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## Modeling and Characterization of Oscillator Circuits by Van der Pol Model using Parameter Estimation

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In this paper, we present a new method for the modeling and characterization of oscillator circuit with a Van-Der Pol (VDP) model using parameter identification. We also discussed and investigated the problem of estimation in nonlinear system based on time domain data. The approach is based on an appropriate state space representation of Van der Pol oscillator that allows an optimal parameter estimation. Using sampled output voltage signal, model parameters are obtained by an iterative identification algorithm based on Output Error method. Normalization issues are fixed by an appropriate transformation allowing a quickly global minimum search. Finally, the proposed estimation method is tested and validated using simulation data from a 1GHz oscillator circuit in GaAs technology.

*Keywords:* Oscillator; Van der Pol Model; parameter estimation; Nonlinear system; modeling; continuous time domain.

### 1. Introduction

The interest in oscillators is motivated, among others, by their recent use in antenna array systems. Coupled oscillators are a simple and efficient method for phase control in microwave antenna arrays due to their synchronization properties<sup>1,2,3,4</sup>. Nevertheless, for the design of an oscillator circuit or an array of coupled oscillators, one must be able to test and implement control laws on simple systems that reproduce well the dynamics of the real system. Classical Van der Pol (VDP) oscillators are usually used to model microwave oscillators. Indeed, in<sup>5,6,7</sup>, R. York made use of simple Van der Pol oscillator to model an array of oscillators coupled through many types of circuits. Hence, this theory provides a full analytical formulation allowing to predict the performances of microwave oscillators arrays. Nevertheless, the identification of the parameters of the VDP oscillator is not always made easily.

## 2 Oscillator Modeling and Characterization using Parameter Estimation

The main difficulty in modeling an oscillating system is caused by its highly and complex non-linear behavior. The parameter computation for strongly non-linear devices is difficult and time consuming.

Parameter identification can be defined as mathematical modeling of a process in order to understand, predict and enhance its dynamic behavior using input and output set of data. Furthermore, such an identification can be carried out using an iterative identification algorithm based on Output Error (OE) method. Output Error techniques are based on iterative minimization of a quadratic criterion, also called cost function, by a Non-Linear Programming (NLP) technique<sup>8,9,10</sup>. This method requires much more computation and do not converge to unique optimum<sup>11,12,13</sup>. Nevertheless, OE methods present very attractive features because the simulation of the output model is based only on the knowledge of the input so that the parameter estimates are unbiased<sup>14,15,16</sup>. Moreover, OE methods can be used to identify non linear systems. For these reasons, the OE methods are more appropriate in microwave systems characterization<sup>17</sup>.

Due to these considerations, the aim of this paper is to present a tool allowing to extract the electrical and nonlinear parameters of the Van der Pol model, reproducing the behavior of a 1.025GHz GaAs FET oscillator circuit, using an identification technique based on Output Error (OE) method.

This paper will be organized as follows. The principles of oscillator circuits and their representation with VDP model in state space will be described in section 2. In section 3, we will introduce and present the proposed method allowing to estimate the involved parameters using Output-Error technique. The application of this technique in the special case of an oscillator circuit will also be investigated. Two examples will then be studied in section 4: a theoretical one to analyze the performances of the method in term of robustness under noisy data and a practical one in order to see the efficiency of the study in a real case.

### 2. Oscillator design and modeling

The basic form of a harmonic oscillator consists in an electronic amplifier connected in a positive feedback loop to a frequency selective network like an *RLC* resonator. Such an architecture is presented in (Fig. 1) where the amplifier, also called the active part, compensates for the tank losses to enable a constant amplitude oscillation.

#### 2.1. VDP model

The models used for real circuits need to take into consideration the energy dissipation and compensation. Thus, the circuit must contain a dynamic element with a negative resistance so that oscillation is maintained. Negative resistance circuit elements are purely theoretical and does not exist as discrete components. However, some elements present, in some parts of their operating characteristic, a negative resistance. The Van der Pol oscillator is considered as a widely used example in

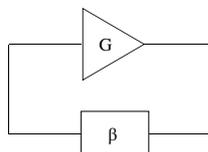


Fig. 1. Block diagram of a feedback oscillator

the literature<sup>18</sup>. It is highly nonlinear and it can exhibit both stable and unstable limit cycles. The main advantages of this model is the simplicity in simulation and high level in understanding and studying nonlinear dynamical phenomena, but also because it contains the necessary and sufficient elements to describe a real oscillator.

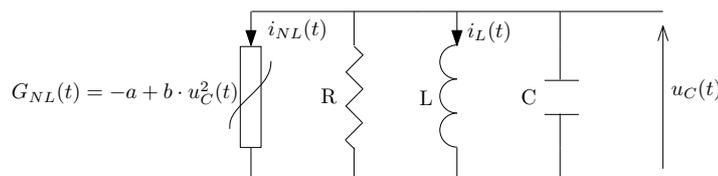


Fig. 2. The Van der Pol Model

The VDP model used contains a nonlinear conductance, noted  $G_{NL}(t)$ , and a  $RLC$  resonator circuit as shown in Fig. (2). The nonlinear conductance,  $G_{NL}(t)$ , models the active part of the oscillator and exhibits the necessary negative resistance region on its operating characteristic. The current  $i_{NL}(t)$  to voltage  $u_C(t)$  relationship describing the behavior of the nonlinear conductance is a cubic equation:

$$i_{NL}(t) = -a \cdot u_C(t) + b \cdot u_C^3(t) = G_{NL}(t) \cdot u_C(t) \quad (1)$$

Since, such an oscillator topology can be modeled by a quasi-linear representation allowing a simple analytical calculation<sup>19</sup>, the negative conductance presented by the active part in order to compensate for the tank losses  $R$  is equal to  $-G_{NL}(A) = -a + \frac{3}{4} \cdot b \cdot A^2$  where  $A$  is the amplitude of the oscillations.

Mathematically, a general form of the second order differential equation that describes the Van der Pol oscillator is:

$$\ddot{u}_C(t) - \mu(1 - u_C^2(t))\dot{u}_C(t) + u_C(t) = 0 \quad (2)$$

where the parameter  $\mu$  is a damping indicator,  $\dot{u}_C(t) = \frac{du_C(t)}{dt}$  and  $\ddot{u}_C(t) = \frac{d^2u_C(t)}{dt^2}$  denote respectively the first and the second derivative of  $u_C(t)$ .

## 2.2. VDP state space representation

The state equation representation of a physical system consists in describing the system as a set of inputs, outputs and state variables related by first-order differential equations. This modeling method, using matrix representation, can be applied for systems with multiple-inputs and multiple-outputs (MIMO) and the model includes the internal state variables and the output variables. Nevertheless, the most important advantage of this modeling form is that the representation consists in simple first order differential equations and provides directly a time-domain solution.

The state vector, noted  $\underline{x}$ , composed of state variables, is a minimum set of variables that are fully describing the system and its response to any given set of inputs. These variables are related to the energy storage elements in a circuit. In this case, the state vector includes two state variables, the current through the inductor  $i_L(t)$  and the voltage across the capacitor  $u_C(t)$ . Using elementary electrical rules (Kirchhoff's and Ohm's laws) and the nonlinear current expression given by (1), we can write the two following differential equations:

$$\begin{cases} C \frac{du_C(t)}{dt} + \frac{1}{R} u_C(t) + i_L(t) = a \cdot u_C(t) - b \cdot u_C^3(t) \\ \frac{di_L(t)}{dt} = \frac{1}{L} u_C(t) \end{cases} \quad (3)$$

For the Van der Pol oscillator, the state space representation of the system, written in matrix form, is the following set of first order differential equations:

$$\begin{cases} \dot{\underline{x}}(t) = \mathcal{A} \cdot \underline{x}(t) + \mathcal{B}(t) \\ y(t) = \mathcal{C} \cdot \underline{x}(t) \end{cases} \quad (4)$$

where  $\underline{x}(t)$  is the state vector such as:

$$\underline{x}(t) = \begin{bmatrix} u_C(t) \\ i_L(t) \end{bmatrix} \quad (5)$$

and the matrices are:

$$\mathcal{A} = \begin{bmatrix} (a - \frac{1}{R}) \cdot \frac{1}{C} & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}, \quad \mathcal{B}(t) = \begin{bmatrix} -\frac{b}{C} \cdot u_C^3(t) \\ 0 \end{bmatrix} \quad \text{and} \quad \mathcal{C} = [1 \quad 0]$$

Let us note that the output signal  $y(t)$  is defined as the state variable of interest which is the output voltage  $u_C(t)$  in our case.

### 3. Parameter identification of the VDP model

Parameter identification is based on the definition of a model. Once the previous mathematical model has been chosen, we can define the parameter vector to be estimated as follows:

$$\underline{\theta} = [a \quad b \quad C]^T \quad (6)$$

where  $[\cdot]^T$  denotes the transposition operation.

The choice of this parameter vector is not random. Indeed, the parameters  $a$  and  $b$  of the non-linear conductance are not easily accessible and thus, need to be identified. Concerning the RLC resonator, designers have *prior* information on the resonator losses, i.e. the  $R$  parameter. Furthermore, the frequency of oscillation is determined by the  $LC$  tank. Hence, knowing one of these two parameters, the frequency of oscillation can be adjusted by varying the other one. In the practical case of a VCO (Voltage Controlled Oscillator), a variable capacitor is used to tune the oscillator. This has led us to the estimation of the parameter  $C$ . Let us note that this method is not restricted to this vector and can be generalized to the inductance  $L$ .

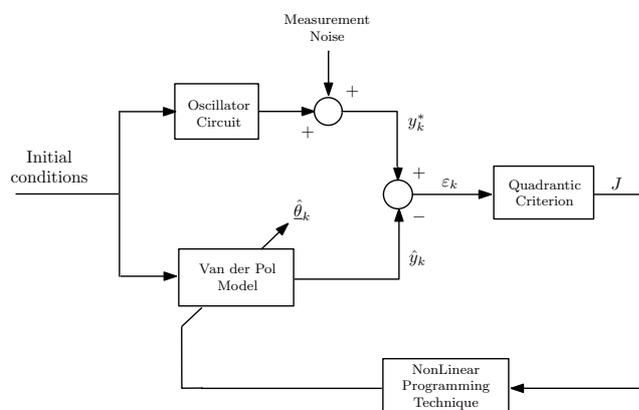


Fig. 3. Principle of Output Error Methods

The block diagram of off-line output error technique is shown in Fig. (3). Let us note that for the oscillator circuit, no input signal is used and only initial values of voltage and current are used to run it. The same values are used in the simulation of the VDP model.

Assuming that we have measured  $K$  values of the output voltage, noted  $y^*(t)$  (where  $t = k.T_s$  for  $k = 1, K$  and  $1/T_s$  is the sampling rate). The identification problem is then to estimate the values of the parameters  $\underline{\theta}$  that minimize the dif-

ference between measured data and estimation. Thus, the output prediction error is defined as follows:

$$\varepsilon_k = y_k^* - \hat{y}_k(\hat{\underline{\theta}}) \quad (7)$$

where the predicted output  $\hat{y}$  is obtained by numerical simulations of the VDP model and  $\hat{\underline{\theta}}$  is an estimation of the true parameter vector  $\underline{\theta}$ .

Parameter estimation with OE technique is based on minimization of a quadratic criterion defined as :

$$J = \sum_{k=1}^K \varepsilon_k^2 = \sum_{k=1}^K \left( y_k^* - \hat{y}_k(\hat{\underline{\theta}}) \right)^2 \quad (8)$$

Since  $\hat{y}(t)$  is nonlinear in parameters  $\hat{\underline{\theta}}$ , the estimation is made iteratively using nonlinear programming techniques. Marquardt's algorithm<sup>20,21</sup> was used to ensure an efficient and rapid convergence even with poor initialization. The parameters to be estimated are updated as follows:

$$\hat{\underline{\theta}}_{i+1} = \hat{\underline{\theta}}_i - \{ [J''_{\theta\theta} + \lambda \cdot I]^{-1} \cdot J'_{\theta} \}_{\hat{\underline{\theta}} = \hat{\underline{\theta}}_i} \quad (9)$$

$J'_{\theta}$  and  $J''_{\theta\theta}$  are respectively the gradient and the hessian such as:

$$J'_{\theta} = -2 \sum_{k=1}^K \varepsilon_k^T \cdot \underline{\sigma}_{y_k, \underline{\theta}}$$

$$J''_{\theta\theta} \approx 2 \sum_{k=1}^K \underline{\sigma}_{y_k, \underline{\theta}} \cdot \underline{\sigma}_{y_k, \underline{\theta}}^T$$

with  $\sigma_{y_k, \underline{\theta}} = \frac{dy}{d\underline{\theta}}$  the sensitivity function,  $\lambda$  the monitoring parameter and  $I$  the identity matrix.

### 3.1. Sensitivity computation

Sensitivity computation is an important point in the identification procedure. Indeed, the sensitivity functions express the effect of a parameter variation on the system output. We can define two types of sensitivity functions:

- $\underline{\sigma}_{y, \underline{\theta}} = \frac{\partial y}{\partial \underline{\theta}}$ : the output sensitivity function used in nonlinear programming algorithm;
- $\underline{\sigma}_{x, \underline{\theta}} = \frac{\partial x}{\partial \underline{\theta}}$ : the state sensitivity function.

The functions can be obtained directly using numerical differentiation methods, but they require high computation time and there is a risk of introducing errors caused by approximations. For this reason, it is recommended to solve, by simulation, the differential system that describes the dynamic of these variables. Thus, for each parameter  $\theta_i$  of the vector  $\underline{\theta}$ , the corresponding sensitivity functions are

determined by partial differentiation of equation (4) so that:

$$\begin{cases} \dot{\underline{\sigma}}_{x,\theta_i} = \mathcal{A} \underline{\sigma}_{x,\theta_i} + \left[ \frac{\partial \mathcal{A}}{\partial \theta_i} \right] \underline{x} + \left[ \frac{\partial \mathcal{B}(t)}{\partial \theta_i} \right] \\ \sigma_{y,\theta_i} = \mathcal{C} \underline{\sigma}_{x,\theta_i} \end{cases} \quad (10)$$

For example, the sensitivity functions for the parameter  $C$  can be obtained by the simulation of the following differential model:

$$\begin{cases} \dot{\underline{\sigma}}_{x,C} = \begin{bmatrix} (a - \frac{1}{R}) \frac{1}{C} & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix} \underline{\sigma}_{x,C} + \begin{bmatrix} (\frac{1}{R} - a) \cdot \frac{1}{C^2} & \frac{1}{C^2} \\ 0 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} \frac{b}{C^2} u_C^3(t) \\ 0 \end{bmatrix} \\ \sigma_{y,C} = [1 \quad 0] \cdot \underline{\sigma}_{x,C} \end{cases} \quad (11)$$

It is important to note that all state space model like VDP model (Eq. 4) and sensitivity function models are solved using the 4<sup>th</sup> order Runge-Kutta method <sup>22</sup>.

### 3.2. Parameter normalization

If a hypothetical situation is studied, the values chosen for the different parameters are close. Nevertheless, in a real situation, the numerical values of the physical system parameters can be highly different, especially for RF systems. This can lead to convergence difficulties that are slowing or even make impossible the process to find the global optimum<sup>17,21</sup>. The solution consist in normalizing the parameters values which means the normalization of the sensitivity functions.

The method is first explained for a single parameter  $\theta_n$ , then it will be extended to the entire vector of parameters  $\underline{\theta}$  to be estimated. Let us consider the parameter  $\theta_n$  with the initial value  $\theta_{n_0}$  so that  $\theta_n = \theta_{n_0} + \Delta\theta_n$ . In this case, estimating the value for  $\theta_n$  is equivalent to estimate the  $\Delta\theta_n$  parameter which represents the difference between the nominal and the initial value.

The  $\Delta\theta_n$  expression depends on the parameter  $\theta_{n_0}$  in the following manner:

$$\Delta\theta_n = \mu_n \theta_{n_0} \quad (12)$$

Thus, for the parameter  $\theta_n$ , we can write the following expression:

$$\theta_n = (1 + \mu_n) \cdot \theta_{n_0} \quad (13)$$

Since the initial value  $\theta_{n_0}$  is known, the estimation of  $\theta_n$  can be made through a change of variables, i.e. by estimating  $\mu_n$ . The sensitivity function according to  $\mu_n$  is written as follows:

$$\sigma_{\mu_n} = \frac{\partial \hat{y}}{\partial \mu_n} = \theta_{n_0} \cdot \frac{\partial \hat{y}}{\partial \theta_n} = \theta_{n_0} \cdot \sigma_{\theta_n} \quad (14)$$

where the sensitivity functions  $\frac{\partial \hat{y}}{\partial \mu_n}$  are with the same magnitude order.

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In order to apply the normalization method for several parameters, all the sensitivity functions are normalized, leading to a well conditioned gradient and Hessian written as:

$$\begin{cases} J'_{\mu_n} = \frac{\partial J}{\partial \mu_n} = -2 \cdot \sum_{k=1}^K \varepsilon_k \cdot \sigma_{k,\mu_n} \\ J''_{\mu_n \mu_m} = \frac{\partial^2 J}{\partial \mu_n \partial \mu_m} = 2 \sum_{k=1}^K \sigma_{k,\mu_n} \cdot \sigma_{k,\mu_m} \end{cases} \quad (15)$$

For the iterative estimation of all parameters of the vector  $\underline{\mu}$ , we apply the Marquardt's algorithm so that:

$$\hat{\underline{\mu}}_{i+1} = \hat{\underline{\mu}}_i - \{[J''_{\underline{\mu}\underline{\mu}} + \lambda I]^{-1} \cdot J'_{\underline{\mu}}\}_{\hat{\underline{\mu}}=\hat{\underline{\mu}}_i} \quad (16)$$

Thus, (Eq. 16) allows to obtain the vector of parameters  $\hat{\underline{\theta}}_i$  so that:

$$\hat{\underline{\theta}}_i = \underline{\theta}_0 \cdot (1 + \hat{\underline{\mu}}_i \cdot I) \quad (17)$$

This method is very simple to implement because it does not change the structure of the algorithm, only the computation of the sensitivity functions changes.

#### 4. Simulation results

The validation method consists in two main parts:

- a study of the robustness technique in presence of stochastic disturbances,
- the characterization possibility of an oscillator using a CFY30 GaAs FET.

##### 4.1. Validation of the technique and investigations

In order to validate the proposed technique, let us consider the identification of a 1.3GHz VDP oscillator of (Fig. (4)) simulated with AGILENT'S ADS software.

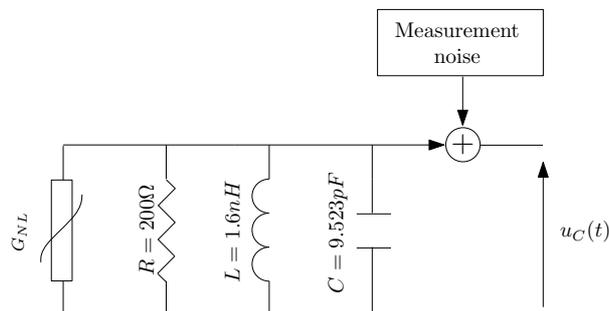


Fig. 4. Simulated circuit with added noise

Thus, no modeling error is introduced because the simulated circuit and the model have the same structure. Only stochastic disturbances effects, due to the

measurement noise with different signal to noise ratio (SNR) values is investigated through Monte Carlo simulations. The nonlinear conductance  $G_{NL}(t)$  is described by the following relation  $G_{NL}(t) = -0.0085 + 0.00071 \cdot u_C^2$ , the measured signal  $u_C(t)$  is sampled with a period of  $T_s = 0.01\text{ns}$  and the number of samples by realization is  $K = 10000$ . The proposed identification algorithm and data treatment is implemented on MATLAB MATHWORKS software.

In order to allow statistical analysis, a zero-mean white noise is added to the output data according to 30dB, 20dB and 10dB of SNR and 100 attempts were performed for each SNR value. Let us note that using noise free measurement, the estimated parameters are identical to the true values.

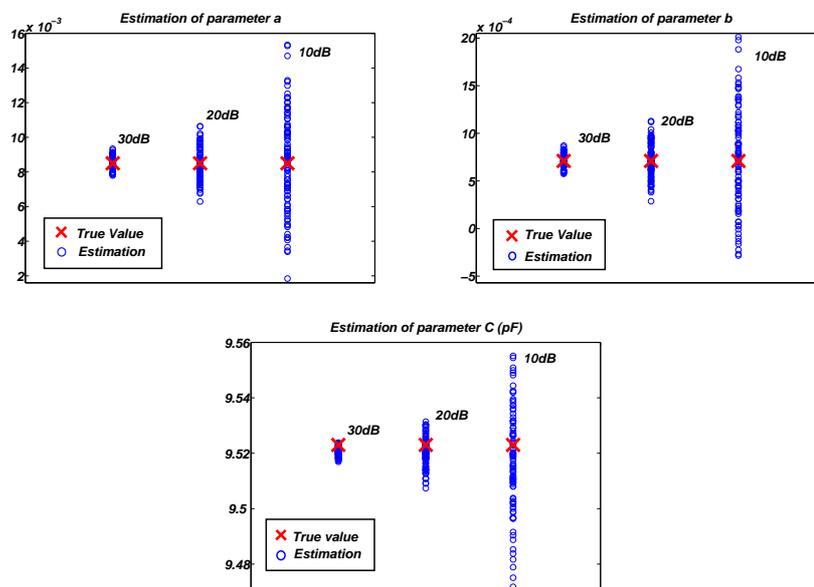


Fig. 5. Projections of estimated values for 100 realizations with different SNR.

Figure (5) shows the estimated values (circle) as well as the true values (cross) for each parameter a, b and C. Furthermore, the estimates have been plotted for different SNR values. It can be seen that the estimates are, on average, close to the exact value, whatever of the noise level. Regarding the accuracy of the estimation, let us note that once the value of SNR is decreasing, the uncertainty range increases for each parameter. Overall, the parameters identification, even in the presence of high noise level, gives satisfactory results.

In order to study the correspondence between the estimated output  $\hat{y}$  and the measured one  $y^*$  and the parameters  $\hat{\theta}$  convergence to true values  $\theta$ , we computed the *Normalized Mean Square Errors* (NMSE) based on the output data and on the

parameters such as:

$$\begin{aligned} \text{NMSE}_{\underline{y},dB} &= 10 \log \left( \frac{(\underline{\hat{y}} - \underline{y}^*)^T \cdot (\underline{\hat{y}} - \underline{y}^*)}{\underline{y}^{*T} \cdot \underline{y}^*} \right) \\ \text{NMSE}_{\underline{\theta},dB} &= 10 \log \left( \frac{(\underline{\hat{\theta}} - \underline{\theta})^T \cdot (\underline{\hat{\theta}} - \underline{\theta})}{\underline{\theta}^T \cdot \underline{\theta}} \right) \end{aligned} \quad (18)$$

The  $\text{NMSE}_{\underline{y},dB}$  criterion is widely used by the radio frequency community, especially for describing the deviation level for time domain measurement. On the other hand, we have deliberately introduced the second criterion which reflects the deviation of the entire parameter space instead of the standard deviation which gives this information but only for a single parameter.

Table 1. Average results for 100 attempts for 10dB of SNR

	Identification results	
	$\theta_{i_{mean}}$	$\sigma_i$
$a = 8.5 \cdot 10^{-3}$	$8.625 \cdot 10^{-3}$	$2.77 \cdot 10^{-3}$
$b = 71 \cdot 10^{-4}$	$73.4 \cdot 10^{-4}$	$5.35 \cdot 10^{-4}$
$C = 9.523 \cdot 10^{-12} F$	$9.517 \cdot 10^{-12}$	$1.874 \cdot 10^{-14}$
$\text{NMSE}_{\underline{y},dB}$	-10.06dB	
$\text{NMSE}_{\underline{\theta},dB}$	-15.02dB	

Table (1) gives the identification results for a 10dB SNR. We can clearly see that the mean of the parameter  $\theta_{i_{mean}}$  is close to the true value with reduced standard deviation  $\sigma_i$ . The normalized error  $\text{NMSE}_{\underline{y},dB}$  on data and on parameters  $\text{NMSE}_{\underline{\theta},dB}$ , presented at the end of the table indicates a low error rate.

## 4.2. Simulation results for a 1.025GHz GaAs FET oscillator circuit

### 4.2.1. Oscillator Design

The schematic of the oscillator used in simulation is shown in Fig. (6). The active part of the oscillator provides the necessary negative resistance to compensate for the resonator's losses. The transistor used is a CFY30 GaAs FET. The total power consumption of the oscillator is 115mW, under  $V_D = 5V$ . The passive part is represented by a RLC parallel resonator that determines the frequency of oscillation. For the tank chosen in this case, the frequency of oscillation has a value of about 1.025GHz and the resulting quality factor of the resonator is 61.35.

A small signal study of the active part was first made. The objective of this part is to define the starting conditions of the VCO and the various constraints that can ensure it will work. The model used for the active part of the oscillator is shown in Fig. (7) where  $C_{gs}$  and  $C_{gd}$  are respectively the gate-source and the

gate-drain capacitances. In steady-state, in order to establish and maintain a stable oscillation, the admittance presented by the active part, expressed as follows, for  $C_{gd} = 0$ , should have a negative real part:

$$Y_e = \frac{g_m}{1 - \omega^2 L_g C_{gs}} + \frac{j\omega C_{gs}}{1 - \omega^2 L_g C_{gs}} = \frac{i_e}{V_e} \quad (19)$$

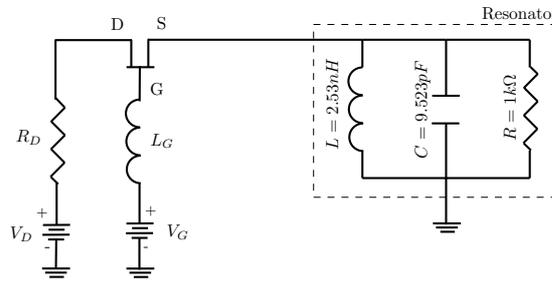


Fig. 6. Oscillator schematic using a CFY30 GaAS FET

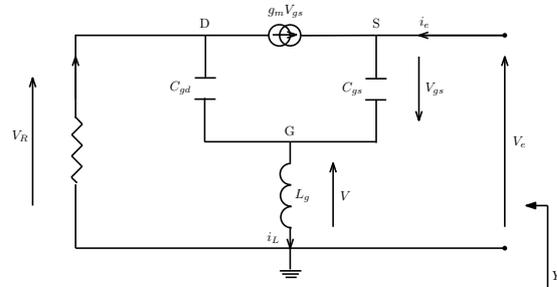


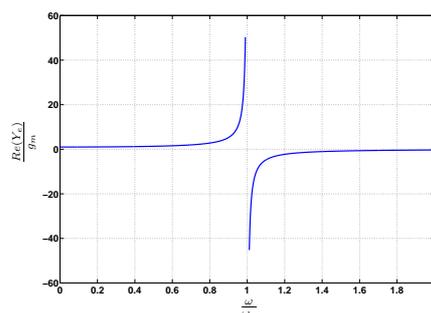
Fig. 7. Small signal model of the active part

In these conditions, Fig. (8) shows the plot of the real part of the admittance  $Y_e$  normalized to  $g_m$  (the transconductance in siemens) versus the pulsation  $\omega$ , normalized with  $\omega_r = \frac{1}{\sqrt{L_g C_{gs}}}$ . Hence, for

- $\omega = 0, Re(Y_e) = g_m$
- $\omega = \left( \frac{1}{\sqrt{L_g \cdot C_{gs}}} \right)^-, Re(Y_e) \rightarrow +\infty$

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- $\omega = \left( \frac{1}{\sqrt{L_g C_{gs}}} \right)^+$ ,  $Re(Y_e) \rightarrow -\infty$
- $\omega \rightarrow \infty$ ,  $Re(Y_e) \rightarrow 0$

Fig. 8. Normalized real part of the admittance  $Re(Y_e)/g_m$  plotted versus  $\omega_e/\omega_r$ 

Therefore, from this curve it can be deduced that, to ensure a negative real part of  $Y_e$ , the value of the inductance  $L_g$  must be chosen so that the resonant frequency of the tank  $L_g C_{gs}$  is less than the desired oscillation frequency. Thus, the ratio  $\frac{\omega}{\omega_r}$  should be greater than unity in order to produce and maintain a constant oscillation. Hence, to ensure proper start-up of the oscillator, the value fo  $L_g$  was chosen to be equal to  $7nH$ .

## 4.2.2. Estimation results

The behavior of the oscillator presented in the previous section is simulated using AGILENT'S ADS software in Transient with a sampling period of 0.01ns. For a sake of clarity, the obtained simulated waveforms will be called the measured data in the following. In general case, the resulting output voltage  $u_C(t)$  for VDP model simulation corresponds to a cosine-type function. To ensure a proper match between the measured data and the estimation during the identification procedure, we made a truncation in output data so that the measured voltage starts with the maximum value, corresponding to cosine type function.

In these conditions, the aim is to find the parameter vector  $\underline{\theta}$  so that the Van der Pol model describes the data collected from the real circuit as accurately as possible. Let us note that no *prior* knowledge is available for  $a$  and  $b$  parameters. After few iterations, the obtained values of parameters are:

$$\begin{cases} a = 6.2 \cdot 10^{-3} \\ b = 1.3 \cdot 10^{-3} \\ C = 9.5 pF \end{cases}$$

Fig. (9) shows the evolution of parameter estimates during identification procedure. We can see that the convergence to optimal values is obtained after 40 iterations.

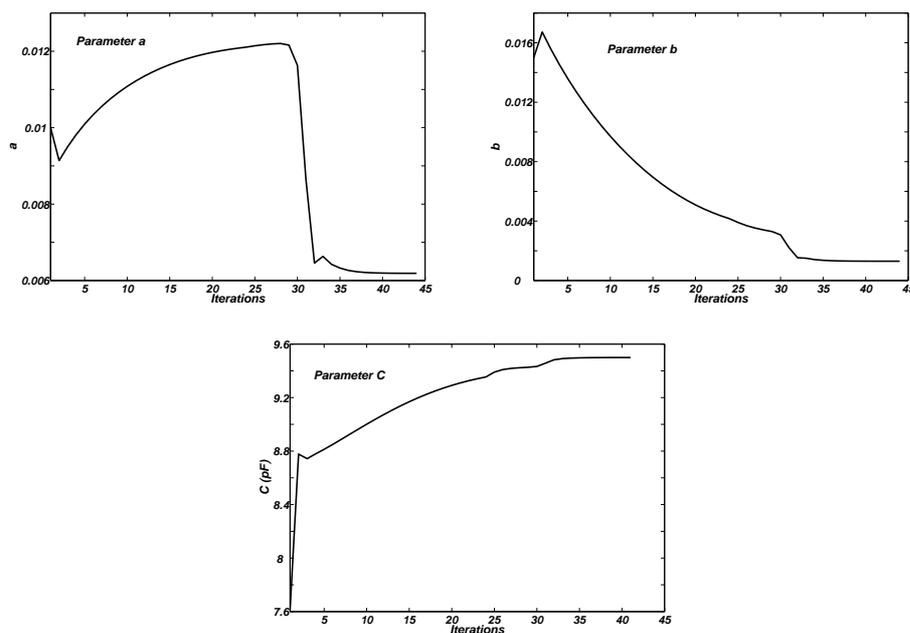


Fig. 9. Evolution of estimations during identification procedure

The comparison between the measured and the estimated responses is shown in figure (10). We can observe good agreement with a maximum error around 0.06V and a computed  $NMSE_{y,dB}$  of  $-32dB$ .

## 5. Conclusion

In this paper, the study and the implementation of an output error approach used for the identification of oscillating circuits is presented. The physical parameters of the corresponding Van der Pol model of an oscillator have been identified, using sampled time data. A brief description of the real system was made and the mathematical model of the considered system is also described. The mathematical equations used for the implementation of the method were developed and the improvement of the algorithm, for RF systems which have a highly different magnitude order of their parameters, was presented.

The simulation results, when the simulated circuit has the same structure as the model in the presence of noise, prove the algorithm convergence and show its

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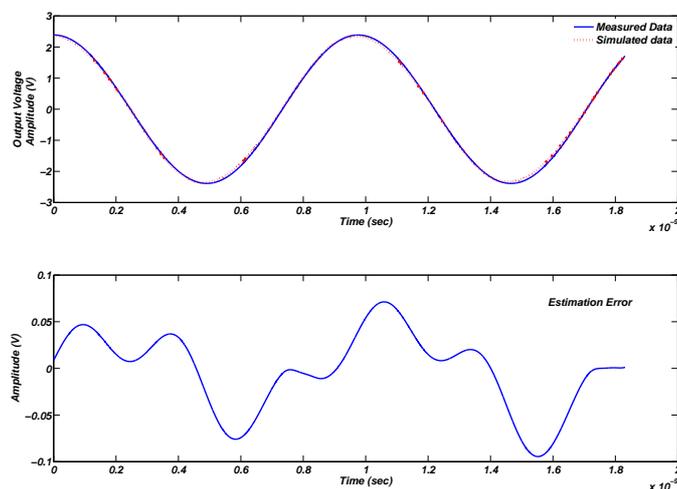


Fig. 10. Comparison between the measured data and the estimations.

robustness under high level of disturbances. A good agreement was found between the response of the circuit and the estimated model, showing the efficiency of such an identification technique. This approach constitutes a simple way to find the Van der Pol model for the large class of oscillating systems.

The investigated optimization technique behave very well and thus it is suitable for characterization of practical RF oscillator circuits with classical VDP model. The next objective is to study how to estimate the parameters of an array of coupled oscillators using the proposed method. Our first investigation in this field seem to be very promising.

## References

1. T. Health, *Beam steering of nonlinear oscillator arrays through manipulation of coupling phases*, IEEE Transactions on Antennas and Propagation, vol. 52, n 7, pp. 1833-1842, July 2004.
2. P. Liao and R.A. York, *Beam Scanning With Coupled VCOs*, In Proc. Antennas Propagation Soc. Int. Symp., vol. 2, pp. 836-839, June 1994.
3. P. Liao and R.A. York, *A new phase-shifterless beam-scanning technique using arrays of coupled oscillators*, IEEE Transactions Microwave theory and techniques, vol. 41, n10, pp. 1810-1815, October 1993.
4. N. Y. Tohme, J.M. Paillot, D. Cordeau, P. Coirault, *Analysis of the Frequency Locking Region of Coupled Oscillators Applied to 1-D Antenna Arrays*, European Microwave Conference, Amsterdam, Holland, pp. 1334-1337, October 2008.
5. R. A. York, *Nonlinear analysis of phase relationships in quasioptical oscillator arrays*, IEEE Transactions on Microwave Theory and Techniques, vol. 41, n10, pp. 1799-1809, October 1993.

6. R. A. York and P. Liao, *Oscillator Array Dynamics with Broadband N-Port Coupling Networks*, IEEE Transactions on Microwave Theory and Techniques, vol. 42, n 11, pp. 2040-2045, November 1994.
7. J. J. Lynch and R. A. York *Synchronization of Oscillators Coupled Through Narrow-Band Networks*, IEEE Transactions on Microwave Theory and Techniques, vol. 49, pp. 237-249, February 2001
8. P. Young, *Parameter estimation for continuous-time models - a survey*, Automatica, VOL. 17, No. 01, pp. 23-39, 1981.
9. I. D. Landau, *Adaptive Control : The model reference approach*, Marcel Dekker. Control and Systems Theory. Vol. 08, 1979.
10. H. Unbehauen and G.P. Rao, *Identification of continuous systems*, Systems and control series. North-Holland, 1987.
11. H. Unbehauen and G.P. Rao, *A review of identification in continuous-time systems*, Annual Reviews in Control 22, pp. 145-171, 1998.
12. M. Mensler, H. Garnier and E. Huselstein, *Experimental comparison of continuous-time model identification methods on thermal process*, In: 12th IFAC Symposium on System Identification. Santa Barbara, California 2000.
13. H. Tulleken, *Grey-box modeling and identification using physical knowledge and bayesian techniques*, Automatica, Vol. 29, N0. 02, pp. 285-308, 1993.
14. E. Walter and L. Pronzato, *Identification of parametric models from experimental data*, Communication and Control Engineering Series, Editions Springer, 1997.
15. L. Ljung, *System identification: Theory for the user*, 2<sup>nd</sup> edition, Prentice Hall, USA, 1999.
16. J. Richalet, A. Rault and R. Pouliquen, *Identification des processus par la méthode du modèle*, Gordon & Breach, Théorie des systèmes, N<sup>o</sup>. 04, 1971.
17. S. Bachir and C. Duvanaud, *New identification procedure for continuous-time radio frequency Power Amplifier model*, Journal of Circuits, Systems and Computers, Vol. 19, N<sup>o</sup> 06, pp. 1259-1274, 2010.
18. B. van der Pol, *The nonlinear theory of electrical oscillations*, proc. IRE, Vol. 22, pp. 1051-1085, sept 1934.
19. M. Odyneec, *RF and Microwave Oscillator Design*, Boston, MA:Artech House, 2002.
20. D. W. Marquardt, *An Algorithm for least-squares estimation of non-linear parameters*, Soc. Indust. Appl. Math, Vol. 11, NO. 2, pp. 431-441, 1963.
21. J-C. Trigeassou, T. Poinot, S. Bachir, *Methods for Electrical Machines: Parameter estimation for knowledge and diagnosis of electrical machines*, Edition ISTE Ltd and John Wiley & Sons Inc;, pp. 207-243, April 2009.
22. L. F. Shampine, *Numerical Solution of Ordinary Differential Equations*, Chapman & Hall, New York, 1994.