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Subproblem $h$-Conform Magnetodynamic Finite Element Formulation for Accurate Model of Multiply Connected Thin Regions

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Abstract — A subproblem $h$-conform eddy current finite element method is proposed for correcting the inaccuracies inherent to thin shell models. Such models replace thin volume regions by surfaces but neglect border effects in the vicinity of their edges and corners. The developed surface-to-volume correction problem is defined as a step of the multiple subproblems applied to a complete problem, consisting of inductors and magnetic or conducting regions, some of these being thin regions. The general case of multiply connected thin regions is considered.

I. INTRODUCTION

Thin shell (TS) finite element (FE) models [1], [2] [3], assume that the fields in the thin regions are approximated by a priori 1-D analytical distributions along the shell thickness. In the frame of the FE method, their interior is thus not meshed and is rather extracted from the studied domain, being reduced to a zero-thickness double layer with interface conditions (ICs) linked to the inner analytical distributions. This means that corner and edge effects are neglected.

To overcome these drawbacks, the subproblem method (SPM) for the $h$-conform FE formulation has been already developed by authors [5] for simply connected TS regions, proposing a surface-to-volume local correction. The method is herein extended to multiply connected TS regions, i.e. regions with holes, for both the associated surface model (alternative to the method in [3]) and its volume correction. The global currents flowing around the holes and their associated voltages are naturally coupled to the local qualities, via some cuts for magnetic scalar potential discontinuities at both TS and correction steps.

A reduced model (SP $q$) with the inductors alone is first considered before adding the TS (SP $p$), followed by the volumic correction SP (SP $k$). From SP $q$ to SP $p$, the solution $q$ contributes to the surface sources (SSs) for the added TS, with TS ICs. From SP $p$ to SP $k$, SSs and volume sources (VSs) allow to suppress the TS and cut discontinuities and simultaneously add the actual volume of the thin region. Each SP requires a proper adapted mesh of its regions. The method is illustrated and validated on a practical problem.

II. FROM THIN SHELL TO VOLUME MODEL

A. $h$-formulation with source and reaction magnetic fields

The magnetic field $h_i$ at step $i$ of the SPM, is defined in a domain $\Omega_i$, with boundary $\partial \Omega_i = \Gamma_i = \Gamma_{k,i} \cup \Gamma_{h,i}$, as

$$h_i = h_{s,i} + h_{c,i} - \text{grad}\phi_i, \text{curl}\phi_i = j_{s,i}, \quad (1a-b)$$

where $h_{s,i}$ is a source magnetic field due to the fixed current density $j_{s,i}$. $h_{c,i}$ is the reaction field in conducting regions $\Omega_{c,i}$ and $\phi_i$ is the reaction magnetic scalar potential in non-conducting regions $\Omega_{k,i}$. Potential $\phi_i$ in $\Omega_{k,i}$ is multivalued (Fig. 1) and made singlevalued via the definition of cuts through each hole of $\Omega_{k,i}$ [6]. The $h_i-\phi_i$ magnetodynamic formulation of the SPs $i = q, p$ and $k$ is obtained from the weak form of Faraday’s equation [5], i.e.

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the TS via \( h_{\ell_{\ell,\ell}} = h_{\ell_{\ell,\ell}} + h_{d_{\ell,\ell}} \), and \( h_{d_{\ell,\ell}} = h_{d_{\ell,\ell}} \) \([2]\). This can be also formulated via a discontinuity \( \phi_{\ell,\ell} = \Delta \phi_{\ell,\ell} + \phi_{d_{\ell,\ell}} \) (Fig. 1), with 
\( \phi_{\ell,\ell} = \phi_{\ell,\ell} + \phi_{d_{\ell,\ell}} \) and \( \phi_{d_{\ell,\ell}} = \phi_{d_{\ell,\ell}} \)._1

The discontinuities \( \phi_{d_{\ell,\ell}} \) of \( \phi_{\ell,\ell} \) are constant on each cut and can be written as

\[
\phi_{\ell,\ell} = \phi_{d_{\ell,\ell}} = \phi_{d_{\ell,\ell}} = I_{\ell},
\]

where \( I_{\ell} \) is the global current flowing around the cut \([4]\). Discontinuities \( \phi_{d_{\ell,\ell}} \) and \( \phi_{d_{\ell,\ell}} \) have to be matched at the TS-cuts intersections.

C. Volume Correction

The obtained TS solution in SP \( p \) is then corrected by SP \( k \) solution that overcomes the TS assumptions \([2]\). The SPM provides the tools to implement such a model refinement, thanks to simultaneous SSs and VSs. A fine volume mesh of the shell is now required and is extended to its neighborhood without including the other regions of previous SPs. This allows to focus on the fineness of the mesh only in the shell. SSs related to ICs \([2]\), \([5]\) compensate the TS and cut discontinuities, i.e., \( \phi_{\ell,\ell} \rightarrow \phi_{d_{\ell,\ell}} \), and \( \phi_{d_{\ell,\ell}} \rightarrow \phi_{d_{\ell,\ell}} \), to suppress the TS representation via SSs opposed to ICs, i.e., \( h_{\ell_{\ell,\ell}} = -h_{\ell_{\ell,\ell}} \), and \( \phi_{d_{\ell,\ell}} = -\phi_{d_{\ell,\ell}} \), and \( \phi_{d_{\ell,\ell}} = \phi_{d_{\ell,\ell}} \), in parallel to VSs \([5]\) in the added volume shell that account for volume change of \( \mu_{p} \) and \( \sigma_{p} \) in SP \( p \) to \( \mu_{k} \) and \( \sigma_{k} \) in SP \( k \) (with \( \mu_{p} = \mu_{0}, \mu_{k} = \mu_{0}, \sigma_{p} = \sigma_{0}, \sigma_{k} = \sigma_{0} \)).

III. APPLICATION

The 3D test problem is based on TEAM problem 7: an inductor placed above a thin plate with a hole (Fig. 2) \((\mu_{\text{plate}} = 1, \sigma_{\text{plate}} = 35.26 \text{ MS/m})\). A SP scheme considers the TS model followed by its volume correction.

Distributions of eddy current densities \( j_{\ell} \) on the TS SP \( p \) and volume correction SP \( k \) are shown in Fig. 2, with plate thickness \( d = 19 \text{ mm} \) and frequency \( f = 200 \text{ Hz} \) (skin depth \( \delta = 6 \text{ mm} \)). The TS error on \( j_{\ell} \), locally reaches 43%. The inaccuracy on the Joule power loss densities of TS SP \( p \) is pointed out by the importance of the correction SP \( k \) (Fig. 3). It reaches several tens of percent along the borders for some critical parameters: e.g., 53% (Fig. 3, top) or 61% (Fig. 3, bottom), for \( \delta = 6 \text{ mm} \) in both cases. Significant errors on the Joule losses for TS SP \( p \) are shown in Table I. They increase with both the thickness of the plate and the frequency.

The SPM allows to accurately correct any TS solution. In particular, accurate correction of eddy current and power loss density are obtained at the edges and corners of multiply connected thin regions. Details on the efficiency of the proposed method and the simplification of SP meshes will be given in the extended paper.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{Thickness} \( d \) (mm) & \textbf{Frequency} \( f \) (Hz) & \textbf{Thin Shell} \( P_{\text{thin}} \) (W) & \textbf{Volume} \( P_{\text{vol}} \) (W) & \textbf{Error} \( \% \) \\
\hline
\textbf{\( d \)} & \textbf{\( f \)} & \textbf{\( P_{\text{thin}} \)} & \textbf{\( P_{\text{vol}} \)} & \textbf{\( \% \)} \\
\hline
1 & 30 & 14.63 & 13.82 & 4.36 \\
19 & 30 & 14.63 & 13.82 & 4.36 \\
2 & 200 & 50.44 & 47.33 & 4.94 \\
19 & 200 & 8.88 & 15.19 & 41.31 \\
\hline
\end{tabular}
\caption{Joule losses in the plate}
\end{table}

\section*{REFERENCES}


