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An hybrid Monte Carlo and Fuzzy Logic Method for Maintenance Modelling

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Abstract

A framework previously proposed by the authors to qualitatively assess the performance of maintenance policies in electrical production plants is summarized in this work; the distinguishing feature of this framework is the characterization of the living conditions of a component by means of Influencing Factors (IFs), i.e., conditioning aspects of the component life that determine the evolution of the degradation mechanisms affecting the component. Modelling of this evolution is addressed via an hybrid Monte Carlo simulation and fuzzy logic scheme which provides the basis for assessing the performance of a maintenance policy.

Keywords: Degradation modelling, Maintenance, Fuzzy Logic, Monte Carlo simulation, Influencing Factors.

1 Introduction

The significant economic impact of maintenance on production and service has led to a strong interest in developing models to support decision makers in their tasks of improving system reliability, preventing the occurrence of accidents and reducing maintenance costs. The output provided by these models are the values of key parameters on which the reliability and the availability of the system depend, and which serve for defining an optimal maintenance strategy in the face of various types of maintenance plans and all other constraints (e.g. safety requirements, budgetary limitations, etc.).

The effectiveness of the models for supporting maintenance decisions increases when these are able to capture the specificity of the components which derives from the particular 'life' (failures, shocks, preventive maintenance actions, unavailability periods, work load profile, etc.) that each of them has experienced [1]. For example, in the electrical industry similar components are installed and used in a large variety

of living conditions (e.g., two transformers of the same electrical network may be installed one on the Alps and one close to the Mediterranean Sea); thus, the more specifically characterized are the living conditions in the model, the more informed can be the supported maintenance decisions.

The issue of giving due account to the influence of covariates (e.g., living conditions) on the evolution of the degradation process of a component has been addressed in a number of works (e.g., [2]-[4]), also from the theoretical point of view (e.g., [5], [6]). However, as remarked in [7], there are few works (e.g., [7], [8]) that focus on the influence of covariates in the modelling of degradation processes, for Condition Based Maintenance (CBM) applications. (CBM is here viewed as a Preventive Maintenance (PM) policy according to which an action is performed only when a monitored index (e.g., degradation state, failure rate, etc.) reaches a predetermined level). Furthermore, the models proposed in these works are all developed within the framework of stochastic processes, and thus rely on a number of parameters which may be difficult to estimate in real applications due to lack of experimental data. Indeed, in practice expert judgement is often the main source of information for these models.

In this context, a novel framework that assesses the effectiveness of a CBM policy by modelling the evolution of degradation mechanisms taking into account the living conditions in which a component works has been proposed and investigated in [9]-[11]. In these works, the fundamental issue of the characterization of the living conditions has been addressed by introducing some Influencing Factors (IFs), i.e. conditioning aspects of the component life, representative of a set of homogeneous variables (physical, environmental, etc.). In particular, five IFs have been considered which, for the sake of clarity, are identified by an index (1, 2, ..., 5):

- IF₁: Environment. It includes the environmental variables (temperature, humidity, vibration, etc) which are expected to influence the degradation and failure behaviour of the component. In general, IF₁ is a re-configurable parameter because some interventions can be done in order to modify its level; for example, the external temperature or humidity can be controlled, if possible, by setting up air conditioning systems, the vibration level can be reduced by performing maintenance actions on the systems causing the vibration, etc.
- IF₂: Operational Mode. The set of variables which influence the stress conditions of the component (e.g. duty cycle, frequency of stops/re-starts, etc); they can be changed during the life time of the component, depending on the demands and opportunities of operation.
- IF₃: Maintenance Policy. It contains all the variables related to the maintenance features (maintenance action effectiveness, etc.) which are often dynamically re-calibrated during the components mission time.
- IF₄: Age. The Age of a component could be different from the calendar time elapsed since it started to work: the effect of some maintenance actions can be accounted by reducing the actual age of the component.
- IF₅: Quality (e.g. the quality level of design, manufacturing, technology, etc.). In general this parameter is fixed: the quality of a device remains constant during its life.

This work is a summary of the framework proposed in [9]-[11] and is organized as follows: a brief description of the framework is provided in Section 2; Section 3 describes the case study on which the proposed methodology is applied and finally, some conclusions are given in the last Section.

2 The Framework

The framework presented in [9]-[11] is partially derived from [12], where a pragmatic approach is proposed for accounting for the component specific living conditions (e.g., environment, working cycles, etc.) by multiplying the base value of the component failure rate by empirical factors. Despite its pragmatism, this approach is not directly applicable in a CBM context which requires the knowledge (even qualitative) of the component degradation level in order to define the most opportune maintenance policy. On the contrary, the approach proposed in [9]-[11] focuses specifically on the modelling of the degradation process affecting the component, taking into account the actual living conditions in which it works.

Figure 1 gives a snapshot of the modelling framework, which is based on three modules:

- Central Module (CM); it defines the IFs that actually influence the considered degradation mechanism.
- Backward Module (BM); the physical variables related to each IF are identified, and the relationships between them and the IF are determined.
- Forward Module (FM): the link between the IFs and the degradation process is defined. The degradation process is described by means of a small number of levels, or degradation ‘macro-states’, each one characterized by a failure rate. The choice of this representation is driven by industrial practice: experts usually adopt a discrete and qualitative classification of the degradation state based on qualitative interpretations of symptoms.

Eliciting information from experts, resorting to the literature, inferring from databases etc. are different ways to address the contents of these modules.

Both the BM and the FM are developed by resorting to FL theory to cope with the scarcity of the data typically available and its qualitative nature. In practice, the IFs are expected to be more easily represented by linguistic variables rather than numeric variables (e.g., ‘the environment is mild’ or ‘the maintenance is efficient’). In this case, fuzzy logic offers the capability of dealing with imprecise variables and linguistic statements provided by experts on the basis of their knowledge and engineering sense of practice.

Furthermore, the typically stochastic behaviour of the living conditions results in stochasticity of the covariates IFs, and thus stochastic transitions between the degradation levels (and associated values of failure rates).

The degradation model can be used to test the effectiveness of a maintenance policy. To do this, the degradation stochastic evolution is simulated by the model and the failure rates associated to the degradation levels evolving in time are input to a Monte Carlo (MC) module which estimates the availability of the system over a specified mission time (Figure 2); through a cost model, the total costs associated to the maintenance policy can then be assessed [9]-[11].

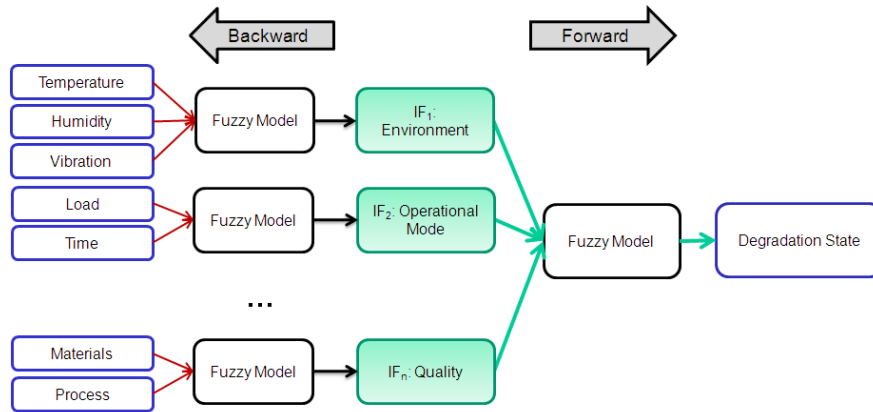


Figure 1: snapshot of the FL degradation model.

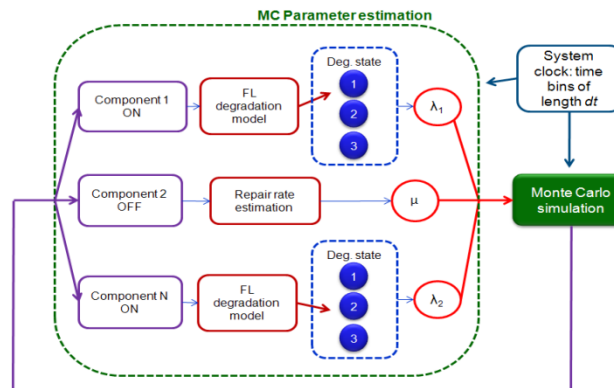


Figure 2: interface between FL degradation models and MC simulation.

3 Case Study

In this Section, the modelling architecture presented above is applied to a case study concerning a Water-Feeding Turbo Pump (WFTP) of a steam generator. A team of experts has identified the degradation processes affecting the components of the WFTP and the associated IFs and symptoms. The present case focuses on the contact fatigue degradation mechanism, which affects the seals of the WFTP. No consideration is given to other degradation processes and their influences that may lead to an acceleration of the degradation process.

The degradation of the seals of the WFTP due to contact fatigue is caused by the development of cracks that affect the ability of the seals to avoid leaks. The creation and propagation of these cracks is a complex physical phenomenon, which has been modelled in a number of different ways ([13] and [14]). According to these models, the degradation is mainly influenced by the loads applied on the component, its constitutive materials and production process, some geometrical factors related to both the shape of the cracks and their position with respect to the direction of the loads etc.

The model presented in this work is based on the assumption that the length of the most critical crack of the component defines its degradation level. Moreover, it is assumed that the length of the crack can only increase in time and maintenance on the component has the effect of decreasing the speed of propagation of the crack but cannot reduce its length. In the modelling, the following three degradation levels are considered (Fig. 3(a)):

1. ‘Good’: the components which are new or almost new. No maintenance actions are foreseen for components in this level and the failure rate is $\lambda=1e-5 h^{-1}$.
2. ‘Medium’: the seals of the WFTP in this state need some actions aimed at decreasing the crack growth rapidity. The failure rate of components in this degradation level is assumed to be $\lambda=5e-4 h^{-1}$.
3. ‘Bad’: if the component is in this degradation level, it is convenient to replace it. The failure rate of the components in this degradation state is $\lambda=1e-3 h^{-1}$.

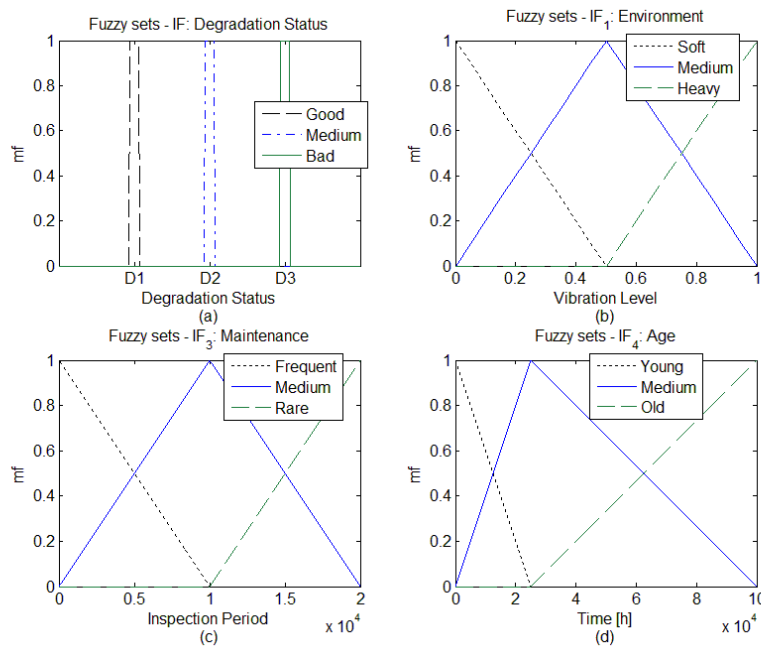


Figure 3: fuzzy sets.

3.1 Central module

The development of the model of the degradation process is based on the identification of the following IFs:

- IF_1 =‘Environment’; in this work it has been assumed that the influence of the environment on the considered degradation mechanism is mainly caused by the vibrations in the location in which the component works. In particular, the input variables of the Backward Module which defines, at a given time t , the relationship between measurable variables and the IF_1 , are the mean values of the frequency and of the amplitude of the fundamental wave, in the time elapsed since the component has started to work. Fig. 3(b) shows the partition of the UoD of the IF_1 environment into the three Fuzzy Sets, ‘Soft’, ‘Medium’ and ‘Heavy’, defined by means of triangular membership functions.
- IF_3 =‘Maintenance’; the component is periodically inspected by operators to control its degradation level. The maintenance policy, a priori established, requires that no maintenance action is performed if the degradation level is ‘Good’ at the occurrence of the inspection whereas a corrective maintenance action is performed if the component is found in level ‘Medium’, and a replacement action is carried out when the component is in the degradation level ‘Bad’. A variation of the frequency of the inspections causes a modification of the degradation process; in particular, the more frequent are the inspections the less is the time in which the degradation advances without any action for reducing its speed. To describe this IF_3 , the three Fuzzy Sets ‘Frequent’, ‘Medium’, ‘Rare’ (Fig. 3(c)) are used on the UoD $[0, 2e4]h$ of the variable inspection frequency.
- IF_4 =‘Age’; it measures the time since the component has been working. The UoD of this IF_4 is the interval $[0, TM]$, with mission time $TM=1e5h$; on this interval, three Fuzzy Sets ‘Young’, ‘Medium’ and ‘Old’ are defined by means of triangular membership functions (Fig. 3(d)). In general, the older the component, the higher its degradation level.

3.2 Backward Module

The tailoring of the BM to the considered case study consists in identifying the physical variables on which the IF_1 depends (the IF_3 and the IF_4 are already directly described by the variables control period and time, respectively). The vibration level, whose range of variability has been arbitrarily set to $[0,1]$, adequately characterizes the defined IF_1 and its value is computed starting from the values of two physical variables measured by means of sensors (e.g., strain gauges): amplitude and frequency of the vibration fundamental wave. In particular, the mean values of these variables in the time elapsed since the system has started to work are given in input to the BM, which links them to the IF_1 by means of a set of fuzzy *if-then* rules (Fuzzy Rule Base, FRB).

Figure 4 shows the fuzzy sets, defined by means of triangular membership functions, partitioning the variables in input to the BM:

- ‘Low’, ‘Medium’ and ‘High’ are the fuzzy sets defined on the UoD $[0,5]mm$ describing the mean value of the amplitude of the fundamental wave;
- ‘Low’, ‘Medium’ and ‘High’ are the fuzzy sets defined on the UoD $[0,200] Hz$ describing the mean value of the frequency of the fundamental wave.

Table 1 shows the rules in the FRB that model the influence of the mean values of the Amplitude and the Frequency on the IF_1 . For example, the bottom-right element of Table 1 represents the rule: *if Amplitude is Low and Frequency is Low then IF_1 is Soft*.

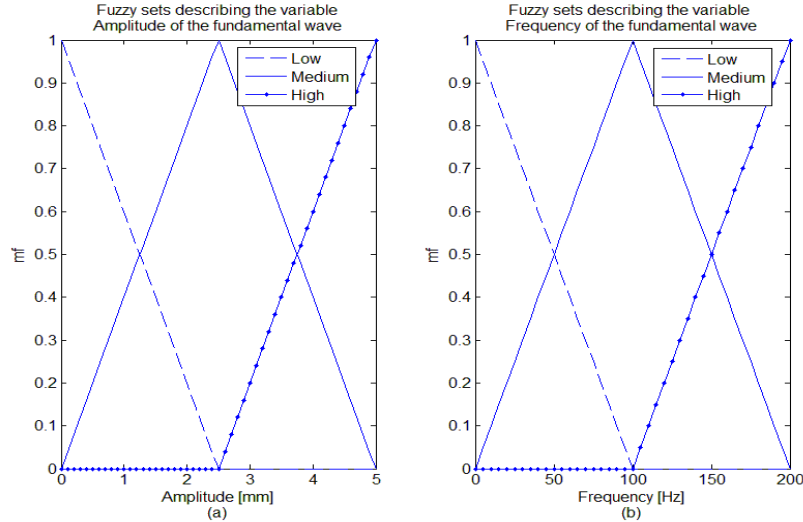


Figure 4: fuzzy sets of the variables in input to the Backward Module.

Table 1: Fuzzy rules defining the relationship between the inputs and the outputs of the BM tailored to the IF_1 .

		Mean frequency of the fundamental wave		
		<i>High</i>	<i>Medium</i>	<i>Low</i>
Mean amplitude of the fundamental wave	<i>High</i>	Heavy	Heavy	Medium
	<i>Medium</i>	Medium	Medium	Soft
	<i>Low</i>	Medium	Soft	Soft

Generally speaking, the vibration in the location in which the system of interest works is caused by other components either because they are degrading (e.g., the increase of the eccentricity of the centre of gravity in rotating machines) or because they have been designed in such a way that a periodic load is applied on the other coupled components (e.g., alternating machines discharging loads on the same basement of the system of interest). Since, in general, the behaviour of both the components producing the vibration and the other components of the overall system (which modify the vibration wave) is stochastic, the vibration profile suffered by the components is also stochastic.

For simplicity, but without loss of generality, in the present case study an arbitrarily chosen vibration profile is assumed in input to the BM, in terms of the mean amplitude and the mean frequency of the fundamental wave (Figure 5).

Such profile “lived” by the component influences its degradation behaviour; the intensity of such influence is assessed by means of the dedicated fuzzy logic model built. Figure 6 shows the activation profile in time of the fuzzy sets Low, Medium

and High, representative of the vibration conditions in terms of mean amplitude (Figure 6, left) and frequency (Figure 6, right) of the fundamental wave. The combination of these activations by the FRB of Table 1 within a Mamdani inference system results in the time profile of the degrees of activation of the Soft, Medium and Heavy levels of IF_1 reported in Figure 7. The Medium level is the most activated for large part of the mission time; Soft and Heavy levels are less activated, and in a similar way. In the first part of the mission time, the rule ‘*if Amplitude is High and Frequency is Low then Environment is Medium*’ has the largest activation degree whereas the rules ‘*if Amplitude is High and Frequency is Medium then Environment is Heavy*’ and ‘*if Amplitude is Medium and Frequency is Low then Environment is Soft*’ are those with largest activation degrees among those with ‘Heavy’ and ‘Soft’ consequents, respectively. With the vibration profile of Figure 5, the two latter rules increase their activation degrees up to the central part of the mission time as the activation degree of the first rule becomes smaller; this leads to the three levels having almost the same degree of activation of about 0.5 at $t=5.5 \cdot 10^4$ h: at this time, there is complete uncertainty on the influence of the IF_1 on the degradation level of the component; then, in the central part of the mission time, the activation degree of the set ‘Medium’ starts again to increase because the activation degree of the rule ‘*if Amplitude is Medium and Frequency is Medium then Environment is Medium*’ becomes larger whereas the degrees of activation of the rules with consequents ‘Heavy’ and ‘Low’ begin to decrease.

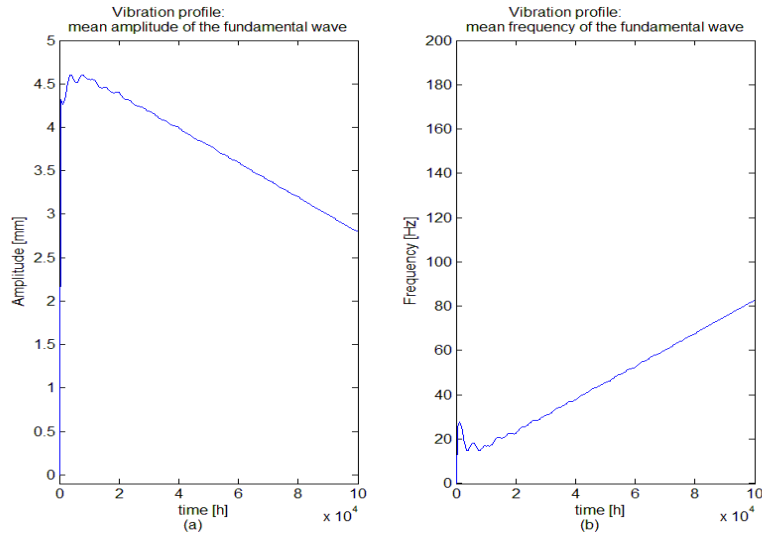


Figure 5: vibration profile applied to the component, in terms of mean amplitude (to the left) and mean frequency (to the right).

3.3 Forward Module

The objective of the Forward Module is to provide a description, in terms of fuzzy rules, of how the IFs impact on the evolution of the degradation process. In other words, a FRB is built which links the identified IFs with the component degradation state and thus its failure rate.

In the considered case study, the Forward Module consists in identifying the failure rate of the seals of the WFTP. More precisely, a fuzzy model has been built based on

rules as, for example: ‘if Environment is Soft and Maintenance is Frequent and Age is Young and Previous Degradation State is Good then Degradation State is Good’. As before, the rules defining the FRB are obtained from expert knowledge. The antecedent ‘Previous Degradation State’ has been introduced in order to ensure that the degradation state does not decrease as the age of the component increases.

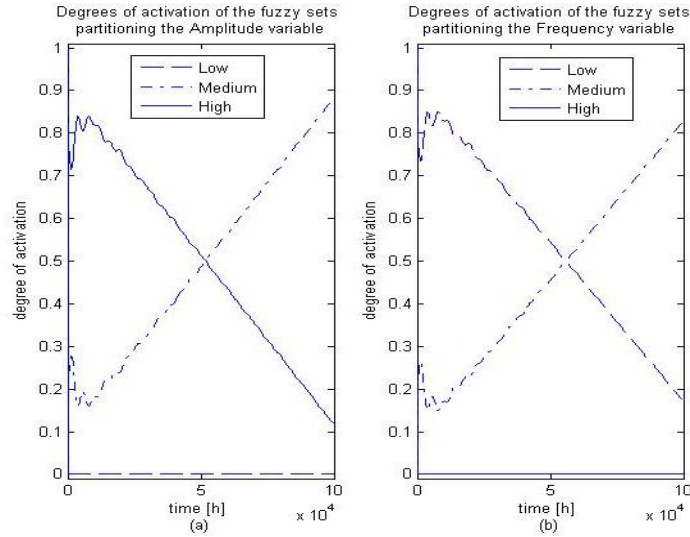


Figure 6: degrees of activation of the fuzzy sets partitioning the variables in input to the Backward Module for the given vibration profile.

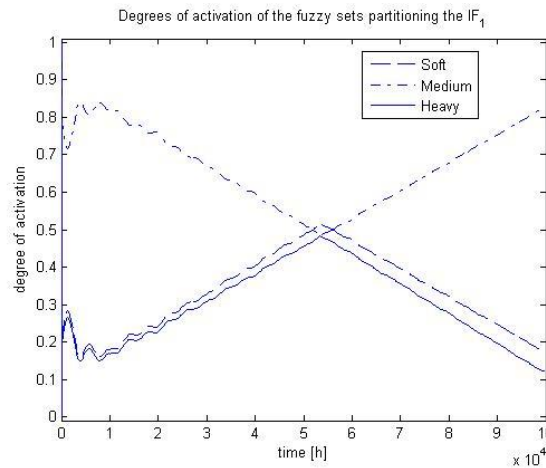


Figure 7: degrees of activation of the fuzzy sets partitioning the IF₁, for the given vibration profile.

The output fuzzy set ‘Degradation State’ is eventually defuzzified to limit the propagation of the uncertainty. Defuzzification is done by simply selecting the degradation state with the highest degree of activation.

Figure 8 shows the application of the proposed model on the component which lives in the environment previously introduced and inspected every 7000h, with no failures during the mission time.

The evolution of IF₁ (Figure 8(a)) and IF₄ (Figure 8(c)) is straightforward until the time instant $t=6.3 \cdot 10^4$ h, when the component is found to be in the degradation state

“Bad” and it is replaced by a new one, whose age is zero and with no accumulated vibration. From that time on, the IF_1 is computed taking into account the vibration suffered by the newly installed component and the IF_4 evolves naturally as its age. The IF_3 (Figure 8(b)) is constant, regardless the replacement of the component, since the maintenance policy is the same throughout the mission time.

Figure 8(e) shows the defuzzified degradation state of the component, which directly determines the failure rate value (Figure 8(d)).

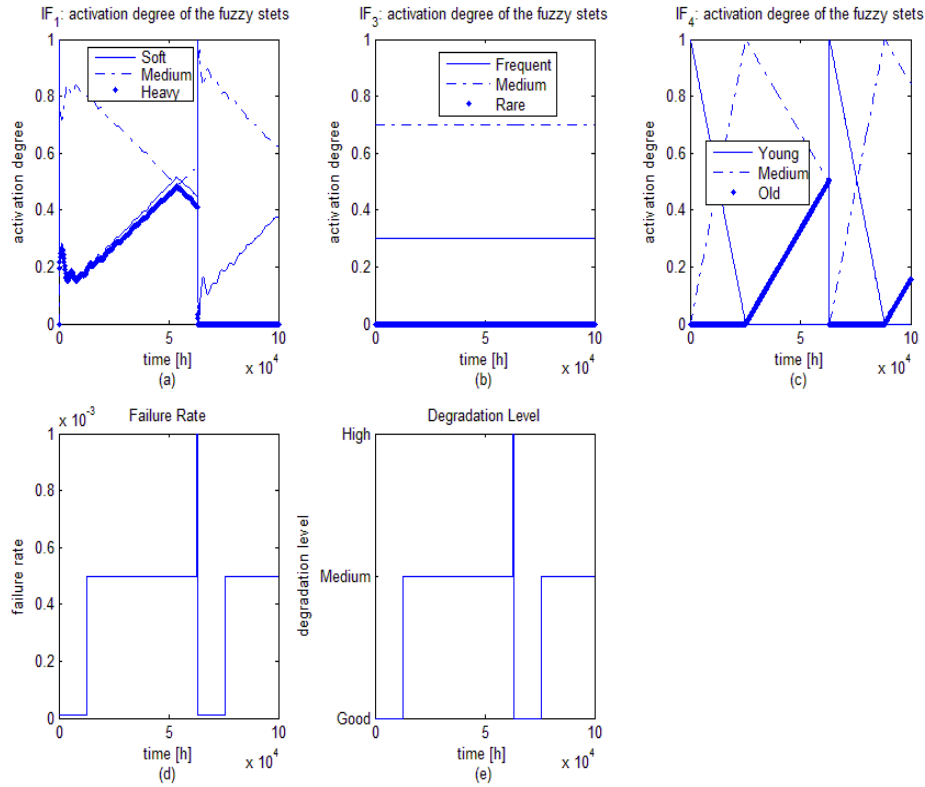


Figure 8: activation degree of the IFs fuzzy sets and of the degradation state, failure rate value and defuzzified degradation state considering a control period of 7000 h when no failure occurs.

3.4 Maintenance policy assessment

In the present Section, the results of the Monte Carlo unavailability estimation of the component are reported and discussed. The computational model has been developed in FORTRAN. Table 2 shows the values of the parameters used in the case study:

Table 2: Monte Carlo parameters.

Parameter	Value
Dt	100 h
Number of MC trials	10000
CPU time (Intel Pentium, 1.6 GHz)	56 s

The instantaneous unavailability of the component with the related 68.3% confidence interval (i.e., plus and minus one standard deviation) is shown in Figure 9.

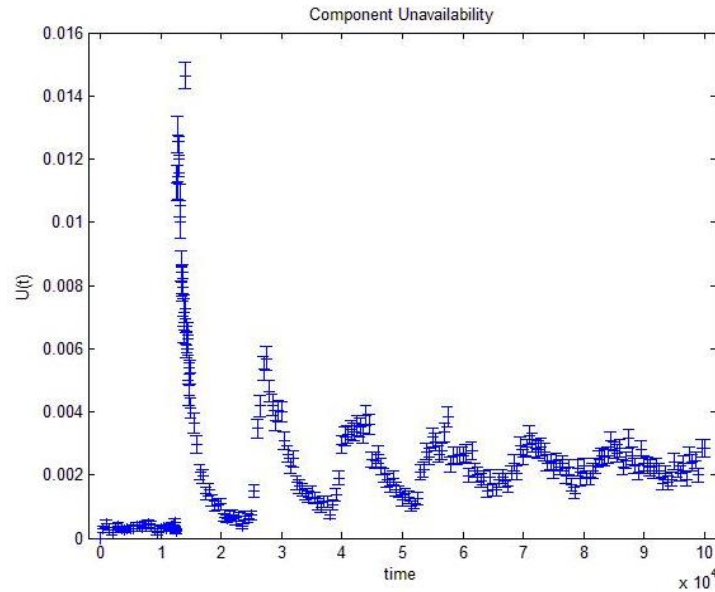


Figure 9: instantaneous component unavailability and its standard deviation.

Two large peaks appear in the first part of the component mission time. The first, at $t=1.26 \cdot 10^4$ h, corresponds to the time instant in which the degradation process has a transition from degradation state 1 to 2, with the failure rate of the component worsening from 10^{-5}h^{-1} to $5 \cdot 10^{-5} \text{h}^{-1}$. After $t=1.26 \cdot 10^4$ h two different conflicting trends are observed:

- an increase in the unavailability due to the contribution of those simulated components which have had a failure before $t=1.26 \cdot 10^4$ h and thus reach the degradation state 2 with a delay;
- a decrease of the unavailability due to the reduced failure rate (10^{-5}h^{-1}) of those simulated components that have undertaken corrective maintenance.

The second effect is prevalent and thus the unavailability decreases. The second peak occurs at $t=1.40 \cdot 10^4$ h, when the first control occurs after the component has entered in degradation state 2 and thus all the simulated components that did not have a failure before are now unavailable, due to the downtime associated to the preventive maintenance action.

Notice that in the considered case study it is extremely unlikely to achieve the degradation state 3: with a failure rate associated to the degradation state 2 equal to $5 \cdot 10^{-4} \text{h}^{-1}$ and a time interval of $\Delta t=4.98 \cdot 10^4$ h between the achievement of the degradation states 2 and 3, the probability of encountering a system in a degradation state 3 is smaller than $\exp(-\lambda \cdot \Delta t) \exp(-5 \cdot 10^{-4} \cdot 4.98 \cdot 10^4)=1.5 \cdot 10^{-11}$. This is the reason of the non-appearance of a peak of unavailability at $t=6.3 \cdot 10^4$ h, at which the component would reach the degradation state 3 (Figure 8).

3.5 Maintenance policy optimization

The proposed framework has been used to optimize the maintenance policy described in Section 4; in particular, the optimization has been performed with respect to the Control Period.

Figure 10 and Figure 11 show the mean unavailability of the component and related 68.3% confidence interval and the maintenance costs for varying values of the Control Period.

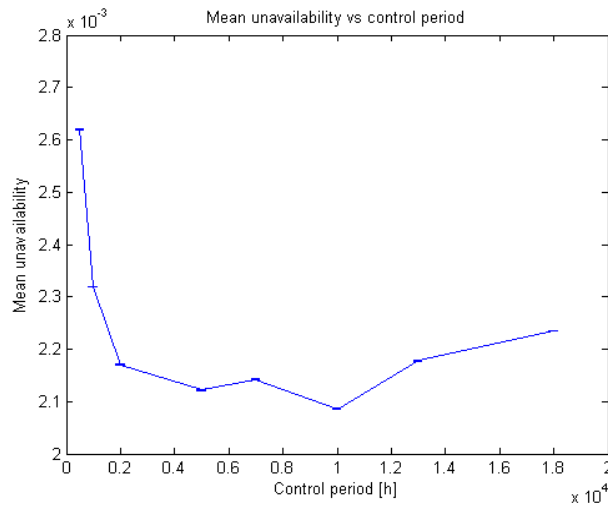


Figure 10: estimated mean unavailability varying the Control Period, with related 68.3% confidence interval.

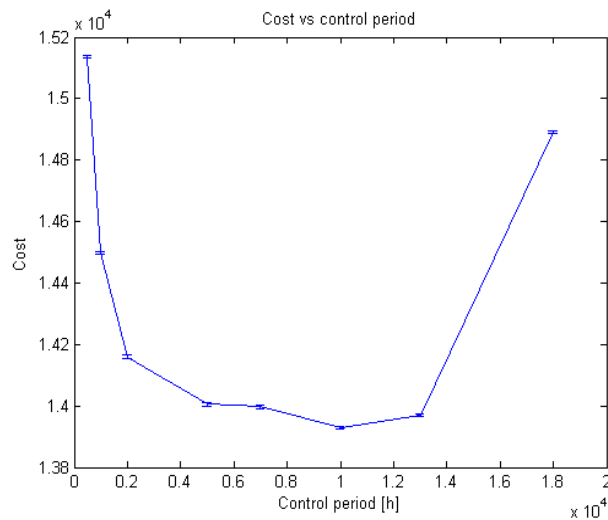


Figure 11: estimated maintenance costs varying the Control Period, with related 68.3% confidence interval.

The mean unavailability shows an initially decreasing trend, with a first minimum in correspondence of a Control Period equal to 5000h and another, deeper one in correspondence of a Control Period of 10000h, after which the trend starts increasing.

The maintenance cost has a similar trend, but with only the minimum in correspondence of a Control Period of 10000h.

One may then conclude that under the considered maintenance policy, the best Control Period is 10000h, with respect to both availability and costs. On the other hand, the relative flatness of the minimum is such that there is a wide interval of Control Period values in which both the mean unavailability and the maintenance cost are small and with little variations, which gives a margin of operational flexibility for choosing the Control Period value also accounting for other criteria (e.g., opportunistic maintenance).

4 Conclusions

Synthesizing from previous works by the authors, a modelling framework has been summarized, which allows assessing the impact of maintenance policies and specific conditions of component operation on the performance of the overall system of which the component is part.

Given the typical lack of experimental evidence on the influence of the component living conditions on its degradation, expert judgment is used within a fuzzy modelling approach. Monte Carlo simulation is then used to assess the goodness of the maintenance policy in terms of system availability.

To illustrate the approach, a previous application to the seals of the WFTP and their degradation due to contact fatigue has been re-proposed.

A number of issues remain open and need to be addressed in future works:

- The original approach includes maintenance as an IF; this requires to jointly model the effects of maintenance on the component degradation together with the effects of the other influencing factors. This may complicate the work of the experts who are requested to provide if-then linguistic rules linking the IFs with the component degradation state. A new approach to the maintenance influence modelling seems in order.
- The case study considered is made up of a single component affected by only one degradation process. The potential of the framework needs to be tested on multi-component and multi-degradation process systems.
- The operation of defuzzification performed on the output of the Forward Module does not propagate the uncertainties affecting the degradation state reached by the component. This leads to MC simulations which sample from exponential distributions without considering the uncertainty of the parameters of those distributions.
- Fuzzy logic framework has been developed by applying the Mamdani inference system. This limits the activation degrees of the degradation states to values smaller than 1, i.e., it is not guaranteed that the maximum of the activation degree of the degradation state is equal to 1. This problem, which leads to a smaller confidence on the degradation state, may be overcome by considering more sophisticated inference systems.

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