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To cite this version:
Jules Sadefo Kamdem. Coefficient of variation and Power Pen’s parade computation. 2011. hal-00586518

HAL Id: hal-00586518
https://hal.archives-ouvertes.fr/hal-00586518
Submitted on 17 Apr 2011

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Coefficient of variation and Power Pen’s parade computation

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April 16, 2011

Abstract

Under the assumption that income $y$ is a power function of its rank among $n$ individuals, we approximate the coefficient of variation and gini index as functions of the power degree of the Pen’s parade. Reciprocally, for a given coefficient of variation or gini index, we propose the analytic expression of the degree of the power Pen’s parade; we can then compute the Pen’s parade.

Key-words and phrases: Gini index, Income inequality, Ranks, Harmonic Number, Pen’s Parade.

JEL Classification: D63, D31, C15.

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1 Introduction

Interest in the link between income and its rank is known in the income distribution literature as Pen’s parade following Pen (1971, 1973). The precedent has motivated research on the relationship between Pen’s parade and the Gini index that is a very important inequality measure. However, this research has so far focused on the linear Pen’s parade for which income increases by a constant amount as its rank increases by one unit (see Milanovic (1997)). Furthermore, a linear pen’s parade does not closely fit many real world income distributions Pen’s parade of which is convex in the absence of negative incomes (i.e., income increases by a greater amount as its rank increases by each additional unit). Mussard et al. (2010) has recently introduced the computation of Gini index with convex quadratic Pen’s parade (or second degree polynomial) parade for which income is a quadratic function of its rank (see also Ogwang (2010)).

This paper extends the computation of gini index by using a more general and empirically more realistic case for which Pen’s parade is a power function. Hence, the Gini indices for a linear Pen’s parade and for quadratic Pen’s parade becomes special cases of our thus introduced power Pen’s parade under some constraints on parameters inducing convexity. Under the assumption that income $y$ is a power function of its rank among $n$ individuals, we approximate the coefficient of variation and gini index as functions of the power degree of the pen’s parade. Reciprocally, for a given coefficient
of variation or gini index, we propose the analytic expression of the degree of the power pen’s parade. It follows that, for a given coefficient of variation or gini index we can compute the pen’s parade. We use the Milanovic (1997) data to illustrate the interest of our results.

The rest of the paper is organized as follows: In Section 2, the specification of an a power Pens parade is provided. In Section 3., the problem of fitting a power Pens parade to real world datasets of Milanovic (1997) is discussed. The concluding remarks are made in Section 4.

2 High degree Power Pen’s Parade

Suppose that positive incomes, expressed as a vector $y$, depend on individuals’ ranks $r_y$ in any given income distribution of size $n$. Suppose that incomes are ranked by ascending order and let $r_y = 1$ for the poorest individual and $r_y = n$ for the richest one. Hence, following Lerman and Yitzhaki (1984), the Gini index may be rewritten as follows:

$$G = \frac{2 \text{cov}(y, r_y)}{n \bar{y}}. \quad (1)$$

Here, $\text{cov}(y, r_y)$ represents the covariance between incomes and ranks and $\bar{y}$ the mean income. It is straightforward to rewrite (13) as:

$$G = \frac{2 \sigma_y \sigma_{r_y} \rho(y, r_y)}{n \bar{y}}, \quad (2)$$

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where $\rho(y, r_y)$ is Pearson’s correlation coefficient between incomes $y$ and individuals’ ranks $r_y$, where $\sigma_y$ is the standard deviation of $y$ and where $\sigma_{r_y}$ is the standard deviation of $r_y$.

Following (2) and under the assumption of a linear Pen’s parade (i.e. $y = a + b r_y$), Milanovic (1997) demonstrates that for a sufficiently large $n$, the Gini index can be further expressed as:

$$G = \frac{\sigma_y}{\sqrt{3y}} \rho(y, r_y) .$$  \hfill (3)

Milanovic’s result is very interesting since it yields a simple way to compute the Gini index. However, as mentioned by Milanovic (1997, page 48) himself, "in almost all real world cases, Pen’s parade is convex: incomes at first rise very slowly, and then their absolute increase, and finally even the rate of increase, accelerates". Thus, $\rho(y, r_y)$ which measures linear correlation will be less than 1. Again, from Milanovic (1997), a convex Pen’s parade may be derived from a linear Pen’s parade throughout regressive transfers (poor-to-rich income transfers). Inspired from Milanovic’s finding, we demonstrate in the sequel, without taking recourse to regressive transfers, that the Gini index can be computed with a quite general nonlinear polynomial function Pen’s parade. The computation of the gini index using a power function of order 2 (i.e. quadratic function) Pen’s parade has been introduced in Mussard et al. (2010). In this paper, we generalize the precede to compute gini index using a power function pen’s parade. Note that a linear and quadratic Pen’s
parade are particular cases our power Pen parade.

2.1 Simple Gini Index with nonlinear power Pen’s Parade

Consider a power function relation between incomes and ranks:

$$y = \sum_{i=0}^{k-1} b_i r^{\alpha_i} + b_k r^{\alpha_k}.$$  \hspace{1cm} (4)

with \(k \in \mathbb{N}^*, \alpha_0 = 0\) and \(\alpha_i \in \mathbb{R}^*_+\) for \(i = 1, \ldots, k\), and

$$\alpha_k > \max\{\alpha_i, i = 1, \ldots, k - 1\}.$$  

The covariance between \(y\) and \(r^{\alpha_j}\) for \(j \in \mathbb{N}^*\) is given by:

$$\text{cov}(y, r^{\alpha_j}) = \sum_{i=1}^{k} b_i \text{cov}(r^{\alpha_i}, r^{\alpha_j})$$  \hspace{1cm} (5)

$$= \sum_{i=1}^{k} \left( \frac{b_i}{n} r^{\alpha_i+\alpha_j+1} - \frac{b_i}{n^2} \sum_{r=1}^{n} r^{\alpha_i} \sum_{r=1}^{n} r^{\alpha_j} \right)$$

$$= \sum_{i=1}^{k} \left[ \frac{b_i}{n} \sum_{r=1}^{n} r^{\alpha_i+\alpha_j+1} - \frac{b_i}{n^2} \sum_{r=1}^{n} r^{\alpha_i} \sum_{r=1}^{n} r^{\alpha_j} \right]$$

The mean income \(\bar{y}\) is then:

$$\bar{y} = \sum_{i=0}^{k} b_i r^{\alpha_i} = \frac{1}{n} \left[ \sum_{r=1}^{n} \sum_{i=0}^{k} b_i r^{\alpha_i} \right]$$  \hspace{1cm} (6)
2.2 The coefficient of variation and Gini computation

Since incomes $y$ are positive, we use (13) by assuming that $b_k > 0$ and $b_j$ for $j = 1, \ldots, k - 1$ are chosen such that $y > 0$. For instance, if $k = 2$ then we can use $b_j$ for $j = 1, \ldots, k$ such that $b_1^2 - 4b_2b_0 < 0$. We are now able to compute the coefficient of variation of incomes as follows:

$$
\frac{\sigma_y}{\bar{y}} = \sqrt{\frac{\sum_{i=1}^{k} \sum_{j=1}^{k} b_i b_j \text{cov}(r^{\alpha_i}, r^{\alpha_j})}{\sum_{i=1}^{k} b_i r^{\alpha_i}}} 
\tag{7}
$$

$$
= \sqrt{\frac{\sum_{i=1}^{k} b_i \sum_{j=1}^{k} b_j \text{cov}(r^{\alpha_i}, r^{\alpha_j})}{\sum_{i=1}^{k} b_i r^{\alpha_i}}} 
\tag{8}
$$

$$
= \sqrt{\frac{\sum_{j=1}^{k} b_j \text{cov}(y, r^2_{\bar{y}})}{\sum_{i=1}^{k} b_i r^{\alpha_i}}} 
\tag{9}
$$

$$
= \sqrt{\sum_{j=1}^{k} b_j \sum_{i=1}^{k} \left[ \frac{b_k}{n} \sum_{r=1}^{n} r^{\alpha_i+\alpha_j+1} - \frac{b_k}{n^2} \sum_{r=1}^{n} r^{\alpha_i} \sum_{r=1}^{n} r^{\alpha_j} \right] \frac{1}{n} \left[ \sum_{r=1}^{n} \left( \sum_{i=0}^{k} b_i r^{\alpha_i} \right) \right]}
\tag{10}
$$

where the variance of $r^{\alpha_k}$ is

$$
\text{cov}(r^{\alpha_k}, r^{\alpha_k}) = \sigma_{r^{\alpha_k}}^2 = \frac{1}{n} \sum_{r=1}^{n} r^{2\alpha_k} - \left( \frac{1}{n} \sum_{r=1}^{n} r^{\alpha_k} \right)^2, 
\tag{10}
$$

the covariance between $r^{\alpha_j}$ and $r^{\alpha_i}$ for $0 < i, j \leq k$, is:

$$
\text{cov} (r^{\alpha_i}, r^{\alpha_j}) = \frac{1}{n} \sum_{r=1}^{n} r^{\alpha_i+\alpha_j} - \frac{1}{n^2} \sum_{r=1}^{n} r^{\alpha_i} \sum_{r=1}^{n} r^{\alpha_j}. 
\tag{11}
$$
After a double summation

\[
\text{cov} \left( \sum_{i=1}^{k} b_i r^{\alpha_i}, \sum_{j=1}^{k} b_j r^{\alpha_j} \right) = \sum_{i=1}^{k} \sum_{j=1}^{k} b_i b_j \left[ \frac{1}{n} \sum_{r=1}^{n} r^{\alpha_i+\alpha_j} - \frac{1}{n^2} \sum_{r=1}^{n} \sum_{r=1}^{n} r^{\alpha_j} \right].
\]  

(12)

**Lemma 2.1** When \( n \to \infty \), for \( q \in \mathbb{R}^*_+ \) and \( r \in \mathbb{N}^* \), we have that

\[
\sum_{r=1}^{n} r^q \equiv \frac{n^{q+1}}{q+1}.
\]  

(13)

Based on the preceding lemma, the variance of \( y \) when \( n \to \infty \) is equivalent to:

\[
\text{cov} \left( \sum_{i=1}^{k} r^{\alpha_i}, \sum_{j=1}^{k} r^{\alpha_j} \right) \equiv \frac{b_k^2}{n} \frac{n^{\alpha_k+\alpha+1}}{\alpha_k+\alpha+1} - \frac{b_k^2}{n^2} \frac{n^{\alpha_k+1}}{\alpha_k+1} \equiv \frac{(n^{\alpha_k} b_k n^{k})^2}{(2\alpha_k+1)(\alpha_k+1)^2}.
\]  

(14)

Therefore when \( n \to \infty \) the standard deviation of \( y \) is equivalent to

\[
\sigma_y \equiv \sqrt{\frac{(n^{\alpha_k} b_k n^{k})^2}{(2\alpha_k+1)(\alpha_k+1)^2}} \equiv \frac{\alpha_k |b_k|}{(\alpha_k+1)\sqrt{2\alpha_k+1}} n^{\alpha_k}.
\]  

(15)

When \( n \to \infty \), the mean of \( y \) is equivalent to

\[
\bar{y} = \frac{1}{n} \sum_{r=1}^{n} \sum_{i=0}^{k} b_i r^{\alpha_i} \equiv \frac{b_k n^{\alpha_k+1}}{n} \frac{n^{\alpha_k}}{\alpha_k+1} \equiv b_k \frac{n^{\alpha_k}}{\alpha_k+1}
\]  

(16)
Thereby, as \( n \to \infty \) the coefficient of variation is equivalent expressed as:

\[
\frac{\sigma_y}{\bar{y}} \equiv \frac{\alpha_k |b_k|}{(\alpha_k + 1)\sqrt{2\alpha_k + 1}} n^{\alpha_k - \frac{2}{2\alpha_k + 1}} \frac{n^{\alpha_k}}{b_k \alpha_k + 1}
\]  

(17)

then we have the following limit which depends on \( k \) and the sign of \( b_k \):

\[
\lim_{n \to \infty} \frac{\sigma_y}{\bar{y}} = \frac{|b_k|}{b_k} \frac{\alpha_k}{\sqrt{2\alpha_k + 1}} = \text{sign}(b_k) \frac{\alpha_k}{\sqrt{2\alpha_k + 1}}.
\]  

(18)

We have then proved the following theorem:

**Theorem 2.1** Under the assumption of a power Pen’s parade, i.e., \( y = \sum_{i=0}^{k} b_i r^i_y \), with \( b_k \neq 0 \), when \( n \to \infty \), the coefficient of variation of the revenue \( y \) has the following limit:

\[
\lim_{n \to \infty} \frac{\sigma_y}{\bar{y}} = \frac{|b_k|}{b_k} \frac{\alpha_k}{\sqrt{2\alpha_k + 1}}
\]  

(19)

On the other hand, following Milanovic (1997):

\[
\lim_{n \to \infty} 2 \frac{\sigma_{ry}}{n} = \lim_{n \to \infty} \sqrt{\frac{n^2 - 1}{3n^2}} = \frac{1}{\sqrt{3}}.
\]  

(20)

**Remark 2.1** The following theorem provide an interesting result in practise. It tells us that, for a given income data such as the coefficient of variation, we can compute the degree of the power Pen’s parade. Therefore, we can compute the Pen’s parade.
Theorem 2.2 For $b_k > 0$, if we know the coefficient of variation $CV_{emp}$ by using incomes data, then we can find the parameter $s = \alpha_k$ as a solution of the following equation:

$$\frac{s}{\sqrt{2 s + 1}} = CV_{emp}. \quad (21)$$

We can then use multiple regression to find $b_i$ and get the Pen’s parade that corresponds to incomes data. After a straightforward calculus, the positive solution of the equation (25) is:

$$\alpha_k = CV_{emp} \left[ CV_{emp} + \sqrt{CV_{emp}^2 + 1} \right]. \quad (22)$$

The product of (20), (18) and $\rho(y, r_y)$ entails the following result:

Theorem 2.3 Under the assumption of a power Pen’s parade, i.e., $y = \sum_{i=0}^{k} b_i r_i^{\alpha_i}$, with $b_k \neq 0$, when $n \to \infty$, the Gini index $G_k$ can be approximately computed as follows:

$$G_{\alpha_k} \simeq \frac{1}{\sqrt{3}} \frac{|b_k|}{b_k} \frac{\alpha_k}{\sqrt{2\alpha_k + 1}} \rho(y, r_y), \quad (23)$$

where $\rho(y, r_y)$ is correlation coefficient between the incomes $y$ and their ranks $r_y$.

Corollary 2.1 For $b_k > 0$, the Gini index is approximately equal to

$$G_{\alpha_k} \simeq \frac{1}{\sqrt{3}} \frac{|b_k|}{b_k} \frac{\alpha_k}{\sqrt{2\alpha_k + 1}} \rho(y, r_y), \quad (24)$$
where $\alpha_k$ is the solution of the following equation:

$$\frac{s}{\sqrt{2s + 1}} = CV_{emp}. \quad (25)$$

where $CV_{emp}$ is the coefficient of variation obtained by using incomes data $y$.

**Remark 2.2** Note that for $\alpha_k = 1$, we have the result of Milanovic (1997), i.e. $G_1 \simeq \frac{\rho(y, r_y)}{3}$ and for $\alpha_k = 2$, we have the result of Mussard et al. (2010), i.e. $G_2 \simeq \frac{2\rho(y, r_y)}{\sqrt{15}}$.

**Remark 2.3** Remarks that the gini index $G_k$ approximation depends on $k$, the sign of $b_k$ and $\rho(y, r_y)$. Since the Gini computation do not depends on $b_i$ for $i = 0, \ldots, k - 1$, we can then compute the Pen’s parade as follows: $y = b_0 + b_k r^{\alpha_k}$. Based on revenues data, it is convenient to use a simple regression to estimate the parameters $\hat{b}_0$ and $\hat{b}_k$. Also we can use the following simplify power Pen’s parade $y = b_0 + b_1 r + b_k r^{\alpha_k}$. If the revenues are ordered as follows: $0 \leq y_1 \leq y_2 \leq \ldots \leq y_n$, then the precede Pen’s parade pass through the origin $(1, y_1)$ implies that $y_1 = b_0 + b_1 + b_k$, $y_2 = b_0 + 2b_1 + 2^{\alpha_k}b_k$ and $y_3 = b_0 + 3b_1 + 3^{\alpha_k}b_k$ where $\alpha_k$ is given by 33. We can the find $b_0$, $b_1$ and $b_k$. 


2.3 Correlation between the incomes and ranks

The covariance between income $y$ and the rank $r_y$ is

$$\text{cov}(y, r_y) = \sum_{i=0}^{k} b_i \left[ \text{cov}(r_{\alpha_i}, r) \right]$$

$$= \sum_{i=1}^{k} b_i \left[ \frac{1}{n} \sum_{r=1}^{n} r_{\alpha_i+1} - \frac{1}{n^2} \sum_{r=1}^{n} r_{\alpha_i} \sum_{r=1}^{n} r \right]$$

$$= \sum_{i=1}^{k} b_i \left[ \frac{1}{n} H[n, -\alpha_i - 1] - \frac{n+1}{n} H[n, -\alpha_i] \right] \tag{26}$$

where

$$H[n, s] = \sum_{r=1}^{n} \frac{1}{r^s}$$

is an harmonic number function for $s > 1$ and $n \in \mathbb{N}^*$. In the simple case where, $y = b_0 + b_1 r_{\alpha_1}$, we have:

$$\text{cov}(y, r_y) = b_1 \left[ \frac{1}{n} H[n, -\alpha_1 - 1] - \frac{n+1}{n} H[n, -\alpha_1] \right] \tag{27}$$

The variance of the income $y$ is:

$$\text{cov}(y, y) = b_1^2 \left[ \frac{1}{n} H[n, -\alpha_1 - 1] - \left( \frac{1}{n} \sum_{r=1}^{n} H[n, -\alpha_1] \right)^2 \right]. \tag{28}$$

The standard deviation of income is:

$$\sigma_y = \sqrt{\text{cov}(y, y)} = |b_1| \sqrt{\frac{1}{n} H[n, -\alpha_1 - 1] - (H[n, -\alpha_1])^2}. \tag{29}$$
The standard deviation of individuals ranks is given by:

$$\sigma_{y} = \sqrt{\frac{1}{n} \sum_{r=1}^{n} r^2 - \left(\frac{1}{n} \sum_{r=1}^{n} r\right)^2} = \frac{(2n+1)(n+1)}{6} - \frac{(n+1)^2}{4}$$ \hspace{1cm} (30)

The correlation coefficient between $y$ and $r$ is:

$$\rho(y, r) = \frac{b_1 \left[ \frac{1}{n} \sum_{r=1}^{n} r^{\alpha_1+1} - \frac{n+1}{n} \sum_{r=1}^{n} r^{\alpha_1} \right]}{|b_1| \sqrt{\frac{1}{n} \sum_{r=1}^{n} r^{\alpha_1+1} - \left(\frac{1}{n} \sum_{r=1}^{n} r^{\alpha_1}\right)^2 \sqrt{\frac{n^2-1}{12}}}} = \frac{b_1 \left[ \frac{1}{n} H[n, -\alpha_1 - 1] - \frac{n+1}{n} H[n, -\alpha_1] \right]}{|b_1| \sqrt{\frac{1}{n} H[n, -\alpha_1 - 1] - (H[n, -\alpha_1])^2 \sqrt{\frac{n^2-1}{12}}}}$$ \hspace{1cm} (31)

**Theorem 2.4** If the income $y = b_0 + b_k r^{\alpha_k}$, with $\alpha_1 > 0$ and $b_k \neq 0$ (i.e. $b_k > 0$), then the Gini index is approximately equal to

$$G_{\alpha_k} \simeq \frac{1}{\sqrt{3}} \frac{\alpha_k}{\sqrt{2\alpha_k + 1}} \left(\frac{1}{n} H[n, -\alpha_k - 1] - \frac{n+1}{n} H[n, -\alpha_k] \right) \sqrt{\frac{1}{n} H[n, -\alpha_k - 1] - (H[n, -\alpha_k])^2 \sqrt{\frac{n^2-1}{12}}}$$ \hspace{1cm} (32)

where

$$\alpha_k = CV_{emp} \left(CV_{emp} + \sqrt{1 + CV_{emp}}\right)$$ \hspace{1cm} (33)

with $CV_{emp}$ being the coefficient of variation obtained by using income data $y$.  

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3 Pen’s parade of different countries data and their Gini indexes

In this section, we use the Milanovic (1997) countries incomes data for the computation of the Pen’s parade and gini index of each country income data. In relation with the parameter $k$ and $b_k > 0$, we have the following table:

Table 1: The Positive solution $s = \alpha_k$ (see (22)) of equation (25) in relation with the empirical coefficient of variation $CV_{emp}$ of each country.

<table>
<thead>
<tr>
<th>$CV_{emp}$</th>
<th>0.43</th>
<th>0.56</th>
<th>0.57</th>
<th>0.60</th>
<th>0.68</th>
<th>0.68</th>
<th>0.71</th>
<th>0.76</th>
<th>1.07</th>
<th>1.63</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_k$</td>
<td>0.653</td>
<td>0.955</td>
<td>0.981</td>
<td>1.060</td>
<td>1.285</td>
<td>1.285</td>
<td>1.375</td>
<td>1.532</td>
<td>2.712</td>
<td>5.774</td>
</tr>
</tbody>
</table>
Table 2: Coefficient of variation and Gini index $G_{\alpha_k}$ for different degree $\alpha_k$ for each country Pen’s parade.

<table>
<thead>
<tr>
<th>Country (year)</th>
<th>n</th>
<th>$\rho(y, r_y)$</th>
<th>$CV_{emp}$</th>
<th>$\alpha_k$</th>
<th>$G_{\alpha_k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hungary (1993; annual)</td>
<td>22062</td>
<td>0.889</td>
<td>0.43</td>
<td>0.653</td>
<td>0.221</td>
</tr>
<tr>
<td>Poland (1993; annual)</td>
<td>52190</td>
<td>0.892</td>
<td>0.56</td>
<td>0.955</td>
<td>0.288</td>
</tr>
<tr>
<td>Romania (1993; annual)</td>
<td>52190</td>
<td>0.892</td>
<td>0.57</td>
<td>0.981</td>
<td>0.288</td>
</tr>
<tr>
<td>Bulgaria (1994; monthly)</td>
<td>8999</td>
<td>0.863</td>
<td>0.60</td>
<td>1.060</td>
<td>0.284</td>
</tr>
<tr>
<td>Estonia (1995; quarterly)</td>
<td>7195</td>
<td>0.889</td>
<td>0.68</td>
<td>1.285</td>
<td>0.308</td>
</tr>
<tr>
<td>UK (1986; annual)</td>
<td>8759</td>
<td>0.871</td>
<td>0.68</td>
<td>1.285</td>
<td>0.342</td>
</tr>
<tr>
<td>Germany (1889; annual)</td>
<td>7178</td>
<td>0.815</td>
<td>0.71</td>
<td>1.375</td>
<td>0.320</td>
</tr>
<tr>
<td>US (1991; annual)</td>
<td>3940</td>
<td>0.744</td>
<td>0.76</td>
<td>1.532</td>
<td>0.305</td>
</tr>
<tr>
<td>Russia (1993-4; quarterly)</td>
<td>16052</td>
<td>0.892</td>
<td>1.07</td>
<td>2.712</td>
<td>0.391</td>
</tr>
<tr>
<td>Kyrgyzstan (1993; quarterly)</td>
<td>16356</td>
<td>0.812</td>
<td>1.63</td>
<td>5.774</td>
<td>0.502</td>
</tr>
</tbody>
</table>

In the precede table, $G_{\alpha_k}$ denotes the estimation of the Gini index by using Milanovic data (1997).

**Remark 3.1** Based on the analysis of the previous table, we can propose to consider power Pen’s parade ($\alpha_k = 0.653$) to compute the Gini index for Hungary (1993, annual), $\alpha_k = 0.955$ for Poland(1993; annual), $\alpha_k = 0.981$ for Romania (1994, monthly), $\alpha_k = 1.060$ for Bulgaria(1994; monthly), $\alpha_k = 1.285$ for Estonia (1995; quarterly), $\alpha_k = 1.285$ for UK (1986; annual), $\alpha_k = 1.375$ for Germany (1889; annual), $\alpha_k = 1.532$ for US (1991; annual),
\( \alpha_k = 2.712 \) for Russia (1993-4; quarterly)) and \( \alpha_k = 5.774 \) for Kyrgyzstan (1993; quarterly).

**Remark 3.2** In our power Pen’s parade, if \( r_y = 1 \), then \( y = y_1 = \sum_{i=0}^{k} b_k \).

In practical applications, the parameters \( \hat{b}_k \) which are estimated from the observed data using multiple regression (OLS) are such that the revenue \( y > 0 \) (i.e. \( b_k > 0 \)). It very important to note that, the correlation coefficient between \( y \) and its rank \( r_y \) depend on \( b_k \). Ignoring this fact will equivalent to arbitrarily restricting the parade to pass through the origin and may result in less accurate estimates of the Gini index.

### 4 Concluding Remarks

In this paper, we proposed the power pen’s parade as an alternative to the linear and quadratic pen’s parade. Following Milanovic (1997), we have proposed another simple way to calculate the Gini coefficient under the assumption of a general power Pen’s parade of order \( k \). By using the data of Table 2, we concluded that the computation of the Gini index of each income data country need to find a specific \( \alpha_k \in \mathbb{R}_+^* \) which is the order of a specific power Pen’s parade. We have therefore compute the Pen’s parade and the gini index for each of the six countries.

It appeared that for a given coefficient of variation of a country incomes data, we found the associated degree of the power Pen’s parade, hence enabling compute the pen’s parade using multiple regression. It also provides
a convenient and straightforward way to compute the Gini index of each
country by using the coefficient of correlation between incomes and ranks.

One immediate and practical implications result from this new Gini ex-
pression. Estimating the coefficients $b_i$ for $i = 1, \ldots, k$, e.g. with Yitzhaki’s
Gini regression analysis or a multiple regression, enables a parametric Gini
index to be obtained that depends on parameters reflecting the curvature of
Pen’s parade, which may be of interest when one compares the shape of two
income distributions.
References


