce bias. Firms in difficulty generally receive help from the facilitator, who does not want participants to give up before the seminar is over. In other words, in situations where a struggling firm would go out of business in real life, in the game, the facilitator tends to delay that outcome as far as possible. He provides assistance in the form of subsidies, or advice, or possibly artificially keeping the firm alive when in reality its financial position would be untenable (long-term negative equity, zero industrial assets, astronomical debts, etc). Such situations remain infrequent, and generally only happen at the end of a seminar. Nonetheless, this intervention could result in overrepresentation of loss-making firms (thick tails on the left of the distribution) in earnings distributions. This bias is limited by eliminating the final year of the game.

### 3.3.2. Impact of methodological limitations on the object of the study

Given the two objectives of this study, the limitations identified may reduce the relevance of the answer it provides for the second objective. It is possible that teaching constraints and economic modeling may, despite the controls applied, influence the earnings distribution pattern. The results of the research on the question of the distribution pattern of “real” earnings must thus be considered in perspective. But there is no reason why these limitations should have any influence...
on discontinuities. In other words, whatever the limitations of the study, the answer it provides to the first question of whether or not there is any discontinuity around the zero earnings level can be considered reliable.

**Part 4: observation of the distribution pattern of unmanaged earnings**

This fourth section presents the results of the study. The two research questions are addressed successively: (1) are there any discontinuities around the zero earnings mark in unmanaged earnings distributions? and (2) what do unmanaged earnings distributions look like?

4.1. Unmanaged distributions show no discontinuities

Figure 5 shows corporate earnings distributions derived from business games, scaled successively by the total assets and sales variables. Unscaled earnings distributions are not shown, as they are not relevant to this study. In the simulations, the 43 business games do not all have identical parameters. There may be considerable differences in the orders of magnitude used by different facilitators. For example, in one game quantities may be expressed in units and prices in euros, while in another with similar parameters for production cost and sales development structure, the production unit may be a batch of 1,000 products, with prices expressed in thousands of euros. In other words, value data are not comparable between games, as there is a possibility they are not expressed in the same units.

![Earnings distribution (simulation)](image)

**Figure 4: Earnings distribution (simulation)**

Distribution of 1,641 earnings (simulation) scaled by Total Assets
Skewness = -0.9
Kurtosis = 3.7
The overall pattern of the distributions is similar for both denominators. Both these distributions are more concentrated than a Normal distribution (kurtosis > 3) and dissymmetrical (skewness < 0). Although they contain data from 43 games, i.e. 43 independent drawings, this is not a large enough number to apply the central limit theorem.

Simple visual observation\(^\text{24}\) shows that there is no discontinuity around the zero earnings mark. This provides an answer to the first research question. To show this result more clearly, the earnings distributions for French, UK and German listed firms from 1992 to 2004 as reported in the Compustat Global Vantage base are traced in Figure 7. All three distributions of reported (and therefore potentially managed) earnings display a large discontinuity at the zero earnings level. There is a clear contrast with the simulated earnings distributions.

\(^{24}\) Visual observation is clear enough to rule out the need for a statistical test, which would be problematic to implement since it requires a non-parametric measure of irregularity, and several methodological limitations have been highlighted in respect of such measures (Glaum, Lichtblau et Lindemann, 2004; Durtschi et Easton, 2005).
Figure 5: Distribution of earnings reported by French, German and UK firms

France: Distribution of 7,742 earnings (scaled by total assets) reported by French listed firms from 1992 to 2004 (source: Compustat Global Vantage database).

United Kingdom: Distribution of 14,028 earnings (scaled by total assets) reported by UK listed firms from 1995 to 2004 (source: Compustat Global Vantage database).
The unmanaged earnings distributions resulting from business game simulations do have some things in common with the reported earnings distributions. They are more concentrated or “pointier” (kurtosis > 3) than Normal distributions, and dissymmetrical with a steeper slope on the left than the right, and more negative values (negative skewness). But despite these similarities, the simulated earnings distributions are more spread out than the reported earnings distributions. This difference may be caused by the smaller number of earnings figures generated by the simulations (1,614 compared to 6,879 to 14,028), but it could also result from the biases identified in section 3 above. Game participants may take more risks in simulations than in real life, and this may lead to overrepresentation of extreme results.

But the most significant difference for the purposes of our study remains the absence of discontinuities in unmanaged earnings distributions, while all the reported earnings distributions show such discontinuities. Finally, as a control, the simulated earnings distributions are traced after elimination of the first and last year of the game. The distribution pattern (Figure 9) is no different, although it is less smooth because of the smaller number of observations (1,054 against 1,614).
4.2. Unmanaged earnings distributions do not follow a Gaussian shape

The similarities observed (Figure 5 and Figure 7) between the distributions of unmanaged earnings and published earnings confirm the relevance of using simulated earnings distributions in seeking to identify the appropriate theoretical distribution law for earnings. Mathematically, identification of a theoretical law of distribution based on an empirical distribution is called “distribution fitting”. There are many distribution fitting softwares that do this. Each one contains a catalogue of laws. For each law, they measure a goodness-of-fit indicator (Kolmogorov-Smirnov distance, Anderson-Darling distance, and Chi-square distance). The laws are ranked, and it is possible to deduce a theoretical distribution (from the laws in the catalogue) that is the best fit to the empirically-observed distribution. But the fit cannot be determined from our aggregate distribution of (scaled) unadjusted earnings, because the law we
are looking for is the distribution law for unmanaged annual earnings. Every year, firms’ earnings correspond to an independent “drawing”. The mathematical process must therefore be conducted in a situation where all other factors are equal, i.e. after eliminating environmental influences on economic performance. The proposed solution is to standardize the annual distributions before aggregating them (Vidal 2008). This part of the study thus requires modification of the database. Each earnings figure is centered on the mean and reduced by the standard deviation for the source annual earnings.\(^{25}\)

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\(^{25}\) Centering is on the mean rather than the median despite the fact that the mean is more sensitive to extreme values, because the number of firms is relatively small in each game (generally 5 to 8). This means that when there is an uneven number of firms (twenty cases), the median is equal to the earnings of one of the firms and as a result the class located exactly in the centre of the distribution after centering on the median would be overrepresented.
Two distributions are shown in Figure 11. The first relates to earnings scaled by total assets, and the second earnings scaled by sales. The fit to Normal distribution is illustrated by the curve superimposed over each graph. Two facts are clearly visible. The first is that the two distributions are almost perfectly identical, whichever variable is used for scaling. There is strong dissymmetry, but the pattern is much less concentrated than in un-standardized distributions.

The second is that the Normal distribution is not a good fit for these distributions, an observation confirmed by statistical tests (p-value is nearer to 0 than 1). The Best Fit software used for distribution fitting (checking against all laws) ranks the best fits as shown in Table 5.

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**Figure 7: Earnings distribution (simulation) after standardization**

In black:
Distribution of the 1,641 earnings figures resulting from the standardized simulations.
In grey: Normal distribution fit for parameters N(0; 0.92194).

In black:
Distribution of the 1,616 earnings figures resulting from the standardized simulations.
In grey: Normal distribution fit for parameters N(0; 0.922).

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26 This software published by Palisade has been totally integrated into the @risk software since 2008 and is no longer available separately.
The proposed distribution laws are ranked according to the Kolmogorov-Smirnov test, which gives more weight to the central portion of the distribution. No law offers a statistically good fit. The Weibull distribution is the best fit, whichever test is used (the value of distance "s" between the fitted distribution and the empirical distribution is almost half the distance measured using Normal distribution), but the significance level remains low. This distribution, bounded to the left, is generally used to study lifetimes (positive values). Its asymmetrical aspect is what makes it a better fit than the Normal distribution, but it remains conceptually ill-suited because it is bounded to the left.

The Logistic distribution comes second in the ranking of fits. It ranks above Normal distribution due to its more concentrated (pointier) shape. But its symmetry (like the Normal distribution) cannot properly reflect the observed pattern of distributions.

Finally, a graphic representation of the standardized (scaled) earnings distribution of French listed firms is drawn up (Figure 8). This distribution shows significant differences from the simulated earnings distributions. First of all, the simulated earnings distributions are less "smooth". This is explained by the smaller number of observations. Simulated distributions are considerably less pointed, due to the strong discontinuity observed around the zero earnings mark in reported earnings, which reinforces concentration (the non-standardized distributions in Figure 5 confirm this).
The final aspect noted is the high dissymmetry in simulated earnings. The peak of the distribution is clearly located among the positive values, whereas it tends to lie around zero (and therefore the median used to center distributions) for reported earnings. In the context of our study this is an unexpected finding, and deserves further exploration. It is as if, in reality, the excess number of firms making very small profits resulted from two separate avoidance mechanisms: avoidance of losses (which leads to the discontinuity at the zero earnings level), and avoidance of above-median earnings (which leads to understatement of results when they are good). This study thus confirms the intuition that firms build up “cushions” in profitable periods and use them when they fall on harder times (Degeorge, Patel and Zeckhauser, 1999).

Conclusion

This study is original, being the first time a laboratory experiment has been presented for analysis of earnings management practices to meet thresholds. Its principal finding is the confirmation of a hypothesis that had not been demonstrated before: unmanaged earnings distributions should not present any discontinuities, and should follow a smooth curve. The second important result of this study is the confirmation that earnings distributions do not appear to follow a Normal distribution. The theoretical distribution pattern is more concentrated, and dissymmetrical. The Weibull distribution (dissymmetrical) and Logistic distribution (concentrated) provide a better fit, but that fit is still imperfect. These results cannot however be generalized due to methodological limitations specific to the practicalities of the experiment. The theoretical distribution of annual earnings remains to be identified. A final finding is unexpected and unusual: this study shows that firms with earnings higher than the average reported earnings tend to manage their earnings downwards.


McNichols M. F. (2003). Discussion of "Why are Earnings Kinky? An Examination of the Earnings Management..."