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Abstract

This paper addresses the two-way implication between punctuality level of public transport and commuter behavior. We consider a modal competition between public transport and an alternative mode. Commuters may choose different strategies to minimize their journey cost. In particular, when the bus becomes less punctual, more potential bus users arrive late at the bus stop. We show that punctuality increases with the alternative mode fare through a price effect. This specificity can be viewed as an extension of the Mohring effect. In the general case, the punctuality of a bus is lower at equilibrium than at optimum. According to the alternative mode operating cost, the bus attracts too many (small cost) or too few (large cost) customers.

Keywords: public transport; reliability; duopoly; welfare; Mohring effect; schedule delay
JEL: R41; R48; D43

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1. Introduction

Despite increasing pollution and congestion in cities, cars remain the most popular mode of transport, because they are usually more convenient than public transport and they keep a strong attractive power due to symbolic and affective motives (Steg, 2005). In the U.S., the predominance of cars is also strengthened, despite the congestion observed on the American highways (The Economist, 2011). Therefore, improving alternative modes of transport and making them attractive is essential in an urban context. Although it has been pointed out that the share of commuters switching from cars to public transport may not be very large (Hensher, 1998), increasing the service quality is still an important determinant of public transport demand (Beirao and Cabral, 2007). Travel time is often presented as the main determinant of trip characteristics.

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Much less focus has been devoted to trip reliability. However, some studies (see eg, Beirao and Cabral, 2007) have shown that users will shift to cars if public transport is not reliable enough. Several studies strongly suggest that reliability (understood as punctuality) of public transport is crucial to leverage the demand (Bates et al., 2001; Hensher et al., 2003; Paulley et al., 2006; Coulombel and de Palma, 2013). In a qualitative review, Redman et al. (2013) claim that reliability is the most important quality attribute of public transport according to users. Ongoing research also tries to show that reliability of public transport may have an impact of the price of land.

The reliability issue does not only affect developing countries, but also developed countries. The example of United States is striking: only 77% of the short-haul trains are punctual, whereas 90% of Europeans trains are on time (The Economist, 2011). Moreover, the reliability of long-distance trains is even worse in the US. This is due to decades of under-investment which have led to infrastructure degradation.[1] For a discussion of the relevance of investment in rail transit system, we refer the reader to Winston and Maheshri (2007).

Although there is a long tradition in studying road reliability, a sensitive lack of research is observed in public transport field (Bates et al., 2001). Studies highlight a valuation of road reliability (Bates et al., 2001; Fosgerau and Karlström, 2010; Fosgerau and Engelson, 2011), others underline the importance of public transport comfort (de Palma et al., 2013) or punctuality (Jensen, 1999), but few works deal with reliability in an analytical way.

This paper focuses on the two-way implication between punctuality level of public transport and (potential) customer behavior. Indeed, on the one hand the punctuality of public transport is a key element of the service quality. The user cost elements, which play an important role in demand analysis, are affected by the punctuality level (Bowman and Turnquist, 1981). The cost of punctuality differs among commuters. It largely depends on the preferred arrival time of commuters. As a consequence, users and potential users choose both the mode of transport and the departure time as a function of punctuality level in public transport. On the other hand, Mohring (1972) has shown that scheduled urban public transport is characterized by increasing returns to scale since the frequency increases with demand. Demand is influential in the service quality offered and the bus company may adapt its punctuality to the level of potential demand. Thus we show that some users may decide to arrive late at the bus stop when punctuality is too low. As a consequence, the bus company itself may become less strict as regards the punctuality. In a nutshell, this means that user behavior (punctuality of users) is influenced by the punctuality of public transport. This generates a vicious circle.

In this paper, we study three situations: (i) the reaction of the bus company when it faces a higher price of the alternative mode, (ii) the gap between the bus punctuality at equilibrium and at optimum and (iii) the equilibrium versus

[1] To address this situation, Mr Obama plans to spend $556 billion for transport over 6 years, according to his 2012 budget.

optimal modal split when punctuality matters.

We consider a duopoly which symbolizes a modal competition between public transport and another mode, which we call taxi. The attention is focused on the monetary impacts of punctuality. We simplify aspects related to engineering. A duopoly is used because determinants of demand for public transport are related to the demand for private transport (Balcombe et al., 2004). Two different types of variables are observed in the model: the public transport punctuality level, which is selected by the bus company and the prices set by the bus and taxi companies. Both have a substantial influence on demand for public transport (Paulley et al., 2006). Unreliability has a strong negative impact because it implies excessive waiting time and uncertainty (Wardman, 2004; Paulley et al., 2006).

Considering commuting trips, preferences can be analyzed with the dynamic scheduling model. In this model, individual's preferences reflect agents tradeoff between travel time, early schedule and late schedule delays. Commuters may choose different strategies to minimize their trip cost. This theory has been first introduced by Vickrey (1969) and then renewed by Arnott et al. (1990). Such analysis is usually specific to road analysis (Fosgerau and Karlström, 2010); here we introduce a waiting time to extend this model to public transport. The French State-owned railroad (SNCF) suggests to reschedule work arrival and departure times in order to reduce congestion (Steinmann, 2013). For the idea of endogenous schedules and private or public bus company, we refer the reader to Fosgerau and Small (2013).

Commuters are differentiated by their preferred arrival time at workplace and by their residential location which is measured as the time to travel to their destination when using the alternative mode. Two different preferred arrival time are considered and the location is uniformly distributed among commuters.

The analysis for the model proceeds in three steps. The first step consists in finding out the modal choice of commuters depending on prices and punctuality for the public transport and the alternative mode. The second step determines which price and punctuality levels are set by companies at equilibrium given the behaviors of commuters identified in step one. The third step is to assess the prices and the punctuality level that minimize the total social cost and to compare these results with the ones derived in step two.

The paper is organized as follows. Section 2 describes the model and the commuter's strategies. Section 3 considers equilibrium and its properties. The gain due the transition from equilibrium to optimum is analyzed in Section 4. A numerical application is provided in Section 5 to illustrate our results. The final section concludes and proposes suggestions for further research.

2. Punctuality in public transport

Our model is based on the monocentric city framework defined by Alonso (1964), Mills (1967) and Muth (1969). All jobs are located in the center of the city, referred to as the central business district (CBD). Consequently, all

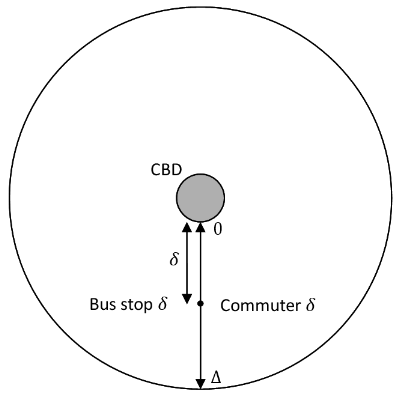


Figure 1: The monocentric city

commuters have to reach the CBD every morning. We focus our analysis on a unique radius of the city, assuming that this radius is representative of the set of radius of the city. We consider an unique road which coincides with this radius. It goes straight from the border of the city to the CBD. The radius is measured in time units and is Δ hours long. An unique bus line and a taxi company serve the CBD by using this road and bus stops are uniformly distributed along the radius of the city. We do not take into account congestion on the road. Thus both mode have the same speed and we refer to a bus stop located at δ hours from the CBD as "bus stop δ". For example, the bus stop Δ is located at the border of the city. Similarly, all commuters live along the radius and we refer to commuters who need δ hours to reach the CBD, whether they use the bus service or the taxi service, as "commuters δ". For each $\delta \in [0; \Delta]$, all commuters δ live at the same place (see Figure 1).

For analytical tractability, we consider a single bus. However this model can be easily adapted to other modes of public transport that run on a schedule. The bus is scheduled to arrive at the CBD at a given time, but it may be late. The lateness probability is not random: the bus company selects its quality service level and applies it in the same manner along the radius. Thus when the bus company chooses to be late, it is late along the whole journey and its lateness is constant over time. Commuters are aware of the punctuality level and adapt their behavior accordingly. In particular they might arrive at the bus stop after the scheduled time even if there is a risk to miss the bus by doing so. This can occur rationally because there is a waiting cost for users. Commuters optimize their tradeoffs between waiting time cost, schedule delay cost and a cost corresponding to the use of an alternative mode, which is the taxi in our model. A commuter may either select *ex ante* the taxi or use the taxi if he misses the bus.

Table 1 presents important notations and their numerical values that will be used in Section 2 and Section 5.We first characterize the network and then the commuter behavior. Finally we characterize the modal split.

Parameter	Comment	Suggested value
T	Scheduled arrival time	-
tt	Bus travel time	25/60 (hour)
x	Lateness	10/60 (hour)
$P \in \left[\frac{1}{2}; 1\right]$	Probability of the bus being late	-
$t_p \in \{T; T+x\}$	Arrival time at the bus stop of the bus	-
$\delta \in [0; \Delta]$	Taxi trip time	(hour)
Δ	Maximal taxi trip time	35/60 (hour)
$t^* \in \{T; T+x\}$	Preferred arrival time of users	-
$t_a \in \{T; T+x\}$	Arrival time at the bus stop of the user	-
$\theta \in \left[\frac{1}{2}; 1\right]$	Share of population in *Group A*	-
α_{bus}	In-bus time cost	15 ($/hour)
α_{taxi}	In-taxi time cost	4 ($/hour)
η	Waiting time cost	20 ($/hour)
β	Early delay cost	10 ($/hour)
γ	Late delay cost	30 ($/hour)
κ	Bus fare	($)
τ	Taxi fare	($/hour)
c	Cost of punctuality (bus)	($)
d	Operating cost per unit of time (taxi)	40 ($/hour)

Table 1: Parameters values

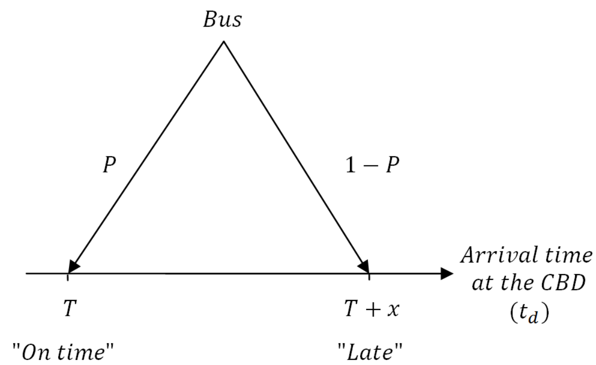


Figure 2: The arrival probabilities of the bus

2.1. Transport supply

Bus stops are uniformly distributed between 0 and Δ. The bus is scheduled to arrive at its destination, the CBD, at time T. As there is no road congestion, it is also scheduled to serve the bus stop δ at time $T-\delta$ and also leaves at time $T-\delta$ i.e there is no transfer cost.[2] The bus company may choose that the bus is late and arrives at CBD time $T+x$. In this case, the bus stops at every bus stop δ at time $T+x-\delta$. The bus arrives at the CBD at time T with probability P and at time $T+x$ with probability $1-P$ (Figure 2). Whatever the bus lateness, the total bus trip time is constant and equal to Δ. The potential lateness is also constant and equal to x.

The probability of the bus being on time is endogenous: the bus company sets its level. It does not depend on traffic conditions, number of passengers or loading time. The worst quality of service occurs when the bus has the same probability of being on time and late. We assume that a regulator imposes this constraint to assure a consistent timetable. [3] The "punctuality level" corresponds to the probability of the bus being on time.

Assumption 1. *The probability P of the bus being on time satisfies the following inequality:*

$$\frac{1}{2} \leq P \leq 1.$$

We assume that there is no capacity constraint in the bus. The bus fare, priced by the bus company, is κ for each passenger.

Commuters have access to an alternative mode of transport. In our model we consider this option as a taxi service, but it can also be walking or personal car use. The taxi company sets a fare τ which corresponds to the price charged per minute of travel.

[2] The loading time is assumed to be set to zero without loss of generality.

[3] Minimal value of P is $1/2$, otherwise we would face another schedule than the expected one.

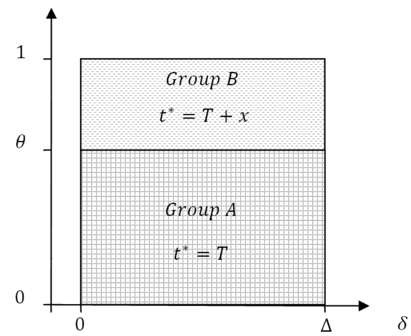


Figure 3: Distribution of taxi trip time in *Group A* and *Group B*

2.2. Demand for bus and taxi

We consider two firms located in the CBD. Firm A employs a part of θ in commuters population and firm B a part of $(1-\theta)$. The share of commuters working for firm A is bigger than the one working for firm B ($\theta \geq 1/2$). The workday in the first firm starts at time T whereas it starts at time $T+y$ in the other firm.[4] This reflects the fact that even though a majority of commuters wishes to arrive at work place at the same time, all commuters have not the same preferred arrival time.

For tractability, we assume that the gap between the beginnings of workday equals to the lateness of the bus ($x=y$). Therefore, each type has a different preferred arrival time denoted $t^* \in \{T; T+x\}$. The first type of commuters (referred to as *Group A*) would rather arrive at time T, and the second one (referred to as *Group B*) at time $T+x$ (see Figure 3).

Commuters locations are uniformly distributed among each group in the same manner (Figure 3) and the distribution is assumed to have a support $[0; \Delta]$ so that $F(0) = 0$ and $F(\Delta) = 1$.

They are assumed to incur a schedule delay cost if traveling at time $t \neq t^*$. There is no transfer cost: commuters do not incur a cost by reaching the bus stop because bus stops are uniformly distributed along the radius where they live.

A commuter has the choice between catching the bus and using the taxi service. However he may miss the bus and then he has to use the taxi service. Indeed we assume the headway is so long that all users who miss the bus prefer to use the taxi service. If he tries to catch the bus, the commuter δ uses the bus stop δ because it minimizes its transfer cost. A commuter δ choosing to catch the bus bears the following schedule delay cost function that is assumed to depend on its arrival time at the bus stop, denoted $t_a \in \{T-\delta; T-\delta+x\}$,

[4]This gap between working start times is conceivable if there is no Marshallian externality between these two firms (see Henderson, 1997).

the arrival time of the bus, denoted $t_p \in \{T-\delta; T-\delta+x\}$, its most preferred trip time, denoted $t^* \in \{T; T+x\}$ as well as on the arrival time at destination of the bus, denoted $t_d \in \{T; T+x\}$:

$$CC_{bus} = \begin{cases} \kappa + \delta\alpha_{bus} + \eta(t_p - t_a) + \beta\left[t^* - t_d\right]^+ + \gamma\left[t_d - t^*\right]^+ & \text{if } (t_a \leq t_p), \\ \delta\left(\alpha_{taxi} + \tau\right) + \gamma\left[t_d - t^*\right]^+ & \text{if } (t_a > t_p), \end{cases}$$

with $[x]^+ = x$ if $x \geq 0$ and 0 if $x < 0$, κ the bus fare, α_{bus} the in-bus time cost, η the waiting time cost, β the early delay cost, γ the late delay cost, α_{taxi} the in-taxi time cost, τ the taxi fare and δ the trip time of commuter δ.

If a commuter chooses from the start to use the taxi service, he incurs the following cost:

$$CC_{taxi} = \delta\left(\alpha_{taxi} + \tau\right),$$

with α_{taxi} the taxi travel time value, τ the taxi fare and δ the taxi trip time.

By considering that the value of time in bus $\delta\alpha_{bus}$ is incurred by every commuter whatever is its choice, we can normalize the cost functions to:

$$CC_{bus} = \begin{cases} \kappa + \eta(t_p - t_a) + \beta\left[t^* - t_d\right]^+ + \gamma\left[t_d - t^*\right]^+ & \text{if } (t_a \leq t_p), \\ \delta\left(\breve{\alpha} + \tau\right) + \gamma\left[t_d - t^*\right]^+ & \text{if } (t_a > t_p), \end{cases} \tag{1}$$

$$CC_{taxi} = \delta\left(\breve{\alpha} + \tau\right), \tag{2}$$

with $\breve{\alpha} = \alpha_{taxi} - \alpha_{bus}$.

Assumption 2. *The cost of waiting one minute for a bus, η, is lower than the cost of being one minute late, γ, and higher than the cost of being one minute early, β:*

$$\gamma \geq \eta \geq \beta.$$

This assumption is consistent with literature valuations (Wardman, 2004).

2.3. Commuters' strategies

Commuters dispose of three different strategies to minimize the cost of a trip. A strategy is defined by an arrival time at the bus stop. Arriving at the bus stop at time T corresponds to *Strategy O* (On-time at the bus stop), arriving at time $T+x$ to *Strategy L* (Late at the bus stop) and *Strategy T* (Taxi) embodies the decision to use the taxi and to not arrive at the bus stop. If a commuter chooses *Strategy O*, he waits until the bus arrives.

As a convention, we assume that a commuter who is indifferent between two strategies has a preference for maximizing its chance to get the bus. The commuter chooses:
Strategy O (arrive at time T) if $EC(O) \leq EC(T)$ and $EC(O) \leq EC(L)$;
Strategy L (arrive at time $T+x$) if $EC(L) < EC(O)$ and $EC(L) \leq EC(T)$;
Strategy T (choose the taxi) if $EC(T) < EC(O)$ and $EC(T) < EC(L)$;
where $EC(i)$ represents the expected cost of strategy i.

Proposition 1. *Under A.1 and A.2, the commuter δ in Group A selects:*

$$\begin{array}{rcl} Strategy\,O\ \ (time\ T) & if & \delta \geq \delta^A_{T,O}, \\ Strategy\,T\ \ (taxi) & if & \delta < \delta^A_{T,O}, \end{array}$$

where $\delta^A_{T,O} \equiv [\kappa + (1-P)\,(\eta+\gamma)\,x]\,/\,(\breve{\alpha}+\tau)$.

Proof. *See AppendixA.* □

For a commuter wishing to arrive at time T, *Strategy L* is never selected. Indeed a commuter chooses *Strategy L* instead of *Strategy T* if he prefers a late bus trip over a taxi trip. However such a commuter prefers an on time bus trip over taxi trip and consequently, he will choose *Strategy O*.

Proposition 2. *Under A.1 and A.2, the commuter δ in Group B selects:*

$$\begin{array}{rcc} Strategy\,O\ \ (time\ T) & if & \delta \geq \delta^B_{L,O}, \\ Strategy\,L\ \ (time\ T+x) & if & \delta \in \left[\delta^B_{T,L};\delta^B_{L,O}\right[, \\ Strategy\,T\ \ (taxi) & if & \delta < \delta^B_{T,L}, \end{array}$$

where $\delta^B_{L,O} \equiv \left[\kappa + \left(\frac{1-P}{P}\eta+\beta\right)x\right]/\,(\breve{\alpha}+\tau)$ *and* $\delta^B_{T,L} \equiv \kappa/\,(\breve{\alpha}+\tau)$.

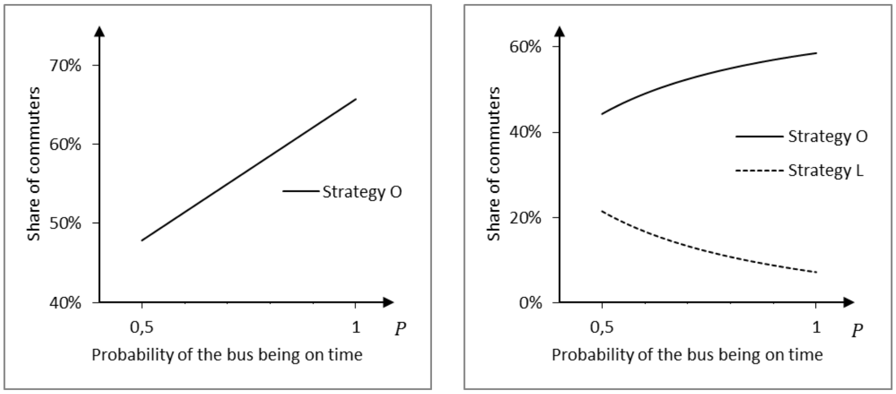
Proof. *See AppendixB.* □

Strategy L is selected by some commuters from *Group B* unlike commuters from *Group A*. It can be explained by the fact that in this case, *Strategy B* corresponds to a possibility of the bus arriving on time. A commuter who prefers an on-time bus trip over a taxi trip, yet prefers a taxi trip more than an early-arrival bus trip, chooses *Strategy L*.

The share of commuters choosing *Strategy T* is independent of the probability of the bus being on time. Indeed P has no influence in the arbitrage between *Strategy L* and *Strategy T*. For commuters in *Group B*, choosing *Strategy L* is equivalent to choosing *Strategy T* except that they take the bus when it is late. Consequently, *Strategy L* is preferred to *Strategy T* as long as the cost of taking the bus when it is late is lower than the cost of taking a taxi. Then this arbitrage is independent of the probability of the bus being on time.

When the punctuality decreases, the share of commuters arriving late at the bus stop increases. The cut in the service quality makes the cost of *Strategy O* higher (because of A.2) and the cost of *Strategy L* smaller (except for commuters living so close to the CBD that a taxi trip is still cheaper than a bus trip, but we do not take account of these commuters because they still prefer *Strategy T*). Then among commuters who chose *Strategy O* before the service quality fall, those living the closest to the CBD were the most indifferent between both strategies and switched from *Strategy O* to *Strategy L*. The bus company may also itself become less strict, and generate a vicious circle.

When the taxi fare, τ, increases, more commuters choose to arrive at the bus stop at T and less commuters choose *Strategy L* and *Strategy T*. This is due to the fact that on the one hand some commuters have a bigger interest to minimize



Group A *Group B*

Figure 4: Share of commuter choosing *Strategy O* and *Strategy L* as a function of P, the probability of the bus being on time ($\kappa = 8$, $\tau = 50$)

the probability of taking the taxi by shifting from *Strategy L* to *Strategy O* and from *Strategy T* to *Strategy L*. On the other hand, the shift from *Strategy L* to *Strategy O* is larger than the one from *Strategy T* to *Strategy L*.

Figure 4 illustrates these results. Other things being equal, the share of commuters arriving at T (and by doing so they are sure to catch the bus) among *Group A* increases from around 40% when $P = 1/2$ to almost 55% when $P = 1$. The share of commuters in *Group B* choosing to arrive late at the bus stop (*Strategy L* depends inversely on the probability of the bus being on time. If the bus arrives later, some users switch from *Strategy O* to *Strategy L* which leaves the bus company no incentive to restore the service quality.

Assumption 3. *The maximum cost of the taxi use, priced at the operating cost, is higher than the cost of the bus use, when priced at zero and when the bus arrives on time with probability* 1/2*:*

$$\Delta\left(\breve{\alpha} + d\right) \geq \frac{1}{2}\left(\eta + \gamma\right) x.$$

Once the commuters strategy are defined, shares of commuters who are at the bus stop at time T or $T + x$ are known. Demands are described by

$$D_{bus} = \theta\left(1 - \frac{\delta^A_{T,O}}{\Delta}\right) + (1-\theta)\left[1 - \frac{\delta^B_{L,O}}{\Delta} + (1-P)\frac{\delta^B_{L,O} - \delta^B_{T,L}}{\Delta}\right], \tag{3a}$$

$$D_{taxi} = \theta\frac{\delta^A_{T,O}}{\Delta} + (1-\theta)\left[P\frac{\delta^B_{L,O} - \delta^B_{T,L}}{\Delta} + \frac{\delta^B_{T,L}}{\Delta}\right]. \tag{3b}$$

Thus the bus (and taxi) patronage depends on the probability of the bus being on time. *Group A* is more sensitive to the service quality than *Group B* (see also Figure 4). This is due to the fact that commuters from *Group A* incur late arrival costs while commuters from $Group\,B$ incur early arrival costs and, as seen in A.2, the penalty for lateness is much higher than the penalty for arriving early at the destination.

3. Competition between bus and taxi companies

In this section, we explore equilibrium pricing and punctuality level in a duopoly competition. We assume that following condition holds:

$$\Delta\left(\breve{\alpha}+\tau\right) \geq \kappa + \frac{1}{2}\left(\eta+\gamma\right)x. \tag{4}$$

This condition assures that the price selected by the bus company is low enough to preserve a demand for bus trips. Thus the demand functions formulation (equations (3a) and (3b)) is still correct. We will check if it holds once the equilibrium values of τ and κ are solved.

Both companies incur a cost. The cost incurred by the bus company depends on the punctuality level and is assumed to be quadratic. It is a sunk cost in the sense of being unrecoverable (Sutton, 1991). The cost of the taxi company linearly depends on the total travel time and can be viewed as an operating cost:

$$Cost_{bus} = \frac{c}{2}P^2, \tag{5a}$$

$$Cost_{taxi} = d * TTT, \tag{5b}$$

with c the punctuality cost, d a cost per hour traveled and TTT the total travel time of the taxi company.

The bus company chooses the bus fare κ and the punctuality level P, so as to maximize its expected profit. From equations (3a) and (5a), the bus company profit can be written as

$$\Pi_{bus} = \kappa D_{bus} - \frac{c}{2}P^2.$$

There exists a unique solution[5] satisfying the first-order conditions $\partial\Pi_{bus}/\partial\kappa = 0$ and $\partial\Pi_{bus}/\partial P = 0$, given by

$$\kappa^e = \frac{1}{2}\left(\check{\triangle}^e - \Gamma^e x\right), \tag{6}$$

$$P^e = \begin{cases} \frac{1}{2} & \text{if } c > c_2^e, \\ \frac{\kappa^e \check{\eta} x}{c\check{\triangle}^e} & \text{if } c \in [c_1^e; c_2^e], \\ 1 & \text{if } c < c_1^e, \end{cases} \tag{7}$$

[5]Second-order conditions are satisfied as $\partial^2\Pi_{bus}/\partial\kappa^2 = -2/\check{\triangle}$ and $\partial^2\Pi_{bus}/\partial P^2 = -c/\check{\triangle}$. The Hessian matrix of second partial derivatives is also negative definite, and the solution is a global maximum.

where $\check{\Delta}^e = \Delta(\check{\alpha}+\tau^e)$, $\Gamma^e = (1-P^e)\eta + (1-\theta)P^e\beta + \theta(1-P^e)\gamma$, $\check{\eta} = \eta - (1-\theta)\beta + \theta\gamma$, $c_1^e \equiv \kappa^e\check{\eta}x/\check{\Delta}^e$ and where $c_2^e \equiv 2c_1^e$.

The price of a minute traveled in a taxi, τ, is set by the taxi company to maximize its profit. From equations (3b) and (5b), taxi profit is given by

$$\begin{aligned}\Pi_{taxi} &= (\tau - d)\left[\theta\int_0^{\delta_{T,O}^A}\delta f(\delta)d\delta\right.\\ &\quad \left.+(1-\theta)\left(\int_0^{\delta_{T,L}^B}\delta f(\delta)d\delta + P\int_{\delta_{T,L}^B}^{\delta_{L,O}^B}\delta f(\delta)d\delta\right)\right].\end{aligned}$$

The level of price satisfying the first-order condition[6] $\partial\Pi_{taxi}/\partial\tau = 0$ is

$$\tau^e = \check{\alpha} + 2d. \tag{8}$$

Condition (4) requires $\check{\Delta}^e \geq \kappa^e + \frac{1}{2}(\eta+\gamma)x$ and yet $\Delta(\check{\alpha}+d) \geq \{P\eta - (1-\theta)P\beta + [1-\theta(1-P)]\gamma\}x/2$. It holds according to A.3.

Note that the probability of the bus being on time in (7) is continuous.

The core component of the bus fare corresponds to the average taxi trip cost cut by the average schedule and waiting time cost incurred by commuters. The bus company takes account of its service quality to remain attractive regarding the alternative mode. As expected, the punctuality decreases when the punctuality cost c increases. Since the punctuality level decreases with the maximal taxi trip time, Δ, a high scatter of commuter's locations makes the service quality regress (see equation (7)). In addition, the longer of the radius where commuters live, the higher is the mark-up for the bus company. The taxi fare is independent of the bus company choices. It only depends on the values of taxi and bus travel time and operating cost. *Ceteris paribus* when d increases, both bus and taxi fares become higher.

There is a unique simultaneous Nash equilibrium which is given by equations (6), (7) and (8).

Proposition 3. *At equilibrium, P^e, the probability of the bus being on time and κ^e, the bus fare, increase with τ, the taxi fare.*

Proof. See AppendixC. □

Consider an initial rise in taxi fare, τ^e, for example due to an increase in the taxi operating cost or in the petrol price. This increase leads to a standard modal shift from taxi service to bus service, other things being equal (see Propositions 1 and 2). Consequently, the cost of the bus punctuality per user decreases. The bus company therefore will have an incentive to increase the punctuality level when τ rises. By doing so, the bus company attracts additional commuters. In this model, an increase in bus patronage improves the service quality of the

[6] Second-order condition requires that $4\check{\alpha} - 2\tau^e + 6d \geq 0$ or $\tau^e \leq 2\check{\alpha} + 3d$.

bus. This can be viewed as an extension of the Mohring Effect (Mohring, 1972) according to which the service quality measured as the frequency increases when the demand for public transport rises.

The increase in the bus fare is explained by two aspects: on the one hand the service quality has been improved, and on the other hand, the rises in the taxi fare increase the average taxi trip cost and therefore the bus fare. There is no strategic complementarity because the taxi company does not react to an change in bus fare (see Vives, 1990).

4. Welfare analysis

Welfare is the sum of the aggregate commuter surplus and the companies profits. Since a cost function is used instead of a surplus function to study the commuter strategies, the social welfare function is defined as the opposite of the social cost function SC which is the difference between aggregate commuter costs and firm profits. From equations of commuter cost (1) and (2), of demand (3a) and (3b), and of companies cost (5a) and (5b), the social cost function can be written as

$$SC = \frac{\Delta \alpha_{bus}}{2} + \theta CC_{\theta=1} + (1-\theta)\, CC_{\theta=0} - \Pi_{bus} - \Pi_{taxi},$$

where

$$\begin{aligned} CC_{\theta=1} &= (\breve{\alpha}+\tau)\int_0^{\delta^A_{T,O}} \delta f(\delta) d\delta \\ &\quad + \{\kappa + [(1-P)(\eta+\gamma)]\, x\} \int_{\delta^A_{T,O}}^{\Delta} f(\delta) d\delta, \end{aligned}$$

and where

$$\begin{aligned} CC_{\theta=0} &= (\breve{\alpha}+\tau)\int_0^{\delta^B_{T,L}} \delta f(\delta) d\delta \\ &\quad + \int_{\delta^B_{T,L}}^{\delta^B_{L,O}} [(1-P)\,\kappa + P\,(\breve{\alpha}+\tau)\,\delta]\, f(\delta) d\delta \\ &\quad + \{\kappa + [(1-P)\,\eta + P\beta]\, x\} \int_{\delta^B_{L,O}}^{\Delta} f(\delta) d\delta. \end{aligned}$$

The social planner chooses the punctuality level P, the bus fare κ and taxi fare τ so as to minimize social cost. The first-order conditions for the socially optimal bus and taxi prices are given by

$$\kappa^o = 0, \tag{9}$$
$$\tau^o = d. \tag{10}$$

As expected, optimal bus and taxi fares equal to the marginal costs incurred by bus and taxi companies. Indeed, as there is no variable cost for the bus, the optimal bus fare is null.

The expression of the optimal punctuality level P^o is not explicit in the general case because the equation to solve is a cube root i.e it has three solutions with only one real.

$$P^o = \underset{P\in\left[\frac{1}{2};1\right]}{\arg\min} SC. \tag{11}$$

However in the extreme case where $\theta = 1$, there exists a unique solution[7] satisfying $\partial SC_{\theta=1}/\partial P = 0$. By using (9) and (10), we obtain, for *Group A*

$$P^o_{\theta=1} = \begin{cases} \frac{1}{2} & \text{if } c > c^o_{2;\theta=1}, \\ \frac{\left[\check{\Delta}^o - (\eta+\gamma)x\right](\eta+\gamma)x}{c\check{\Delta}^o - [(\eta+\gamma)x]^2} & \text{if } c \in \left[c^o_{1;\theta=1}; c^o_{2;\theta=1}\right], \\ 1 & \text{if } c < c^o_{1;\theta=1}, \end{cases} \tag{11a}$$

where $\check{\Delta}^o = \Delta\left(\check{\alpha} + \tau^o\right)$, $c^o_{1;\theta=1} \equiv (\eta+\gamma)\,x$,
$c^o_{2;\theta=1} \equiv \left[2\check{\Delta}^o - (\eta+\gamma)\,x\right](\eta+\gamma)\,x/\check{\Delta}^o$ and where $c^o_{1;\theta=1} < c^o_{2;\theta=1}$. Note that the probability of the bus being on time when $\theta = 1$ is continuous.

We generalize the above result to the other extreme case where $\theta = 0$ in the following conjecture.

Conjecture 1. *For Group B ($\theta = 0$), the punctuality level of the bus $P^o_{\theta=0}$ weakly decreases when the cost of reliability c increases. There are two critical values of c, $c^o_{1;\theta=0}$ and $c^o_{2;\theta=0}$ with $c^o_{1;\theta=0} \leq c^o_{2;\theta=0}$ such that:*

$$P^o_{\theta=0} = \begin{cases} \frac{1}{2} & \text{if } c > c^o_{2;\theta=0}, \\ 1 & \text{if } c < c^o_{1;\theta=0}. \end{cases} \tag{11b}$$

with $c^o_{1;\theta=0} < c^o_{2;\theta=0}$.

Equations (9), (10) and (11) provide the values at optimum in the general case. Equations (11b) and (11a) point out the optimal punctuality level in extreme cases.

The optimal probability of the bus being on time has the same properties we describe in Section 3: it decreases when the punctuality cost c or the travel time of the commuter living the farthest Δ increases. The important observation is that the optimal probability of the bus being on time does not necessarily equal to 1. It may be lower than 1 and even equal to 1/2 under some conditions. Critical values $c^o_{1;\theta=0}$ and $c^o_{2;\theta=0}$ are expected because $P^o_{\theta=0} \in [1/2; 1]$. The above conjecture is illustrated in Figure 5.

[7] Second-order condition is verified as $c\check{\Delta}^o \geq [(\eta+\gamma)\,x]^2$.

From now on, as the expression of P^o is not explicit and $P^o = \theta P^o_{\theta=1} + (1-\theta) P^o_{\theta=0}$, properties of the optimal probability of the bus being on time will be addressed separately according to the structure of the population. The two extreme cases $\theta = 1$ and $\theta = 0$ are highlighted, even if $\theta \geq 1/2$.

Proposition 4. *For Group A ($\theta = 1$), the punctuality level of the bus is higher at optimum than at equilibrium.*

Proof. *See AppendixE.* □

Commuters in *Group A* want to arrive at T therefore the later is the bus, the more commuters incur a cost. The bus company wishes to maximize the probability of the bus being on time at equilibrium, as the social planner does at optimum, while taking into account the punctuality cost per user incurred by the bus company. The difference between equilibrium and optimum bus punctuality is mainly explained by a price-effect. Indeed, the gap between the bus fare relative to the taxi fare is much higher at equilibrium than at optimum. Thus other things being equal, the bus company attracts less customers at equilibrium than at optimum. Consequently, the bus company has to reduce the bus punctuality at equilibrium more than the social planner does at optimum to keep the punctuality cost per user small enough. This result is summarized in Proposition 4.

As there is no explicit expression for P^o and $P^o_{\theta=0}$, a discussion with a figure is provided in Section 5.

Proposition 5. *For Group A ($\theta = 1$), if the taxi operating cost d is higher than d_1^c, the bus patronage is higher at optimum than at equilibrium.*
When $d \leq d_1^c$, the bus patronage is higher at optimum than at equilibrium if and only if the cost of punctuality for the bus company is small enough ($c \leq c_1^c$).[8]

Proof. *See AppendixF.* □

As the expression of $P^o_{\theta=0}$ is not explicit, the analysis is more difficult for *Group B*. However we formulate a proposition, as well as a conjecture.

Proposition 6. *For Group B ($\theta = 0$), if the taxi operating cost d is smaller than d_2^c* (higher than d_3^c, resp.)*, the bus patronage is smaller* (resp. higher) *at optimum than at equilibrium.*[9]

Proof. *See AppendixG.* □

We conjecture the variations in demand for *Group B* when $d \in [d_2^c; d_3^c]$.

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[8]The critical value of the taxi operating cost d is $d_1^c = \frac{3(\eta+\gamma)x}{8\Delta} - \breve{\alpha}$. The critical value of the punctuality cost c_1^c is defined as the unique solution of $D^o_{\theta=1} = D^e_{\theta=1}$.

[9]The critical values of the taxi operation cost d are $d_2^c = -\left(\frac{1}{2}\eta - \frac{7}{2}\beta\right) x/2\Delta - \alpha_{taxi}$ and $d_3^c = (2\eta + \beta)\, x/2\Delta - \alpha_{taxi}$, with $d_2^c < d_3^c$.

Conjecture 2. *For Group B, when $d \in [d_2^c; d_3^c]$, the bus patronage is higher at optimum than at equilibrium if the punctuality cost for the bus company is small enough ($c > c_2^c$).*[10]

This conjecture is discussed in AppendixH. The basic idea in Propositions 5 and 6 and in Conjecture 2 is that when the taxi operating cost is small, the bus company tends to underprice which consequently attracts too many customers. As the taxi operating cost is high, the bus company overprices. This is due to the fact that the bus fare highly depends on the taxi fare (see equation (6)).

The equilibrium modal split meets the optimal modal split under two conditions. First the taxi operating cost d has to be included between the two critical values we defined. Then the punctuality cost incurred by the bus company c has to equal a critical value. If the taxi operating cost is higher than the interval defined by critical values, the optimal modal split is reached by a partial commuters shift from taking a taxi to taking a bus. This shift can also be in the opposite direction if the taxi operating cost is smaller than the critical interval. This reflect the fact that the bus company underprovides quality relative to the social optimum when c is small (see De Borger and Van Dender, 2006, for a detailed discussion).

The taxi operating cost corresponds to the traditional costs as fuel or insurance, but it may also be viewed as an extra tax set by the planner to account for the externalities such as pollution or noise. In this sense, the operating cost trend should be growing and in the long run, the bus patronage would increase at the expense of the taxi service.

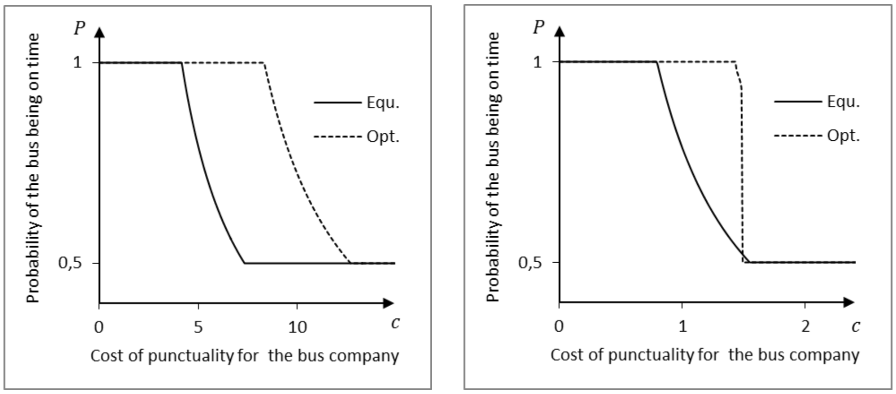
5. Numerical application

We develop an applied case to illustrate previous theoretical findings. Numerical results are obtained with values specified in Table 1. The studied case is related to a 25 minutes bus trip. The bus has a probability P of being on time and a probability $1 - P$ of being 10 minutes late at departure. The commuter living the farthest from their trip destination has a taxi trip time equal to 35 minutes. We consider a uniform distribution of the taxi trip time. The operating taxi cost d is constant and equal to 40 \$/hour. Lastly, cost parameters α_{bus}, α_{taxi}, η, β and γ are equal to 15, 4, 20, 10 and 30 \$/hour, resp. Each variable is drawn depending on the reliability cost for the bus c.

A reminder to the readers, P^e and P^o are respectively the probability of the bus being on time at equilibrium and at optimum. As expected, the probability of the bus being on time decreases when the reliability cost increases (see Figure 5). The more expensive the punctuality is, the less interesting is the reliability for both the bus company and the social planner.

As indicated in Proposition 4, the probability of the bus being on time when $\theta = 1$ is higher at optimum than at equilibrium. The opposite extreme

[10] The critical value of reliability c_2^c is the unique solution of $D_{\theta=0}^o = D_{\theta=0}^e$.



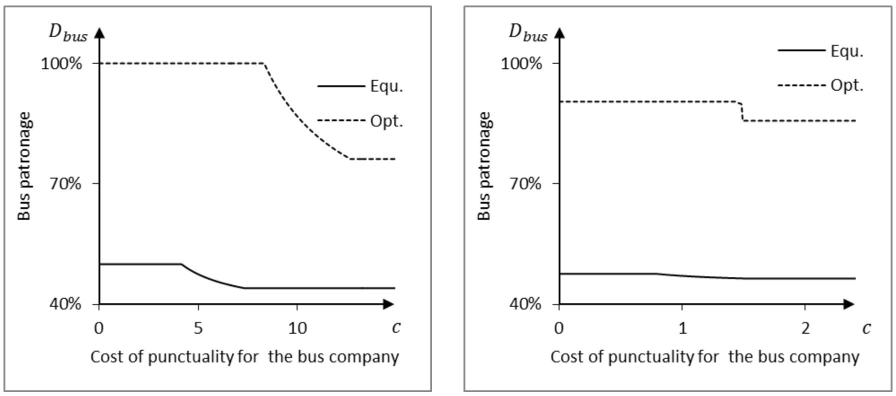
$Group\,A(\theta = 1)$ $\qquad$ $Group\,B(\theta = 0)$

Figure 5: Probability of the bus being on time as a function of the punctuality cost c

case where $\theta = 0$ is more complex as P^o is not continuous. It seems that the probability of the bus being on time is higher at optimum than at equilibrium when c is small and that after a critical value of c this relation is inverted. Probabilities of the bus being on time are higher when $\theta = 1$ than when $\theta = 0$. This is due to the fact that users from *Group A* are more sensitive to unreliability because when the bus is late they incur a late delay cost which is higher than the waiting time cost incurred by commuters from *Group B*. Thus when $\theta = 1$, the bus company needs to maintain a better level of service than when $\theta = 0$ in order to keep their patrons. An important observation is that the optimal punctuality may be very low and even equal to 0.5 which is the worst reliability level. Indeed, since the reliability cost is not too high, the social planner makes the bus company increase the punctuality of the bus to minimize the cost born by users. However, if the punctuality cost for the bus is too high, it is socially better to share cost with users by making or allowing the bus to be late.

Two points are especially interesting in Figure 6. First, the bus patronage is weakly decreasing when the punctuality cost increases. This drop is higher at optimum than at equilibrium. Along with Figure 5 we note that the punctuality has a strong effect on demand. The variations of the bus patronage corresponds to the variations of the bus punctuality. When the bus punctuality is stable, the split between the bus and the taxi is constant.Secondly, note that the demand for the bus is higher at optimum than at equilibrium in both extreme cases. Regarding Proposition 5, this example illustrates the common case where the bus patronage is higher at optimum than at equilibrium. At equilibrium the bus patronages is sub-optimal. Too much commuters use the taxi service because catching the bus is too expensive and the bus is not reliable enough.

Table 2 provides the values of main variables when $c = 5$ and $\theta = 0.75$. The probability of the bus being on time at equilibrium equals to 0.72 (note that this measure is consistent with observed average lateness in Paris Area (STIF, 2013a)). The bus fare may seem high, but it is not surprising as we do not take



Group $A(\theta = 1)$ *Group* $B(\theta = 0)$

Figure 6: Bus patronage as a function of the punctuality cost c

Variable	Equilibrium	Optimum
Probability of the bus being on time	0.72	0.88
Bus fare	16.9$	0$
Taxi fare	70$/hour	40$/hour
Bus patronage	47%	96%
Social gain	-	42%

Table 2: Values of main variables when $c = 5$ and $\theta = 0.75$

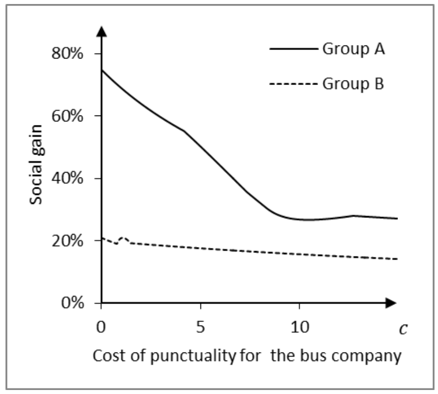


Figure 7: Relative social gain compared to equilibrium as a function of the punctuality cost c

into account subsidies. Indeed for example in Paris Area in 2010, monetary public transport revenues equal to 29.7% of total operating cost (STIF, 2013b). The optimum is reached by increasing the reliability at its maximal level and decreasing prices. Consequently, the bus patronage becomes much higher and the social gain is about 42%.

The relative social gain is computed as the ratio of the absolute gain, due to the transition from equilibrium to optimum, to the absolute social cost at equilibrium (see Figure 7). Such curves allow to determine when the gain is high enough to justify public intervention: the lower the punctuality cost is, the more useful is public intervention. Indeed when c is high (see equations 7, 11a and 11b), punctuality at equilibrium and at optimum is the same. The only difference between equilibrium and optimum is the modal split, but the gain due to this difference is gradually offset by the growing punctuality cost. Consequently, for both $Group\,A$ and $Group\,B$, the social gain tends to 0 when the punctuality cost, c, tends to infinity. The cut in social gain is faster for $Group\,A$ because variation in patronage is more sensitive with respect to the rise in c.

The brief application in this section illustrates that the effectiveness of public intervention varies according to punctuality cost. In the more general and realistic case, a stronger intervention seems useful in relation to the current situation.

6. Conclusion

The modeling of the bus punctuality reported here has provided an improved understanding of the two-way implication between punctuality level of public transport and customer public transport use. Commuters develop adaptive strategies to fit the transport system. Thus a rise in the fare of a mode decreases the patronage for this mode. In particular, an increase in the taxi fare rises the share of commuters arriving on time at the bus stop because they wish to minimize the probability of missing the bus. Moreover when the bus company becomes less strict as regards punctuality, more bus users will prefer to arrive late at the bus stop. Then the bus company is not incited to maintain a high level of reliability. This can generate a vicious circle. We also appreciate the efficiency of the punctuality when it is viewed as an instrument of service quality that can be adapted to fit and regulate the public transport patronage.

The main findings of this paper follow. At equilibrium, the probability of the bus being on time increases with the price of the alternative mode. The service quality reacts well to a rise in the taxi fare. Indeed, a new market share of commuters is assailable with a reasonable effort in terms of service quality. Compared with the optimum, buses are very often too late at equilibrium. Commuters bear the cost of this extra-lateness, because they have to wait for the bus more often or take the taxi which is expensive. However, it does not mean that the bus should not be late. Indeed if the cost of the punctuality is too high relative to the cost of the alternative mode, a late bus is socially preferable. Finally, we find that the sign and the amplitude of the gap between the equilibrium and optimal modal split first depends on the cost of the alternative mode and secondly on the punctuality cost incurred by the bus company. Nevertheless, in the more general and realistic case the bus patronage seems under-optimal.

Several elements remain to be addressed. Considering risk averse users would change users strategies and affect the punctuality. It should be interesting to include congestion on road networks and in the bus. Congestion on the road would make the taxi journey longer and unpredictable, whereas congestion in the bus (understood as crowding) would accentuate the cost incurred by users. Finally, introducing the bus punctuality in a bus transit line with several stops and several buses (de Palma and Lindsey, 2001) will improve the modeling by introducing a snowball effect: if a bus is late, its lateness increases along its journey.

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AppendixA. Proof of Proposition 1

We wish to compare expected costs of *Strategys A*, *B* and *O*, denoted $EC\,(O)$, $EC\,(L)$ and $EC\,(T)$ respectively, to define a choice rule for a commuter in *Group A*. From equations (1) and (2), we can write:

$EC\,(O) = \kappa + (1-P)\,(\eta+\gamma)\,x,$
$EC\,(L) = P\delta\,(\breve{\alpha}+\tau) + (1-P)\,(\kappa+\gamma x),$
$EC\,(T) = \delta\,(\breve{\alpha}+\tau).$

Therefore we have

$$EC\,(O) \leq EC\,(L) \text{ iff } \delta \geq \frac{\kappa + \left(\frac{1-P}{P}\right)\eta x}{\breve{\alpha}+\tau} \equiv \delta^A_{L,O}, \tag{A.1}$$

$$EC\,(T) < EC\,(L) \text{ iff } \delta < \frac{\kappa+\gamma x}{\breve{\alpha}+\tau} \equiv \delta^A_{T,L}, \tag{A.2}$$

$$EC\,(T) < EC\,(O) \text{ iff } \delta < \frac{\kappa + (1-P)\,(\eta+\gamma)\,x}{(\breve{\alpha}+\tau)} \equiv \delta^A_{T,O}. \tag{A.3}$$

We use A.1 and A.2 to rank $\delta^A_{L,O}$, $\delta^A_{T,L}$ and $\delta^A_{T,L}$:

(i) $\delta^A_{T,L} \geq \delta^A_{L,O} \iff \gamma \geq \left(\frac{1-P}{P}\right)\eta \iff \frac{\gamma}{\eta} \geq \frac{1-P}{P}$ which is true since $\frac{\gamma}{\eta} > 1 \geq \frac{1-P}{P}$ by A.1 and A.2.

(ii) $\delta^A_{T,L} \geq \delta^A_{T,O} \iff \gamma \geq (1-P)\,(\eta+\gamma) \iff P \geq 1 - \frac{\gamma}{(\eta+\gamma)}$ which is true since $\frac{\gamma}{(\eta+\gamma)} > \frac{1}{2}$ by A.2.

(iii) $\delta^A_{T,O} \geq \delta^A_{L,O} \iff \eta+\gamma \geq \frac{1}{P}\eta \iff P \geq \frac{\eta}{(\eta+\gamma)}$ which is true since $\frac{\eta}{(\eta+\gamma)} < \frac{1}{2}$by A.2.

Therefore $\delta^A_{L,O} \leq \delta^A_{T,O} \leq \delta^A_{T,L}$.

Strategys B is chosen if and only if $\delta < \delta^A_{L,O}$ and $\delta \geq \delta^A_{T,L}$. As $\delta^A_{L,O} \leq \delta^A_{T,L}$, *Strategys B* is dominated and never chosen by commuter in *Group A*. Figure A.8 summarizes results of the proof.

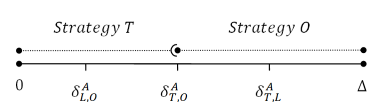

Figure A.8: Strategy choice of a commuter in *Group A* depending on the taxi trip time δ.

AppendixB. Proof of Proposition 2

We wish to compare expected costs of *Strategys A*, *B* and *O*, denoted $EC\,(O)$, $EC\,(L)$ and $EC\,(T)$ respectively, to define a choice rule for a commuter in *Group B*. From equations (1) and (2), we can write:

$EC(O) = \kappa + [(1-P)\eta + P\beta]x,$
$EC(L) = P\delta(\check{\alpha} + \tau) + (1-P)\kappa,$
$EC(T) = \delta(\check{\alpha} + \tau).$

Therefore we have

$$EC(O) \leq EC(L) \text{ iff } \delta \geq \frac{\kappa + \left(\frac{1-P}{P}\eta + \beta\right)x}{\check{\alpha} + \tau} \equiv \delta^B_{L,O}, \tag{B.1}$$

$$EC(T) < EC(L) \text{ iff } \delta < \frac{\kappa}{\check{\alpha} + \tau} \equiv \delta^B_{T,L}, \tag{B.2}$$

$$EC(T) < EC(O) \text{ iff } \delta < \frac{\kappa + [(1-P)\eta + P\beta]x}{\check{\alpha} + \tau} \equiv \delta^B_{T,O}. \tag{B.3}$$

We use A.1 and A.2 to rank$\delta^B_{L,O}$, $\delta^B_{T,L}$ and $\delta^B_{T,L}$:

(i) $\delta^B_{T,L} \leq \delta^B_{L,O} \iff 0 \leq \left(\frac{1-P}{P}\eta + \beta\right)$ which is true.

(ii) $\delta^B_{T,L} \leq \delta^B_{T,O} \iff 0 \leq (1-P)\eta + P\beta$ which is true.

(iii) $\delta^B_{T,O} \leq \delta^B_{L,O} \iff (1-P)\eta + P\beta \leq \frac{1-P}{P}\eta + \beta \iff 1 \geq \frac{1}{P}$ which is true since $\frac{1}{P} \geq 1$ by A.1.

Therefore $\delta^B_{T,L} \leq \delta^B_{T,O} \leq \delta^B_{L,O}$. Figure B.9 summarizes results of the proof.

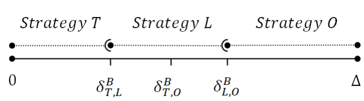

Figure B.9: Strategy choice of a commuter in *Group B* depending on the taxi trip time δ.

AppendixC. Proof of Proposition 3

We wish to show that P^e, the probability of the bus being on time at equilibrium, and κ^e, the bus fare, increase with τ the taxi fare. We first show that $\partial P^e/\partial\tau \geq 0$ (i) and that $\partial\kappa^e/\partial\tau \geq 0$ (ii). Then we check that boundaries of interval, c^e_1 (iii) and c^e_2 (iv), increase with τ. Let us recall expressions of equilibrium variables (see equations (6) and (**??**)):

$$\begin{aligned}
\kappa^e &= \frac{1}{2}\left\{\check{\triangle}^e - [(1-P^e)\eta + (1-\theta)P^e\beta + \theta(1-P^e)\gamma]x\right\}, \\
P^e &= \begin{cases} \frac{1}{2} & \text{if } c > c^e_2, \\ \frac{\kappa^e\check{\eta}x}{c\check{\triangle}^e} & \text{if } c \in [c^e_1; c^e_2], \\ 1 & \text{if } c < c^e_1, \end{cases}
\end{aligned}$$

where $\check{\triangle}^e = \Delta(\check{\alpha}+\tau^e)$, $\check{\eta} = \eta - (1-\theta)\beta + \theta\gamma$, $c_1^e \equiv \kappa^e\check{\eta}x/\check{\triangle}^e$ and where $c_2^e \equiv 2c_1^e$. By substituting κ^e in P^e, we obtain

$$P^e = \begin{cases} \frac{1}{2} & \text{if } c > c_2^e, \\ \frac{\left[\check{\triangle}^e - (\eta+\theta\gamma)x\right]\check{\eta}x}{2c\check{\triangle}^e - (\check{\eta}x)^2} & \text{if } c \in \left[c_1^e; c_2^e\right], \\ 1 & \text{if } c < c_1^e, \end{cases}$$

where $c_1^e = \left[\check{\triangle}^e - (1-\theta)\beta x\right]\check{\eta}x/2\check{\triangle}^e$ and where $c_2^e = \left(\check{\triangle}^e - \check{\eta}x\right)\check{\eta}x/2\check{\triangle}^e$. We now derive P^e, κ^e, c_1^e and c_2^e on τ^e.

(i) $\frac{\partial P^e}{\partial \tau^e} = \Delta\check{\eta}x\frac{2c(\eta+\theta\gamma)x-(\check{\eta}x)^2}{\left[2c\check{\triangle}^e-(\check{\eta}x)^2\right]^2}$ so $\frac{\partial P^e}{\partial \tau} \geq 0$ if $c \geq \frac{(\check{\eta}x)^2}{2(\eta+\theta\gamma)x}$. Let us substitute c by c_1^e the minimal value of the interval $[c_1^e; c_2^e]$. Thus

$$c_1^e - \frac{(\check{\eta}x)^2}{2(\eta+\theta\gamma)x} = \frac{\check{\eta}x\left[\check{\triangle}^e - (\eta+\theta\gamma)x\right](1-\theta)\beta x}{2\check{\triangle}^e(\eta+\theta\gamma)x}.$$

Yet $\frac{\check{\eta}x(1-\theta)\beta x}{2\check{\triangle}^e(\eta+\theta\gamma)x} \geq 0$ and $\check{\triangle}^e - (\eta+\theta\gamma)x \geq 0$ by A.3. We therefore have $\frac{\partial P^e}{\partial \tau} \geq 0$;

(ii) $\frac{\partial \kappa^e}{\partial \tau^e} = \frac{\Delta}{2} + \frac{\partial P^e}{\partial \tau^e}\frac{\check{\eta}x}{2} \geq 0$ by A.2,

(iii) $\frac{\partial c_1^e}{\partial \tau^e} = \frac{(1-\theta)\beta x\check{\eta}x}{2\Delta(\alpha_{taxi}+\tau^e)^2} \geq 0$,

(iv) $\frac{\partial c_2^e}{\partial \tau^e} = \frac{(\check{\eta}x)^2}{2\Delta(\alpha_{taxi}+\tau)^2} \geq 0$.

P^e, the probability of the bus being on time at equilibrium, and κ^e, the bus fare, increase well with τ^e the taxi fare.

AppendixD. Proof: optimal bus and taxi fare

The social planner chooses the punctuality level P, the bus fare κ and taxi fare τ so as to minimize social cost. The first-order conditions for the socially optimal bus and taxi prices are given by

$$\frac{\partial SC}{\partial \kappa} = \frac{\kappa(\check{\alpha}+d) - (\tau-d)\Gamma x}{\Delta(\check{\alpha}+\tau)^2} = 0, \tag{D.1a}$$

$$\frac{\partial SC}{\partial \tau} = \frac{(\tau-d)A - \kappa(\check{\alpha}+d)(\kappa+\Gamma x)}{\Delta(\check{\alpha}+\tau)^3} = 0, \tag{D.1b}$$

where $\Gamma = (1-P)\eta + (1-\theta)P\beta + \theta(1-P)\gamma$, $A = \kappa\Gamma x + \chi$ and $\chi = \theta\left[(1-P)(\eta+\gamma)x\right]^2 + (1-\theta)P\left[\left(\frac{1-P}{P}\eta+\beta\right)x\right]^2$. Therefore from (D.1a) and (D.1b)

$$\kappa^o = \frac{(\tau^o-d)\Gamma x}{(\check{\alpha}+d)}, \tag{D.2a}$$

$$\tau^o = \frac{\kappa^o(\check{\alpha}+d)(\kappa^o+\Gamma x)}{A\Delta(\check{\alpha}+\tau^o)^3} - d. \tag{D.2b}$$

By substituting (D.2a) into (D.2b), the first-best optimal bus and taxi prices can be written as[11]

$$\begin{aligned}\kappa^o &= 0;\\ \tau^o &= d.\end{aligned}$$

AppendixE. Proof of Proposition 4

We wish to show that the probability of the bus being on time is higher in the optimal situation than in equilibrium when $\theta = 1$. For that, we need to show that the result of $P^o_{\theta=1} - P^e_{\theta=1}$ is positive (i) and that the limits of the variation intervals are well sorted i.e. $c^o_{1;\theta=1} \geq c^e_{1;\theta=1}$ (ii) and $c^o_{2;\theta=1} \geq c^e_{2;\theta=1}$(iii):

(i) $P^o_{\theta=1} - P^e_{\theta=1} = \frac{c\check{\Delta}(\eta+\gamma)x\left[\check{\Delta}-(\eta+\gamma)x\right]}{\left\{2c\check{\Delta}-[(\eta+\gamma)x]^2\right\}\left\{c\check{\Delta}-[(\eta+\gamma)x]^2\right\}} \geq 0$ by A.3;

(ii) $c^o_{1;\theta=1} - c^e_{1;\theta=1} = \frac{\check{\Delta}(\eta+\gamma)x}{2\check{\Delta}} \geq 0$ by A.3;

(iii) $c^o_{2;\theta=1} - c^e_{2;\theta=1} = \frac{\left[2\check{\Delta}-(\eta+\gamma)x\right](\eta+\gamma)x}{2\check{\Delta}} \geq 0$ by A.3.

The probability of the bus being on time well and truly is higher in the optimal situation than in equilibrium.

AppendixF. Proof of Proposition 5

The idea of the proof is that the difference between optimal demand for the bus and equilibrium demand for the bus is a function of c, the bus punctuality cost and d the taxi operating cost. Throughout this proof we consider the extreme case where $\theta = 1$. Let us recall the expression of demand for the bus function:

$$D_{bus} = \int_{\delta^A_{T,O}}^{\Delta} f(\delta)d\delta,$$

where $\delta^A_{T,O} = \left[\kappa + (1-P)(\eta+\gamma)x\right]/(\breve{\alpha}+\tau)$. We can define

$$\begin{aligned}\overline{D} \equiv D^o_{bus} - D^e_{bus} &= 1 - \frac{\kappa^o + (1-P^o)(\eta+\gamma)x}{\Delta(\breve{\alpha}+\tau^o)}\\ &\quad - \left(1 - \frac{\kappa^e + (1-P^e)(\eta+\gamma)x}{\Delta(\breve{\alpha}+\tau^e)}\right),\end{aligned}$$

where $\kappa^o = 0$, $\tau^o = d$, $\kappa^e = \frac{1}{2}\left[\Delta(\breve{\alpha}+\tau^e) - (1-P^e)(\eta+\gamma)x\right]$ and $\tau^e = \breve{\alpha}+2d$. We therefore have

$$\overline{D} = \frac{2\Delta(\breve{\alpha}+d) - (3+P^e-4P^o)(\eta+\gamma)x}{4\Delta(\breve{\alpha}+d)}.$$

[11] Second-order conditions are satisfied as they require $(\alpha_{taxi}+d) \geq 0$ and $A \geq 0$.

Since P^e and P^o are functions of c (equations (7) and (11a)), we derive $\overline{D}$ on c. For that, we need to know the order of c_1^e, c_2^e, c_1^o and c_2^o. We know that $c_1^e \leq c_2^e$ and $c_1^o \leq c_2^o$.

$$
\begin{aligned}
c_1^o - c_2^e = &\quad (\eta + \gamma)\, x - \frac{2\kappa^e \check{\eta} x}{\check{\triangle}^e}, \\
\Longleftrightarrow \quad c_1^o - c_2^e = &\quad (\eta + \gamma)\, x \left[1 - \frac{(1-P)\, 2\kappa^e}{\check{\triangle}^e}\right], \\
\Longleftrightarrow \quad c_1^o - c_2^e = &\quad (\eta + \gamma)\, x \left[1 - (1-P)\left(1 - \frac{\Gamma^e x}{\check{\triangle}^e}\right)\right] \geq 0.
\end{aligned}
$$

We therefore have $c_1^e \leq c_2^e \leq c_1^o \leq c_2^o$ and we distinguish between five sub-cases defined depending on the position of c relatively to c_1^e, c_1^o, c_2^e and c_2^o. Indeed the expression of the derivative is different according to the value of c.

(i) If$c \leq c_1^e$ then $P^e = P^o = 1$ and $\partial\overline{D}/\partial c = 0$.

(ii) If $c \in [c_1^e; c_2^e]$ then $P^o = 1$ and $\partial\overline{D}/\partial c \geq 0$.

(iii) If$c \in [c_2^e; c_1^o]$ then $P^o = 1$, $P^e = \frac{1}{2}$ and $\partial\overline{D}/\partial c = 0$.

(iv) If $c \in [c_1^o; c_2^o]$ then $P^e = \frac{1}{2}$ and $\partial\overline{D}/\partial c \leq 0$.

(v) If $c \geq c_2^o$ then $P^e = P^o = \frac{1}{2}$ and $\partial\overline{D}/\partial c = 0$.

Critical values of $\overline{D}(c)$ follow:

$\overline{D}(c_1^e) = \frac{1}{2}$,

$\overline{D}(c_2^e) = \frac{1}{2} + \frac{(\eta+\gamma)x}{8\Delta(\check{\alpha}+d)}$,

$\overline{D}(c_2^o) = \frac{1}{2} - \frac{3(\eta+\gamma)x}{8\Delta(\check{\alpha}+d)}$,

where $\bar{\Delta} = \Delta(\alpha_{taxi} + d)$.

The variations of the difference between optimal demand for the bus and equilibrium demand for the bus are described in Table F.3.

c	0		c_1^e		c_2^e		c_1^o		c_2^o		∞
$\frac{\partial \overline{D}}{\partial c}$		0		$+$		0		$-$		0	
$\overline{D}\left(c\right)$		$\longrightarrow$	$\overline{D}\left(c_1^e\right)$	$\nearrow$	$\overline{D}\left(c_2^e\right)$	$\longrightarrow$		$\searrow$	$\overline{D}\left(c_2^o\right)$	$\longrightarrow$	

Table F.3: Variation table of the difference between optimal demand for the bus and equilibrium demand for the bus depending on the cost of reliability.

We know that $\overline{D}\left(c_1^e\right) \geq 0$ and $\overline{D}\left(c_1^e\right) \geq \overline{D}\left(c_1^o\right)$. According to Table F.3, we can distinguish between two cases where the difference between optimal and equilibrium demand for the bus is positive. First, if the minimum value of the difference,$\overline{D}\left(c_2^o\right)$ is positive, the difference is positive. Second, if this minimum value of the difference is negative, then as $\overline{D}\left(c_2^e\right) \geq 0$ and $\overline{D}\left(c\right)$strictly decreases between c_1^o and c_2^o, there exists a unique value of c denoted c^c for which $D^o_{bus} \underset{c=c^c}{=} D^e_{bus}$. The difference is then positive if $c \leq c^c$.

One critical value of the taxi operating cost d_1^c may be defined such that

$$\begin{aligned}
\overline{D}\left(c_2^o\right) \geq 0 \quad &\Longleftrightarrow \quad 2\Delta\left(\breve{\alpha} + d\right) - \left(3 + P^e - 4P^o\right)\left(\eta + \gamma\right)x \geq 0, \\
&\Longleftrightarrow \quad d \geq \frac{3\left(\eta + \gamma\right)x}{4\Delta} - \breve{\alpha} \equiv d_1^c.
\end{aligned}$$

We can now write

$$\overline{D}\begin{cases} \geq 0 & \begin{array}{l} \text{if} \quad d \geq d_1^c, \\ \text{or if} \quad d < d_1^c \quad \text{and } c \leq c^c, \end{array} \\ < 0 & \text{if} \quad d < d_1^c \quad \text{and } c > c^c \ . \end{cases} .$$

AppendixG. Proof of lemma 6

The idea of the proof is that the difference between optimal demand for the bus and equilibrium demand for the bus is a function of the cost of the bus reliability c and the operating cost of taxi d. We deal with the case where $\theta = 0$. Let us recall expressions of the demand function:

$$D_{bus} \quad = \quad \left(1 - P\right)\int_{\delta^B_{T,L}}^{\delta^B_{L,O}} f(\delta)d\delta + \int_{\delta^B_{L,O}}^{\Delta} f(\delta)d\delta,$$

where $\delta^B_{L,O} = \left[\kappa + \left(\frac{1-P}{P}\eta + \beta\right)x\right] / \left(\breve{\alpha} + \tau\right)$ and $\delta^B_{T,L} = \kappa / \left(\breve{\alpha} + \tau\right)$. We therefore have:

$$\overline{D} \equiv D^o_{bus} - D^e_{bus} \quad = \quad \frac{2\Delta\left(\breve{\alpha} + d\right) + \left(4P^o_{\theta=0} - P^e_{\theta=0}\right)\left(\eta - \beta\right)x - 3\eta x}{4\Delta\left(\breve{\alpha} + d\right)}.$$

Then

$$\overline{D} \geq 0 \iff d \geq \frac{3\eta x - \left(4P^o_{\theta=0} - P^e_{\theta=0}\right)(\eta - \beta)\, x}{2\Delta} - \breve{\alpha}.$$

Considering $\max\left(4P^o_{\theta=0} - P^e_{\theta=0}\right) = \frac{7}{2}$ and $\min\left(4P^o_{\theta=0} - P^e_{\theta=0}\right) = 1$, we can define d_2^c and d_3^c such as if $d \leq d_2^c$ then $\overline{D} \leq 0$ and if $d \geq d_3^c$ then $\overline{D} \geq 0$. Consequently we have $d_2^c = -\left(\frac{1}{2}\eta - \frac{7}{2}\beta\right) x/2\Delta - \breve{\alpha}$ and $d_3^c = (2\eta + \beta)\, x/2\Delta - \breve{\alpha}$. We may write:

$$\overline{D} \begin{cases} \geq 0 & \text{if } d \geq d_3^c, \\ \leq 0 & \text{if } d \leq d_2^c. \end{cases}$$

AppendixH. Discussion of Conjecture 2

With values specified in Table 1, we can draw the curve of the difference between optimal demand for the bus and equilibrium demand for the bus depending on the operating taxi cost d in Figure H.10. When c is small, $P^o = P^e = 1$ and when c is large, $P^o = P^e = 1/2$. The lemma 6 is illustrated. $\overline{D}$ functions are first negative then positive. Moreover they increase with d. The sign of $\overline{D}$ between d_2^c and d_3^c depends on the values of P^o and P^e which depend on c (see Equations (7) and (11b)). Therefore we conjecture that between d_2^c and d_3^c, $\overline{D}$ is positive when $c \leq c_2^c$ and negative when $c > c_2^c$, where c_2^c is defined as the unique solution of $D^o_{\theta=0} = D^e_{\theta=0}$.

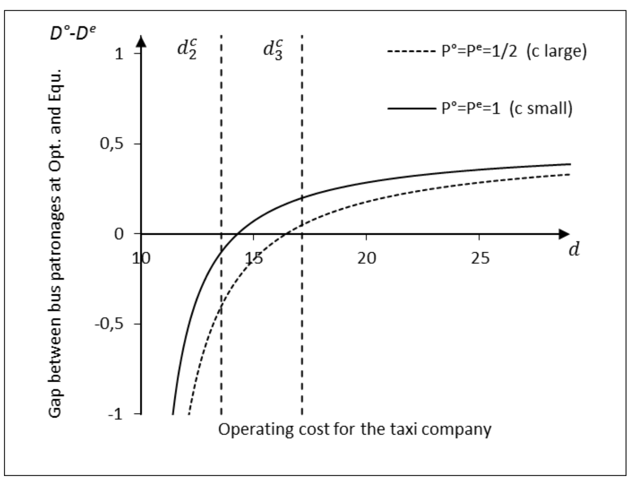

Figure H.10: Difference between optimal demand for the bus and equilibrium demand for the bus depending on operating taxi cost d for *Group B*.