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Integration of a new hysteresis model in the Finite Elements method

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Abstract—In this work we present an original magnetic hysteresis model with parameters fitted from measured reversal curves. This model is used together with the Finite Element method for studying a structure composed of a magnetic core with air-gap excited by a coil.

I. INTRODUCTION

The modeling of hysteresis phenomena is a complicated subject: indeed many models exist, but nowadays none is universally adopted as the “best” model. One of the main problems with all these models is the difficulty to determine the required parameters from experimental measurements [1], [2]. Most approaches are scalar neglecting the vectorial nature of electromagnetic fields. Thus they are not adapted to represent structures where the magnetic field has not a unique direction, like in proximity of air-gaps in rotating machines, or in the T-joints of transformers magnetic circuits. Furthermore, the few more sophisticated vectorial models like [3] fail also to correctly represent phenomena like DC demagnetization. Last but not least, some of them are so complex that their integration into a Finite Elements (FE) code is far from being evident.

In this preliminary work we present a new scalar hysteresis model, the parameters of which can be easily determined from the slopes of First Order Reversal Curves (FORC). The main advantage of the proposed model is that no optimization is required, and we avoid thus solving ill-posed problems. This model is used for simulating a 2D structure by using the FE method.

II. THE MODEL OF HYSTERESIS

The main assumption of this model is based on experimental considerations. For many soft magnetic materials (FeSi NO, FeSi GO, NiFe alloys, . . .), we notice that the slope $\frac{\partial h}{\partial b}$ (or $\frac{\partial b}{\partial h}$) at a given point inside the major hysteresis loop depends mainly on the coordinates of the point $(h, b)$ and on the sign of $\frac{dh}{dt}$ due to symmetry properties. Indeed, the slope for an increasing excitation field at a given point $(h_i, b_i)$ is exactly the opposite of the slope of the point $(-h_i, -b_i)$ for a decreasing excitation field.

Taking into account this fact, we measure a number of FORC (see Fig. 1), and evaluate their slope $\frac{dh}{db}$ in a discrete set of points. This experimental procedure allows us to “explore” the major loop, and to obtain enough information for characterizing the magnetic material as long as $(h, b)$ lays inside the major loop loop. For any point outside the major loop (where no data is available), it is assumed that the slope is equal to that of the nearest point in the major loop.

In our model the differential reluctivity $\frac{\partial b}{\partial h}$ (or, equivalently, the differential permeability $\frac{\partial h}{\partial b}$) is computed in any point $(h_0, b_0)$ from the collected data by using bilinear interpolation:

$$\left(\frac{\partial h}{\partial b}\right) = f \left(h_0, b_0, \text{sign} \left(\frac{dh}{dt}\right)\right)$$

The magnetic field $h$ is computed by (numerically) integrating (1):

$$h = \int_{b_0}^{b} \left(\frac{\partial h}{\partial b}\right) db$$

Note that $\text{sign} \left(\frac{dh}{dt}\right) = \text{sign} (b - b_0)$.

III. APPLICATION TO THE FINITE ELEMENT METHOD

The model is integrated in the Finite Element method. The non linear system is solved by means of the Newton-Raphson method [4]. In this work we assume that each of the component of the magnetic field is independent from the others, and it is governed by the same scalar model. We use the classical magnetostatic a formulation, weak form of the Ampère law (curl $\mathbf{h} = \mathbf{j}$):

$$(\mathbf{h}, \text{curl} \mathbf{a'})_{\Omega} - (\mathbf{j}_s, \mathbf{a'})_{\Omega_s} = 0 \ \forall \mathbf{a'} \in F^1$$

where $\mathbf{j}_s$ is the imposed current density in domain $\Omega_s \subset \Omega$, $\mathbf{a}$ is the unknown magnetic vector potential, $\mathbf{b} = \text{curl} \mathbf{a}$, and the space $F^1$ is a discrete approximation of $H(\text{curl}, \Omega)$.

Fig. 1. Major loop and First Order Reversal Curves
\( \Omega \) denotes a volume integral in \( \Omega \) of the product of scalar or vector fields. In practice \( a \) is discretized with edge basis functions \( s_i(x) \):

\[
a = \sum_i a_i s_i(x)
\]

where \( a_i \) are the coefficients to be determined. In linear materials (3) is simply written by replacing: \( h = \frac{1}{\mu} \text{curl} \ a \).

In the hysteretic (or more general non linear) materials, we apply the Newton-Raphson method where the magnetic field is approximated by a first-order Taylor development:

\[
h \simeq h_0 + \sum_i \frac{\partial h}{\partial a_i} \Delta a_i
\]

with \( \Delta a_i = a_i - a_{0,i} \) the increment of \( a_i \). This equation can be further developed by explicitly introducing the differential reluctivity, which is obtained through (1):

\[
\frac{\partial h}{\partial a_i} = \left( \frac{\partial h}{\partial b} \right) \frac{\partial b}{\partial a_i} = \left( \frac{\partial h}{\partial b} \right) \text{curl} \ s_i(x)
\]

Finally the equation to be assembled in the stiffness matrix is:

\[
(h_0, \text{curl} \ a')_\Omega + \sum_i \left( \frac{\partial h}{\partial b} \text{curl} \ s_i(x), \text{curl} \ a' \right)_\Omega \Delta a_i
\]

\[
- (j_s, a')_\Omega = 0 \quad \forall a' \in \mathbf{F}^1
\]

At each non linear iteration, this equation is solved with respect of the increment \( \Delta a_i \).

**Fig. 2.** Flux lines of the magnetic flux density in the computational domain (1/2 of the geometry is represented).

**Fig. 3.** Hysteresis cycle computed in a point of the “U”-core.

### IV. Application

We test our method with a simple 2D simulation of a magnetic circuit composed of a “I” core, a “U” core separated by an air gap. A sinusoidal excitation is provided by a coil wound around the “I” core (Fig. 2). The hysteresis model has been implemented in the FE software GetDP [5]. The flux lines and the magnetic flux density in the computational domain at a given time step are depicted in Fig. 2 up. The hysteretic cycle computed along one period at one point in the middle of the “U”-core is shown in 2 down as well. This numerical result looks “reasonable”, but unfortunately it has not yet been validated, due to the lack of experimental measurements – they will be included in the full paper.

### V. Conclusion

An original and simple static hysteresis model has been successfully implemented in a 2D FEM based on the slopes of FORC. The on-going work aims at developing a “true” vectorial extension of the present model. This is a challenge from the theoretical point of view. Moreover, special attention is devoted to the building of a suitable experimental system for obtaining the indispensable measurements.

### References


