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***Abstract*—We challenge the validity of Dempster-Shafer Theory by using an emblematic example to show that DS rule produces counter-intuitive result. Further analysis reveals that the result comes from a understanding of evidence pooling which goes against the common expectation of this process. Although DS theory has attracted some interest of the scientific community working in information fusion and artificial intelligence, its validity to solve practical problems is problematic, because it is not applicable to evidences combination in general, but only to a certain type situations which still need to be clearly identified.**

Keywords: Dempster-Shafer Theory, DST, Mathematical Theory of Evidence, belief functions.

I. Introduction

Dempster-Shafer Theory (DST), also known as the Theory of Evidence or the Theory of Belief Functions, was introduced by Shafer in 1976 [1], based on Dempster's previous works [2]–[4]. This theory offers an elegant theoretical framework for modeling uncertainty, and provides a method for combining distinct bodies of evidence collected from different sources. In the past more than three decades, DST has been used in many applications, in fields including information fusion, pattern recognition, and decision making [5].

Even so, starting from Zadeh's criticism [6]–[8], many questions have arisen about the validity and the consistency of DST when combining uncertain and conflicting evidences expressed as basic belief assignments (bba's). Beside Zadeh's example, there have been several detailed analysis on this topic by Lemmer [9], Voorbraak [10] and Wang [11]. Other authors like Pearl [12], [13] and Walley [14], and more recently Gelman [15], have also warned the "belief function community" about the validity of Dempster-Shafer's rule (DS rule for short) for combining distinct pieces of evidences based on different analyses and contexts. Since the mid-1990's, many researchers and engineers working with belief functions in applications have observed and recognized that DS rule is problematic for evidence combination, specially when the sources of evidence are high conflicting.

In response to this challenge, various attempts have been made to circumvent the counter-intuitive behavior of DS rule. They either replace Dempster-Shafer's rule by alternative rules, listed for example in [16] (Vol. 1), or apply novel semantic interpretations to the functions [16]–[18].

Before going further in our discussion, let us recall two of Shafer's statements about DST:

> The burden of our theory is that this rule [Dempster's rule of combination] corresponds to the pooling of evidence: if the belief functions being combined are based on entirely distinct bodies of evidence and the set Θ discerns the relevant interaction between those bodies of evidence, then the orthogonal sum gives degree of belief that are appropriate on the basis of combined evidence. [1] (p. 6)
>
> This formalism [whereby propositions are represented as subsets of a given set] is most easily introduced in the case where we are concerned with the true value of some quantity. If we denote the quantity by θ and the set of its possible values by Θ, then the propositions of interest are precisely those of the form "The true value of θ is in T," where T is a subset of Θ. [1] (p. 36)

These two statements are very important since they are related to two fundamental questions on DST that are central in this discussion on the validity of DS theory:

1) What is the meaning of "pooling of evidence" used by Shafer? Does it correspond to an experimental protocol?
2) When "the true value of θ is in T" is asserted by a source of evidence, are we getting absolute truth (based on the whole knowledge accessible by everyone eventually) or relative truth (based on the partial knowledge accessible by the source at the moment)?

This paper starts with a very emblematic example to show what we consider as really problematic in DS rule behavior, which corresponds to the possible "dictatorial power" of a source of evidence with respect to all others and thus reflecting the minority opinion. We demonstrate that the problem is in fact not merely due to the level of conflict between sources to combine, but comes from the underlying interpretations of *evidence* and *degree of belief* on which the combination rule is based. Such interpretations do not agree with the common usage of those notions where an opinion based on certain evidence can be revised by (informative) evidence from other sources.

This work is based on our preliminary ideas presented in the Spring School on Belief Functions Theory and Applications (BFTA) in April 2011 [19], and on many fruitful discussions with colleagues using belief functions. Their stimulating comments, especially when they disagree, help us to clarify and present our ideas.[1] In Section II we briefly recall basics of DST and DS rule. In Section III, we describe the example and its strange (counter-intuitive) result. In Section IV we present a general analysis on the validity of DST, and we conclude our analysis in Section V.

II. Basics of DST

Let $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ be a frame of discernment of a problem under consideration containing n distinct elements θ_i, $i = 1, \dots, n$.

A basic belief assignment (*bba*, also called a belief mass function) $m(.) : 2^\Theta \rightarrow [0,1]$ is a mapping from the power set of Θ (i.e. the set of subsets of Θ), denoted 2^Θ, to $[0,1]$, that must satisfy the following conditions: 1) $m(\emptyset) = 0$, i.e. the mass of empty set (impossible event) is zero; 2) $\sum_{X \in 2^\Theta} m(X) = 1$, i.e. the mass of belief is normalized to one. Here $m(X)$ represents the mass of belief exactly committed to X. An element $X \in 2^\Theta$ is called a focal element if and only if $m(X) > 0$. The set $\mathcal{F}(m) \triangleq \{X \in 2^\Theta | m(X) > 0\}$ of all focal elements of a bba $m(.)$ is called the core of the bba. By definition, a Bayesian bba $m(.)$ is a bba having only focal elements of cardinality 1. The vacuous bba characterizing full ignorance is defined by $m_v(.) : 2^\Theta \rightarrow [0;1]$ such that $m_v(X) = 0$ if $X \neq \Theta$, and $m_v(\Theta) = 1$.

From any bba $m(.)$, the belief function $Bel(.)$ and the plausibility function $Pl(.)$ are defined as $\forall X \in 2^\Theta : Bel(X) = \sum_{Y | Y \subseteq X} m(Y)$ and $Pl(X) = \sum_{Y | X \cap Y \neq \emptyset} m(Y)$. $Bel(X)$ represents the whole mass of belief that comes from all subsets of Θ included in X. $Pl(X)$ represents the whole mass of belief that comes from all subsets of Θ compatible with X (i.e., those intersecting X).

The DS rule of combination [1] is an operation denoted $\oplus$, which corresponds to the normalized conjunction of mass functions. Based on Shafer's description, given two independent and distinct sources of evidences characterized by bba $m_1(.)$ and $m_2(.)$ on the same frame of discernment Θ, their combination is defined by $m_{DS}(\emptyset) = 0$, and $\forall X \in 2^\Theta \setminus \{\emptyset\}$

$$m_{DS}(X) = [m_1 \oplus m_2](X) = \frac{m_{12}(X)}{1 - K_{12}} \qquad (1)$$

where

$$m_{12}(X) \triangleq \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = X}} m_1(X_1) m_2(X_2) \qquad (2)$$

corresponds to the conjunctive consensus on X between the two sources of evidence. K_{12} is the total *degree of conflict* between the two sources of evidence defined by

$$K_{12} \triangleq m_{12}(\emptyset) = \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = \emptyset}} m_1(X_1) m_2(X_2) \qquad (3)$$

When $K_{12} = m_{12}(\emptyset) = 1$, the two sources are said in total conflict and their combination cannot be applied since DS rule (1) is mathematically undefined, because of 0/0 indeterminacy [1]. DS rule is commutative and associative, which makes it attractive from engineering implementation standpoint, since the combinations of sources can be done sequentially instead globally and the order doesn't matter. Moreover, the vacuous bba is a neutral element for the DS rule, i.e. $[m \oplus m_v](.) = [m_v \oplus m](.) = m(.)$ for any bba $m(.)$ defined on 2^Θ, which seems to be an expected[2] property, i.e. a full ignorant source doesn't impact the fusion result.

The conditioning of a given bba $m(.)$ by a conditional element $Z \in 2^\Theta \setminus \{\emptyset\}$ has been also proposed by Shafer [1]. This function $m(.|Z)$ is obtained by DS combination of $m(.)$ with the bba $m_Z(.)$ only focused on Z, i.e. such that $m_Z(Z) = 1$. For any element X of the power set 2^Θ this is mathematically expressed by

$$m(X|Z) = [m \oplus m_Z](X) = [m_Z \oplus m](X) \qquad (4)$$

It has been proved [1] (p. 67) that this rule of conditioning expressed in terms of plausibility functions yields to the formula

$$Pl(X|Z) = Pl(X \cap Z)/Pl(Z) \qquad (5)$$

which is very similar to the well-known Bayes formula $P(X|Z) = P(X \cap Z)/P(Z)$. Partially because of this, DST has been widely considered as a generalization of Bayesian inference [3], [4], or equivalently, that probability theory is a special case of the Mathematical Theory of Evidence when manipulating Bayesian bba's.

Despite of the appealing properties of DS rule, its apparent similarity with Bayes formula for conditioning, and many attempts to justify its foundations, several challenges on the theory's validity have been put forth in the last decades, and remain unanswered. For instance, an experimental protocol to test DST was proposed by Lemmer in 1985 [9], and his analysis shows an inherent paradox (contradiction) of DST. Following a different approach, an inconsistency in the fundamental postulates of DST was proved by Wang in 1994 [11]. Some other related works questioning the validity of DST based on different argumentations have been listed in the introduction of this paper.

In the following, we identify the origin of the problem of DS rule under the common interpretation of the "pooling" of evidence, and why it is very risky to use it in very sensitive applications, specially where security, defense and safety are involved.

[1] Our presentation is not based on a previous statistical argumentation developed in [20], since it appears for some strong proponents of DST as an invalid approach to criticize DS rule. In this paper we adopt a simpler argumentation based only on common sense and simple considerations manipulating witnesses reports.

[2] A detailed discussion about this "expected" property can be found in [20].

III. A simple example and its strange result

To see the problem in combining evidence with DS rule, let us analyze an emblematic example. Consider a frame of discernment with three elements only, $\Theta = \{A, B, C\}$, satisfying Shafer's request, i.e. the elements of the frame are truly exhaustive and exclusive. As in Zadeh's example, we interpret the problem as medical diagnosis, where A, B and C correspond to three distinct pathologies (say A = brain tumor, B = concussion and C = meningitis) of a patient. In such a situation, it is reasonable to assume that these pathologies do not occur simultaneously, so Shafer's assumptions truly hold.

We suppose that two distinct doctors (or more generally, two witnesses) provide their own medical diagnostic (or more generally, a testimony) of the same patient, based on their own knowledges and expertises, after analyzing symptoms, IRM images, or any useful medical results. The diagnostics (testimonies) of the two distinct sources of evidences correspond to the two non-Bayesian bba's given by the doctors listed in Table I. The parameters a, b_1, and b_2 can take any value, as long as $a \in [0, 1]$, $b_1, b_2 > 0$, and $b_1 + b_2 \in [0, 1]$.

Focal elem. \ bba's	$m_1(.)$	$m_2(.)$
A	a	0
$A \cup B$	$1-a$	b_1
C	0	$1 - b_1 - b_2$
$A \cup B \cup C$	0	b_2

Table I
INPUT BBA'S $m_1(.)$ AND $m_2(.)$.

The two distinct sources are assumed to be truly independent (the diagnostic of Doctor 1 is done independently of the diagnostic of Doctor 2 and from different medical results, images supports, etc, and conversely) so that we are allowed to apply DS rule to combine the two bba's $m_1(.)$ and $m_2(.)$. Both doctors are also assumed to have the same level of expertise and they are equally reliable. Note that in this very simple parametric example the focal elements of bba's are not nested (consonant), and there really does exist a conflict between the two sources (as it will be shown in the derivations). It is worth to note also that the two distinct sources are truly informative since none of them corresponds to the vacuous belief assignment representing a full ignorant source, so it is reasonable to expect for both bba's to be taken into account (and to have an impact) in the fusion process. Here we use the notion of "conflict" as defined by Shafer in [1] (p. 65) and recalled by (3).

When applying DS rule of combination, one gets:

1) Using the conjunctive operator:

$$m_{12}(A) = a(b_1 + b_2) \tag{6}$$

$$m_{12}(A \cup B) = (1-a)(b_1 + b_2) \tag{7}$$

$$K_{12} = m_{12}(\emptyset) = 1 - b_1 - b_2 \qquad \text{(conflicting mass)} \tag{8}$$

2) and After normalizing by $1 - K_{12} = b_1 + b_2$, the final result is as follows:

$$\begin{aligned} m_{DS}(A) &= \frac{m_{12}(A)}{1-K_{12}} = \frac{a(b_1+b_2)}{b_1+b_2} \\ &= a = m_1(A) \end{aligned} \tag{9}$$

$$\begin{aligned} m_{DS}(A \cup B) &= \frac{m_{12}(A \cup B)}{1-K_{12}} = \frac{(1-a)(b_1+b_2)}{b_1+b_2} \\ &= 1 - a = m_1(A \cup B) \end{aligned} \tag{10}$$

Surprisingly, after combining the two sources of evidences with Dempster-Shafer's rule, we see that in this case the medical diagnostic of Doctor 2 doesn't count at all, because one gets $m_{DS}(.) = m_1(.)$. Though Doctor 2 is not a fully ignorant source and he/she has same reliability as Doctor 1, nevertheless his/her report (whatever it is when changing values of b_1 and b_2) doesn't count. We see that the level of conflict $K_{12} = 1 - b_1 - b_2$ between the two medical diagnostics doesn't matter in fact in the DS fusion process, since it can be chosen at any high or low level, depending on the choice of $b_1 + b_2$. Based on DST analysis, the Doctor 2 plays the same role as a vacuous/ignorant source of evidence even if he/she is informative (not vacuous), and truly conflicting (according to Shafer's definition) with Doctor 1.

This result goes against common sense. It casts serious doubt on the validity of DS rule, as well as its usefulness for applications, and interrogates on the real meaning of Shafer's pooling of evidence process. This example seems more crucial than the examples discussed in the existing literature in showing intolerable flaws in DST behavior, since in this example the level of conflict (whatever it is) between the sources doesn't play a role at all, so that it cannot be argued that in such a case DS must not be applied because of the high conflicting situation. In fact such a situation can occur in real applications and is not anecdotal, and the results obtained by DS rule can yield dramatical consequences. From Zadeh's example [6] and all the debates about it in the literature, it has been widely (though not completely) admitted that DS is not recommended when the conflict between sources is high. Our example brings out a more important question since it reveals that the problem of the behavior of DS rule is not due to the (high) level of conflict between the sources, but from something else — we can choose a low conflict level, but the result is still the same, so the problem remains.

We can see the situation better by generalizing from this example. What make this example special and emblematic of DS behavior is the fact that $Pl_1(C) = 0$. It not only means that Doctor 1 completely rules out the possibility of C, but also that this opinion cannot be changed by taking new evidence into consideration. This is the case, because according to Shafer's definition [1] (p. 43), $Pl_1(C) = 0$ means for every $X \in 2^\Theta$ that $X \cap C \neq \emptyset$, $m_1(X) = 0$. When DS rule is applied to combine $m_1(.)$ and an arbitrary $m'(.)$, for every $Y \in 2^\Theta$ that $Y \cap C \neq \emptyset$, $m_{DS}(Y) = 0$, because it is the sum of some products, each of them take one of the above $m_1(X)$ as a factor. Consequently, $Pl_{DS}(C) = 0$, no matter what the other body of evidence is. Actually in such situations DS rule doesn't perform a fusion between sources' opinions, but an exclusion, ruling out the conflicting hypothesis considered by the second source.

Put it in another way, the effective frame of discernment of Doctor 1 is not really $\{A, B, C\}$, but $\{A, B\}$, because the pathology C has been ruled out of the frame by Doctor 1, since the focal elements of $m_1(.)$ are A and $A \cup B$ only. The above analysis tells us that when different supports (i.e. sets of focal elements) are combined according to DS rule, the resulting bba will be defined in the intersection of the supports of each source, under the condition that it is not empty (otherwise the evidence is total conflicting, and the rule is not applicable). Furthermore, all of the original bba will be normalized on this common support before being combined. This is the very fundamental principle on which is based DST and the combination of evidence proposed by Shafer.

More precisely in our example, the adjusted bba $m'_2(.)$ of Doctor 2 is described in Tables II- III and IV.

Focal elem. \ bba's	$m_1(.)$	$m_2(.)$
A	a	0
$A \cup B$	$1-a$	b_1
$C \equiv \emptyset$	0	$1-b_1-b_2$
$A \cup B \cup \emptyset = A \cup B$	0	b_2

Table II
STEP 1 OF ADJUSTMENT OF $m_2(.)$.

Focal elem. \ bba's	$m_1(.)$	$m_2(.)$
A	a	0
$A \cup B$	$1-a$	b_1+b_2
$C \equiv \emptyset$	0	$1-b_1-b_2$

Table III
STEP 2 OF ADJUSTMENT OF $m_2(.)$.

Focal elem. \ bba's	$m_1(.)$	$m'_2(.)$
A	a	0
$A \cup B$	$1-a$	$\frac{b_1+b_2}{1-(1-b_1-b_2)} = 1$

Table IV
ADJUSTED AND NORMALIZED BBA'S $m_1(.)$ AND $m'_2(.)$.

After this adjustment, the bba $m'_2(.)$ of Doctor 2 becomes the vacuous bba, which has no impact to the result. This perfectly explains the result produced by DS rule, but doesn't suffice to fully justify its real usefulness for applications.

In general, given two frames of discernment to be combined, if one is a proper subset of the other, the result is asymmetric — the smaller frame always wins the competition, though the other one does not always become vacuous.

Again, here we see that the result is not from any specialty of our emblematic example, but directly from conjunctive nature of the DS rule. As Shafer wrote: "A basic idea of the theory of belief functions is the idea of evidence whose only direct effect on the frame Θ is to support a subset A_1, and an implicit aspect of this idea is that when this evidence is combined with further evidence whose only direct effect on Θ is to establish a compatible subset A_2, the support for A_1 is inherited by $A_1 \cap A_2$." [21]

Now the fundamental question becomes: should evidence combination be treated in this way?

IV. Evaluating the validity of DST

After sharing the above result we found with other researchers in the field, we got three types of response, which can be roughly categorized as:

1) This result does not show that DST is wrong, but that there are situations where it is not applicable. This example contains conflicting evidence, so DST should not be applied.
2) This result does not show that DST is wrong, and this result is exactly the correct one. It is your intuition that is wrong.
3) This result shows that DST is wrong, since it is unreasonable to let one expert's opinion to completely suppress the other opinions.

The first response is not very satisfactory because it tells us that DST should not be applied when evidences conflict. If we admit such a response, what is the real purpose in using DS rule in practical applications using belief functions, since most of them do involve conflicting sources? In agreeing with the first response, we see that DS rule reduces to the strict conjunctive rule which should be used only in limited cases where there is no conflict between sources. It is not obvious to see why the conjunctive rule even in these cases is well-adapted for the pooling of evidence. In fact, in the context on no conflicting sources, the conjunctive rule corresponds just to the selection of the most specific source, rather than a combination (pooling) of evidences.

Each of the two last responses is supported by a long argument, which sounds reasonable until they are put together — how can we have such different opinions on such a simple example? Can DST be used to combine them to provide a final conclusion based on the pooled evidence?

Instead of trying to apply DS rule (if possible) or to analyze the above responses one by one, we will temporarily step back from this concrete case, and discuss a meta-level problem first, that is, when a mathematical theory is applied to a practical situation, how to decide the validity of this application? In what sense the result is "right" or "wrong"?

Of course, there are some trivial cases where the solution is obvious. If the result is deterministic and there is an objective way to check it, then the conclusion is conclusive. Unfortunately, in the field of uncertain reasoning, it is not that simple. In the above example, we cannot use the disease the patient has (assume we finally become certain about it) to decide whether DST is correctly applied to it, though it may influence our degree of belief about the theory. Actually, this is exactly how "evidence" is different from "proof" in deciding the truthfulness of a conclusion — while a proof can determine the truth-value of a statement *conclusively*, evidence can only do so *tentatively*, because in realistic situations there is always further evidence to come.

Another relatively simple situation is that an internal inconsistency is found in the mathematical theory. In that case the theory is clearly "wrong", and is not good for any normal usage. This is not the case here, neither. There are inconsistencies founded about DST, such as [11], but it is between the theory and its semantic interpretations (that is, between what it is claimed to do and what it actually does), rather than within the (uninterpreted) mathematical structure of the theory.

What we are facing is a more complicated situation, where the result produced by a theory "sounds wrong", that is, it conflicts with our intuition, experience, or belief. DST is not the only theory that has run into this kind of trouble, and there are indeed three logical possibilities, as represented by the responses listed previously. What to make the situation more complicated is the existence of two types of researchers, with very different motivations in this context:

- **A**: There are people who start with a domain problem, which is called "belief revision", "evidential reasoning", "data fusion", and so on, by different researchers. They are looking for a mathematical tool for this job.
- **B**: There are people who start with a mathematical model that has some properties they like, DST in this case, and are looking for proper practical applications for it.

In general, both motivations are legitimate, but it is crucial that they should not be confused with each other. We belong to Type **A**, and are evaluating DST with respect to the problem we have in mind, to which DST is often claimed to be a solution. For this reason, we argue that DST failed to do the job. Some objection to our conclusion comes from people of Type **B**, to them DST can be called "wrong" only when an internal inconsistency is found, otherwise the theory is always correct, and all mistakes are cased by its human users. Here we are not criticizing DST in that sense. Using the above example, we conclude DST to be "wrong" because it fails to properly handle evidence combination, or in other words, what it claims to do does not match what it actually does, as the defect proved in [11].

To support our conclusion with evidence (rather than with intuition), we start from an analysis of the task of "evidence combination" (or call it "data fusion"). As mentioned above, "evidence" has an impact on "degree of belief" in a system doing evidential reasoning, like "proof" has on "truth-value" in a system using classical logic, except here the impact is tentative and inconclusive (i.e. it doesn't provide an absolute truth). This is exactly why evidence combination becomes necessary (while there is no corresponding operation in classical logic) — in a system that is open to new evidence, it needs to use new evidence to adjust its degree of belief, and the "rule" here should be similar to the rule used to merge the opinions of different experts. In both cases, each opinion has some evidential support, though none of them can be treated as absolutely certain.

This is according to the above understanding of "evidence combination" that DST's result in the above example is considered as "wrong", simple because it allows certain opinion to become immune to revision. To be concrete, what if the previous example consists of 100 doctors, and all of them, except Doctor 1, consider C the most likely disease the patient has, though they cannot completely rule out the possibility of A and B. On the other hand, Doctor 1, for some unspecified reason, considers C impossible, and A more likely than B. In this case, DST will still completely accept Doctor 1's opinion, and ignore the judgment of the other 99 experts. We don't believe anyone will consider this judgment reasonable.

Based on conjunction, DS rule supports the dictatorial power of a source, by accepting the minority opinion as effective solution for "pooling" evidences, no matter that the general a priori assumption applying DS rule is all sources of information are equally reliable, which means all sources' opinions should be taken into account on equal terms. From a theoretical point of view, we don't think this type of belief should be allowed in evidential reasoning; from a practical point of view, such a treatment can lead to serious consequences, since it means that some errors in one evidence channel cannot be corrected by other channels, no matter how many and how strong.

To us, the only possible way to justify DST in similar situations is to change what we mean by "evidence combination". According to Shafer's treatment, "evidence combination" becomes a process similar to constraint satisfaction, where each piece of evidence put some absolute restriction on where the final result can be, and their combination corresponds to "to reach a consensus by mutual constraining". According to this interpretation, Doctor 1 has the right to suppress all the other opinions and therefore can dictates his opinion. If we want to consider each doctor's opinion as absolute truth (following Shafer's interpretation), though sometimes underspecified, then the result becomes acceptable. But in this case, the validity and usefulness of DS rule is strongly conditioned by the justification of the fact that each doctor does really have access to the absolute truth on the proposition under consideration. How can this be done in practice? From what knowledge can a doctor get an absolute truth on a proposition? The answers to these very important questions for validating DS rule haven't been given in the literature so far (to the authors knowledge).

Furthermore, if every doctor is allowed to claim this kind of absolute truth, there is nothing preventing different doctors from announcing different "truths", which leads to "total conflict" situation that cannot be resolved by Dempster's rule. Therefore, the theory faces a paradox: it must either ban the claim of any unrevisable belief, or find a way to handle the conflict among such beliefs. To accept unrevisable beliefs only from a single source does not sound reasonable.

The difference between the two interpretations of "evidence combination" are semantic and philosophical. According to our interpretation, when there are competing opinions supported by distinct evidences, none of them has "absolute truth", but each has some "relative truth", with respect to the supporting evidence, so in the combination process all the opinions can be more or less revised, and the result is usually a compromise; According to Shafer's interpretation, if one source considers an element in the frame of discernment as impossible, this judgment will be taken as absolute truth, and is therefore unrevisable by the other opinions.

Though it is possible to imagine certain situations, such as Shafer's "random coding" scenario [21], where DST can produce reasonable results, we believe our interpretation of "evidence combination" better matches the common sense meaning of the phrase, as well as the most practical needs in this domain.

It is true that every mathematical theory has its limited applicable domain, and we are not demanding DST to be "universal". However, here the situation is that DST is often presented as a general mechanism for evidential reasoning. Even though it has been widely acknowledged in the community that DST cannot properly handle (highly) conflicting evidence, its cause has not been clearly analyzed, nor is the applicable situations of the theory clearly specified. The above analysis answers these questions: conflicting evidence (whatever they are, in high or in low conflict) cannot be handled well by DST, since they cannot be seen as "partial truth" anymore.

The last important point to underline is the about DS conditioning rule (4) and the formula (5) for conditional plausibility. Let consider Θ and two bba's $m_1(.)$ and $m_2(.)$ defined on 2^Θ and their DS combination $m_{DS}(.) = [m_1 \oplus m_2](.)$ and let assume a conditioning element $Z \neq \emptyset$ in 2^Θ and the bba $m_Z(Z) = 1$, then $m_{DS}(.|Z) = [m_{DS} \oplus m_Z](.) = [m_1 \oplus m_2 \oplus m_Z](.)$. Because $m_{DS}(.) = [m_1 \oplus m_2](.)$ is inconsistent with the probability calculus [10], [11], [14], [15], [20], then $m_{DS}(.|Z)$ is also inconsistent. Therefore for any X in 2^Θ, the conditional plausibility $Pl(X|Z)$ expressed by $Pl(X|Z) = Pl(X \cap Z)/Pl(Z)$ (with apparent similarity with Bayes formula) obtained from $m_{DS}(.|Z)$ is not compatible with the conditional probability as soon as several sources of evidences are involved.

V. Conclusions

In this paper, through a very simple example, we have shown and explained what we consider as a very serious flaw of DS reasoning, which has generated strong controversies in the last three decades. The problem is: given the mathematical property of the combination rule, in certain situation the judgment expressed by a single information source will be effectively treated as absolute truth that will dominate the final result, no matter what judgments the other sources have. Such a result is in total disagreement with the common-sense notion of "evidence combination", "information fusing", or whatever the process is called, because in such a process, each information or evidence source should always be considered only as having local or relative truth. In summary, we believe DST has been often and widely used in situations where it should not, and such applications are wrong. After several decades of existence, proponents of DST need to clearly identify the situations where its model may be truly applicable and what real experimental "pooling" of evidence process DS rule corresponds to. This question is not what this paper is discussing, but is left for future research and discussions.

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