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ANALYTICAL INVESTIGATION AND EXPERIMENTAL APPLICATION
OF THE SOURCE MODULATION TECHNIQUE TO MEASURE \( \rho/\beta_{\text{eff}} \)

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Abstract

In recent years great interest has been displayed, worldwide, for Accelerator Driven Subcritical reactors (ADS) to incinerate the minor actinides generated by the existing energy producing reactors. In sub critical systems, the effective neutron multiplication factor is lower than 1.0 and the neutrons otherwise required to maintain the chain reaction, can be put to other uses, in particular, the destruction of nuclear wastes such as minor actinides (MA). One of the major advantages of such ADS systems is that it can be operated with very high MA content without jeopardizing the overall safety due to a small effective delayed neutron fraction, a small Doppler temperature coefficient and possibly also a large void coefficient depending on the chosen coolant. This enhanced safety however prerequisites at all time a sufficient subcriticality margin. Reliable reactivity monitoring techniques are hence required to achieve this goal. The MUSE-4 program is a series of low power experiments carried out at the CEA-Cadarache MASURCA facility to investigate the various methods leading to the measurement of the reactivity level and associated kinetic parameters such as the effective delayed neutron fraction. The aim of this paper is to present the results obtained with a method which directly gives the ratio, for a sub critical assembly, between the reactivity \( \rho \) and the effective delayed neutron fraction \( \beta_{\text{eff}} \). By combining these results to those obtained with the \( k_p \)-method for the prompt neutron multiplication coefficient, we have access to the parameters which govern the prompt and the slow kinetics of a sub critical assembly. These parameters can be obtained without reference to any calibration measurement in critical configuration. It opens the way to the control of larger subcritical demonstrators which are operating with fuels which cannot be used in critical reactor, and, thanks to subcriticality, which are characterized by a deterministic safety.

Keywords

prompt jump, subcritical system, reactivity measurement, delayed neutron fraction
In recent years great interest has been displayed, worldwide, for Accelerator Driven Sub critical reactors (ADS) to incinerate the minor actinides generated by the existing energy producing reactors. In sub critical systems, the effective neutron multiplication factor is lower than 1.0 and the neutrons otherwise required to maintain the chain reaction, can be put to other uses, in particular, the destruction of nuclear wastes such as minor actinides (MA). One of the major advantages of such ADS systems is that it can be operated with very high M.A content without jeopardizing the overall safety due to a small effective delayed neutron fraction, a small Doppler temperature coefficient and possibly also a large void coefficient depending on the chosen coolant. This enhanced safety however prerequisites at all time a sufficient subcriticality margin. Reliable reactivity monitoring techniques are hence required to achieve this goal. The MUSE-4 program [1] is a series of low power experiments carried out at the CEA-Cadarache MASURCA facility to investigate the various methods leading to the measurement of the reactivity level and associated kinetic parameters such as the effective delayed neutron fraction. This program has been conducted within the frame of a large international collaboration subsidized by the 5th European Framework (EURATOM). The aim of this paper is to present the results obtained with a method which directly gives the ratio, for a sub critical assembly, between the reactivity \( \rho \) and the effective delayed neutron fraction \( \beta_{\text{eff}} \).

2 EXPERIMENTAL SETUP

We will give only the main characteristics of the experimental facility because an exhaustive description of the whole assembly can be found elsewhere [1,2].

The MUSE-4 experiment has investigated the coupling of the MASURCA multiplying medium to neutron sources of 2.6 and 14 MeV provided by the GENEPI pulsed neutron generator. The MASURCA reactor facility is dedicated to the neutronic studies of fast lattices. The fuel is a mixture of uranium and plutonium oxide (MOX), with a basic cell composed of equal amounts of solid Na and fuel, representative of a fast Pu burner core (Pu enrichment of \( \sim 25\% \) with \( \sim 18\% \) content of \( ^{239}\text{Pu} \)) with sodium coolant. The axial and radial reflectors consist of steel and Na (75%/25%). The core is cooled by air and the maximum operating power of the facility is limited to 5 kWth (when critical).

Several sub critical configurations (SC0, SC2 and SC3) with a multiplication factor \( k_{\text{eff}} \) of respectively close to 1, 0.97 and 0.95, obtained by replacing peripheral fuel assemblies by reflector cells, were studied, with special attention paid to the kinetic behavior of the system. In the last configuration SC3, some of the sodium rods were replaced by lead in order to study the influence of a lead cooled ADS (SC3-Pb). The GENEPI pulsed neutron generator was designed and built specifically for the MUSE-4 experiment. Its main characteristic is to deliver very intense (around 40 mA peak current) and short (<1 \( \mu \)s) pulses of deuterons, with a very short falling time. The 250 keV deuteron beam is guided inside the reactor in a thimble, arriving in the middle of the core where a titanium-deuterium or a titanium-tritium target is located. In the following experiment the tritium target is used, providing about \( 3 \times 10^6 \) neutrons per pulse by \( \text{T(d,n)} \) reactions (with a fresh target). The repetition rate is adjustable from a few Hz up to 4 kHz.
The static and dynamic measurements performed during the MUSE-4 program are summed up in Ref.[2] with an exhaustive description of the measurements techniques.

3 OVERVIEW OF THE SOURCE MODULATION TECHNIQUE

The source modulation technique (SMT) is in principle a source variation technique. In such a technique the response of the reactor to a change in external source intensity is analyzed to obtain information about the reactivity level.

The source modulation technique is based on an external pulsed source with a fixed frequency operating in the kHz range connected to a subcritical core with a fixed reactivity level. During this stationary condition (on the "macroscopic" time scale level), the associated flux level $P_H$ (averaged over a repetition period) is measured. While the sub-critical level is kept the same, the frequency of the external source is then suddenly changed to a much lower frequency. Immediately after this drop in source frequency the flux level $P_c$ (averaged over a repetition period) is measured. When the precursor population is adjusted to the new (lower) source frequency level, the stationary flux level $P_L$ (averaged over a repetition period) is also measured.

By averaging the measured count-rate over the repetition period, the microstructure of the source pulsation will be masked (see Fig. 1). In the following we will convert the time microstructure of the source pulsation to a time macrostructure.

**Microscopic Time Structure ("ms"-scale)**

**Macroscopic Time Structure ("s"-scale)**

Fig. 1: Illustration of the reactor response to Dirac pulses in two different time scales, microstructure and macrostructure, in the source modulation technique.

3.1 Conversion of the time microstructure to the time macrostructure

To simplify the mathematical treatment the source pulses of about 1 µs pulse width will be represented by Dirac pulses. Hence, the source will be represented by a series of Dirac pulses with a fixed frequency. From basic reactor theory we know that in the case of point kinetics with one group of delayed neutrons the reactor response to a Dirac pulse is given by:
This response contains a fast decaying part \( p_p(t) \) due to the decay of the prompt neutron population and a much slower decaying part \( p_d(t) \) due to the decay of the neutron precursor population. Since the source frequency can easily be chosen to be much slower than the decay constant of \( p_p(t) \) and much faster than the decay constant of \( p_d(t) \), one obtains in the case of a series of Dirac pulses the typical superimposed signal. Since the prompt part \( p_p(t) \) of every pulse is died out when the next Dirac pulse is sent, the response after the \( n \)-th Dirac pulse will contain again the same prompt contribution, but also a second contribution which is due to the superposition of all previous delayed contribution. The total response after an infinite number of preceding Dirac pulses will therefore be given by:

\[
p_{\infty}(t) = p_{\infty}(t) + p_{\infty}(t)
\]

\[= S e^{\frac{\rho-\beta}{\lambda}} + \sum_{n=0}^{\infty} S \frac{\lambda \beta \Lambda}{(-\rho + \beta)} e^{\frac{-\rho \lambda n}{\rho+\beta}} \]  

(2)

When we examine in more detail the infinite sum of delayed contributions, we obtain the following expression:

\[
p_{\infty}(t) = S \frac{\lambda \beta \Lambda}{(-\rho + \beta)} e^{\frac{-\rho \lambda}{\rho+\beta}} \sum_{n=0}^{\infty} e^{\frac{(-\rho) \lambda n}{\rho+\beta}} = S \frac{\lambda \beta \Lambda}{(-\rho + \beta)} e^{\frac{-\rho \lambda}{\rho+\beta}} \sum_{n=0}^{\infty} e^{\frac{(-\rho) \lambda T}{\rho+\beta}}
\]

(3)

Taking into account the numerical values for the different parameters in MUSE (and for a sub critical system driven by a pulsed source with a high frequency) the approximation \( e^{-x} \approx 1 - x \) is valid. In that case, the last expression further reduces to:

\[
p_{\infty}(t) \approx S \frac{\lambda \beta \Lambda}{(-\rho + \beta)} e^{\frac{-\rho \lambda}{\rho+\beta}} \frac{-\rho + \beta}{(-\rho) \lambda T} = S \frac{\beta \Lambda}{(-\rho + \beta)(-\rho) T} e^{\frac{-\rho \lambda}{\rho+\beta}}
\]

(4)

Hence, the total response after an infinite number of preceding Dirac pulses will therefore be given by:

\[
p_{\infty}(t) = p_{\infty}(t) + p_{\infty}(t)
\]

\[= S e^{\frac{\rho-\beta}{\lambda}} + S \frac{\beta \Lambda}{(-\rho + \beta)(-\rho) T} e^{\frac{-\rho \lambda}{\rho+\beta}} \]  

(5)
Since during the measurement of the count-rate the considered integration time interval is much larger than the repetition period of the pulsed source, the measured average prompt contribution will be given by the average of the prompt contribution over the period $T$:

$$P_p = \frac{1}{T} \int_0^\infty S e^{-\frac{t}{\Lambda}} dt = \frac{\Lambda}{(-\rho + \beta)T} \quad (6)$$

Since the decay of the precursor population over the repetition period can to a first good approximation be considered as being negligible, the measured average delayed contribution will be given by:

$$P_d(t) = \frac{1}{T} \int_0^T S \frac{\beta \Lambda}{(-\rho + \beta)(-\rho)T} e^{-\frac{t}{\Lambda}} dt \approx S \frac{\beta \Lambda}{(-\rho + \beta)(-\rho)T} \quad (7)$$

The average count-rate over the period $T$ for a source being stationary during a long time is therefore given by:

$$P = P_p + P_d = S \frac{\Lambda}{(-\rho + \beta)T} + S \frac{\beta \Lambda}{(-\rho + \beta)(-\rho)T} = S \frac{\Lambda}{T(-\rho)} \quad (8)$$

One notices that the conventional formula for the subcritical multiplication of a stationary with source strength $S/T$ is found. The microstructure has been masked and results in a macrostructure with a stationary source with an averaged source strength of $S/T$.

### 3.2 Determination of the reactivity based on the different average count-rates

In the source modulation technique, three different average count-rates are measured. The average count-rate at the stationary flux level for a high source frequency is first measured. Subsequently, the high frequency is changed to a low frequency and the average count-rate immediately after the change is determined. Finally, when the precursor population is stabilized, the average count-rate at the stationary flux level for a low source frequency is measured.

Based on equation (8) we obtain for the high stationary count-rate level:

$$P_H = S \frac{\Lambda}{(-\rho)T_{\text{high}}} \quad (9)$$

For the low stationary count-rate level a similar expression is found:
The average count-rate immediately after the change in frequency is characterized by:
- The presence of the average delayed count-rate during the high count-rate level
- The presence of the average prompt count-rate during the low count-rate level
- The absence of the average delayed count-rate during the low count-rate level

Hence, the average count-rate $P_c$ immediately after the change in frequency is given by:

$$P_c = P_{H,d} + P_{L,p} = S \frac{\beta \Lambda}{(-\rho + \beta)(-\rho)T_{low}} + S \frac{\Lambda}{(-\rho + \beta)T_{high}}$$ (11)

Based on these measured count-rates we can define the following quantity:

$$\frac{P_{H} - P_c}{P_c - P_L} = (-\rho)$$ (12)

The measurement of the count-rate levels $P_H$, $P_c$ and $P_L$ hence allows the determination of the reactivity expressed in dollars.

4 DETERMINATION OF THE EFFECTIVE DELAYED NEUTRON FRACTION

Based on the measurement of the $\rho/\beta_{\text{eff}}$ ratio with the Source Modulation Technique, the effective delayed neutron fraction $\beta_{\text{eff}}$ can be derived in various sub critical configurations if the corresponding reactivity level is known. To yield the reactivity we will determine the effective prompt neutron multiplication coefficient $k_p$ which is closely related to the reactivity by means of:

$$\rho = \frac{k_{p,\text{eff}}}{k_p} - 1 + \beta$$ (13)

Based on equations (12) and (13), the reactivity level and the effective delayed neutron fraction can be derived.

When measuring the effective prompt neutron multiplication constant, one has to pay attention that the correct definition and interpretation is used. In literature, not always an explicit distinction is made between the two definitions of the prompt multiplication coefficient $k_p$:
- the source prompt neutron multiplication coefficient $k_{p,s}$ is linked to the multiplication of a particular neutron source and defined by the expression:
  $$\frac{1}{1 - k_{p,s}} = 1 + k_1 + k_2 + k_1k_2k_3 + \ldots$$ (14)
  $k_i$ being the prompt neutron multiplication of the $i^{th}$ generation

- the effective prompt neutron multiplication coefficient defined by the expression:
  $$k_{p,\text{eff}} = \lim_{i \to \infty} k_i$$ (15)

This coefficient is an intrinsic characteristic of the multiplying region of the reactor, it defines the safety margin to criticality and is the solution of the eigenvalue problem of the static, homogeneous transport equation
Since the ability to monitor this quantity \( k_{p,\text{eff}} \) (or shorter \( k_p \)) is a central issue, different techniques have been investigated during the MUSE program. We have chosen to use one of the most promising techniques in this respect: the \( k_p \)-method [3-6]. Since it is not our aim in this paper to focus on this technique which moreover was extensively discussed elsewhere, we will only recall here the basic principles.

In the \( k_p \)-method a model is proposed which takes into account the distribution of the neutron generation times following a fission \( P(\tau) \), \( \tau \) being the time between the creation of a neutron and the creation of another neutron resulting from a fission induced by the first neutron. This distribution can easily be obtained by Monte-Carlo simulation for a stabilized neutron source.

From that definition we deduce that
\[
\int P(\tau)d\tau = k_p \quad (16)
\]
and thus we can normalise \( P(\tau) \) to \( k_p = 1 \). With that normalised distribution \( P'(\tau) \), we have access to the number of neutrons in the core at any time for any \( k_p \) value, summing the contribution of each generation:
\[
N(t) = k_p P'(t) + k_p^2 P'(t)*P'(t) + k_p^3 P'(t)*P'(t)*P'(t) + .. \quad (17)
\]
where * denotes the convolution operator. The decrease rate \( \alpha_{kp}(t) \) can then be calculated for different \( k_p \) values from the logarithmic derivative:
\[
\alpha_{kp}(t) = \frac{1}{N(t)} \frac{dN}{dt} \quad (18)
\]
These different functions \( \alpha_{kp}(t) \) can then be compared with the one obtained from the experimental \( N(t) \) distribution. The one which fits best the experiment determines the \( k_p \) value of the reactor.

In Table 1, the values of the (effective) prompt neutron multiplication coefficient are given for two of the three configurations of interest. It is these values, combined with the \( \rho/\beta_{\text{eff}} \) measured values, which will be used in the next section, to get the effective delayed neutron fraction and the reactivity level.

Table 1: Prompt neutron multiplication factor for two of the three different core configurations of interest.

<table>
<thead>
<tr>
<th>configuration</th>
<th>SC0</th>
<th>SC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_p )</td>
<td>0.989 ± 0.001</td>
<td>0.967 ± 0.002</td>
</tr>
</tbody>
</table>

5 APPLICATION IN THE MUSE EXPERIMENT FOR DIFFERENT SUB CRITICAL Configurations

5.1 Practical implementation of the experimental technique

As explained in paragraph 3 the modulation of the source is obtained experimentally by varying the repetition rate of the neutron generator: during an irradiation time of 200 s
the generator is operating at 4 kHz and during the next 200 s, after a very short transient (~50 ns), it is operating at 300 Hz. The return phase to the high frequency is adapted to the reactor doubling time (~6 s) to avoid the drop of the safety rods; it is also a reason for not switching off the source, in order to decrease the time needed to reach the high frequency regime. To make this special frequency piloting a programmable voltage generator was used. Fig. 2 shows the operating repetition rate of the neutron source and the counting rate of a U5 fission chamber located in the core of the MASURCA reactor for the SC0 configuration. The dependence of the reactor power on the source intensity illustrates the source driven character of the assembly: neglecting the intrinsic source contribution, the ratio of the two asymptotic flux levels are equal to the frequencies ratio. On the other hand, the prompt and delayed neutron flux decreases are clearly seen.

After summation of several cycles we obtain one time spectrum from which the quantities involved in the $\rho/\beta_{\text{eff}}$ calculation can be extracted, as shown in Fig. 3. The detectors used to obtain the results shown in the next section are CEA U5 fission chambers loaded with 920 µg (SC0, SC2) and 15 mg (SC3-Pb). The counting rate at 4 kHz was about 200 counts/s in the SC2 configuration, and 2700 counts/s in the SC3-Pb. The SC2 data of the Fig. 4 represents 90 cycles of 400 s (about two days of MASURCA reactor) and the SC3-Pb data is the sum of 9 cycles of 800 s (a half of a reactor day): in that case the number of delayed neutrons is the lowest, consequently the counting rate at the low frequency level needs to be known as accurately as possible, that is why we increased each phase duration to 400 s.
Fig. 3: $P_h$, $P_l$, $P_c$ quantities required for the $-\rho/\beta_{\text{eff}}$ calculation (summation over all measured transients, here for the SC0 configuration).

5.2 Results and Analysis

Fig. 4 shows the time averaged counting rates obtained after summation over all the measured transients, for SC0, SC2 and SC3-Pb configurations. As expected from the previous analysis, the relative contribution of the delayed component strongly depends on the sub criticality level of the core configuration. To obtain the $P_c$ value and minimize its statistical error, the neutron counting rate decrease (due to precursor decreases) is fitted by the sum of six exponentials and a constant. The amplitudes of the exponentials are free parameters but their effective time constants $1/w_j$ are set by solving the inhour equation with six delayed neutron groups (assuming $\rho$ is constant and its order of magnitude is known). The value of the fit at the frequency transition gives the $P_c$ value.

![Graph showing neutron count rates for SC0 and SC2 configurations](image)

Fig. 4: Neutron count rates for the SC0 and SC2 configurations (sum of all measured transients) and logarithmic zoom on the transitions.

In Table 2 are given the $\rho/\beta_{\text{eff}}$ values obtained with the standard Modified Source Multiplication method [2, 5] and the values provided by the SMT. Both show quite good agreement for the three configurations. Table 2 gives also the $\beta_{\text{eff}}$ results for two of the three sub critical configurations obtained by combining the $\rho/\beta_{\text{eff}}$ values provided by the SMT with the $k_p$ values in Table 1 obtained with the $k_p$-method. They show an excellent agreement with the value $(327 \pm 3 \text{ pcm})$ calculated [2] using the ERANOS code and the JEFF-3 library.
Table 2: Reactivity values in $ obtained in this work and from the reference Modified Source Multiplication method [2, 5]. Effective delayed neutron fractions deduced from prompt multiplication factor measurements [4, 6] are also given.

<table>
<thead>
<tr>
<th>configuration</th>
<th>SCO</th>
<th>SC2</th>
<th>SC3-Pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho/\beta_{\text{eff}}$ (MSM)</td>
<td>$-1.86 \pm 0.1$</td>
<td>$-8.7 \pm 0.6$</td>
<td>$-11.1 \pm 0.9$</td>
</tr>
<tr>
<td>$\rho/\beta_{\text{eff}}$ (this work)</td>
<td>$-1.960 \pm 0.004$</td>
<td>$-9.23 \pm 0.12$</td>
<td>$-10.7 \pm 0.5$</td>
</tr>
<tr>
<td>$\beta_{\text{eff}}$ (pcm)</td>
<td>$336 \pm 17$</td>
<td>$329 \pm 15$</td>
<td>-</td>
</tr>
</tbody>
</table>

6 DISCUSSION

The Source Modulation Technique is obviously closely related to the classical Source Jerk (SJ) experiments. However, due to the sudden change of source strength, the SMT results are free of correction factors induced for S.J. experiments by the translation time of the source [7]. This technique, which was developed for a pulsed neutron source, can easily be extrapolated to the case of a continuous beam in an ADS. Indeed, the asymptotic count rate $C$ is proportional to the source strength $S$ through the relationship:

$$C = \varepsilon \frac{S}{(1 - k_s)} = \varepsilon \frac{1}{(1 - k_s)} \times \frac{I}{e} N_{\text{source/ion}}$$

(19)

where $\varepsilon$ is the detector efficiency, $k_s$ is the source multiplication factor, $e$ the elementary charge, $I$ the ion beam intensity, and $N_{\text{source/ion}}$ the number of source neutrons created per ion. By means of a sudden beam intensity decrease or switch off, we can then imagine to deduce the count rate at low intensity from the high regime count rate and the knowledge of the source beam intensity $I$, if all other parameters are assumed constant. Under these conditions, the low intensity phase duration could be sufficiently reduced to avoid temperature effects that the change of reactor power might induce. Theoretically the knowledge of the inherent source intensity should also be required, but practically, in a power ADS, the external source intensity should be much larger than the inherent one, leading to neglect the latter.

This method, so-called “prompt jump technique” requires experimental validation (comparison between different detector types and locations, investigation to what extent the low or zero intensity phase duration can be reduced...), which is the subject of a future program at the zero-power YALINA facility (Sosny, Belarus), in the frame of the 6th EURATOM FP “IP EUROTRANS” (ECATS).

7 CONCLUSION

In the framework of the MUSE project aimed at investigating different reactivity monitoring techniques for ADS, we have developed a source modulation technique, closely related to the classical Source Jerk techniques. This technique allows us to determine the ratio of the absolute reactivity $\rho$ to the effective delayed neutron fraction $\beta_{\text{eff}}$. By combining these results to those obtained with the kp-method for the prompt
neutron multiplication coefficient, we have access to the parameters which govern most relevant for safety issues. These parameters can be obtained without reference to any calibration measurement in critical configuration. It opens the way to the control of larger sub critical demonstrators which are operating with fuels which cannot be used in critical reactor, and, thanks to sub criticality, which are characterized by a deterministic safety. This kind of systems is an attractive option for nuclear waste incineration and provides a unique opportunity to improve the social acceptability of nuclear energy.

Acknowledgement

We want to thank F.Mellier, P.Chaussonnet, J.-M.Laurens and all the MASURCA reactor operating team for their constant help during the MUSE program. We are also very grateful to M.Fruneau for making possible the source modulation of GENEPI, and to B.Geslot for making the acquisition in the SC3-Pb configuration.

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