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Radio Labelings of Distance Graphs

Roman Čada 1, Jan Ekstein 1, Přemysl Holub 1, and Olivier Togni 2

1 University of West Bohemia, Pilsen, Czech Republic
2 LE2I, University of Bourgogne, France

Extended Abstract

A radio $k$-labeling of a connected graph $G$ is an assignment $f$ of non negative integers to the vertices of $G$ such that
$$|f(x) - f(y)| \geq k + 1 - d(x, y),$$
for any two distinct vertices $x$ and $y$, where $d(x, y)$ is the distance between $x$ and $y$ in $G$. The radio $k$-labeling number $r_k(G)$ of $G$ is the minimum of $\max_{x, y \in V(G)} |f(x) - f(y)|$ over all radio $k$-labelings $f$ of $G$.

The study of radio $k$-labelings was initiated by Chartrand et al. [1], motivated by radio channel assignment problems with interference constraints.

Except for paths [1, 3] and cycles [5], radio $k$-labelings have been investigated mainly for fixed values of $k$. This problem generalizes both the classical proper vertex-colouring problem (when $k = 1$) and the well studied $L(2,1)$-labeling problem (when $k = 2$). The other values of $k$ considered were when $k$ is close to the diameter of the graph. The interested reader is referred to surveys [2, 7] and recent papers [6, 8] for complementary results.

For a set of positive integers $\{d_1, d_2, \ldots, d_t\}$, the (infinite) distance graph $D(d_1, d_2, \ldots, d_t)$ has the set $\mathbb{Z}$ of integers as vertex set, with two distinct vertices $i, j \in \mathbb{Z}$ being adjacent if and only if $|i - j| = d_t$, for some $t$.

Concerning radio $k$-labelings of distance graphs, the only known results are for $k = 2$ and mainly for 4-regular distance graphs [4, 9]. Moreover, for the path $P_n$ of order $n$ (a finite subgraph of $D(1) = P_{\infty}$), the following bounds were proved in [1, 3]: for any $n > 3$ and any $1 \leq k \leq n - 3$,
$$\frac{k^2 + 4}{2} \leq r_k(P_n) \leq \frac{k^2 + 2k}{2}, \text{ if } k \text{ is even},$$
$$\frac{k^2 + 1}{2} \leq r_k(P_n) \leq \frac{k^2 + 2k - 1}{2}, \text{ if } k \text{ is odd};$$
and it was conjectured in [3] that the upper bound is the exact value of the radio $k$-labeling number when the length of the path is large enough.

We prove the following results:
$$\frac{k^2}{2} + \frac{1}{2} \leq r_k(D(1, 2, \ldots, t)) \leq \begin{cases} \frac{k^2}{2} + \frac{1}{2}k, & \text{when } k \text{ is odd}, \\ \frac{k^2}{2} + \frac{1}{2}k, & \text{when } k \text{ is even}. \end{cases}$$
$$\frac{k^2}{2} - P_2(t)k + P_3(t) \leq r_k(D(1, t)) \leq \frac{k^2}{2}, \text{ for } t \geq 3 \text{ and odd } k,$$
$$\frac{k^2}{2} - Q_2(t)k + Q_3(t) \leq r_k(D(t - 1, t)) \leq \frac{k^2}{2} + k - \frac{13}{2}, \text{ for } t \geq 3 \text{ and odd } k;$$

where $P_i(t)$ and $Q_i(t)$ denote polynomials of variable $t$ of degree $i$.

For each upper bound, we have found a corresponding coloring sequence with the desired number of labels while lower bounds were obtained by bounding the upper traceable number of the distance graphs by a function of the same parameter on the infinite path.
References


