Option Pricing under GARCH models with Generalized Hyperbolic distribution (II) : Data and Results

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Abstract

In this paper, we provide a new dynamic asset pricing model for plain vanilla options and we discuss its ability to produce minimum mispricing errors on equity option books. The data set is the daily log returns of the French CAC 40 index, on the period January 2, 1988, October 26, 2007. Under the historical measure, we estimate an EGARCH model with Generalized Hyperbolic innovations, using this dataset. We showed in Chorro, Guégan and Ielpo (2008) that when the pricing kernel is an exponential affine function of the state variables, the risk neutral distribution is unique and implies again a Generalized Hyperbolic dynamic, with changed parameters. Thus, using this theoretical result associated to Monte Carlo simulations, we compare our approach to natural competitors in order to test its efficiency. More generally, our empirical investigations analyze the ability of specific parametric innovations to reproduce market prices in the context of the exponential affine specification of the stochastic discount factor.

Keywords: Generalized Hyperbolic Distribution - Option pricing - Incomplete market - CAC 40 - GARCH models

JEL classification: G 13 - C 22 - G 22.

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1 Introduction

In this paper, we provide a new dynamic asset pricing model for plain vanilla options and we discuss its ability to produce minimum mispricing errors on equity option books. The data set is the daily log returns of the French CAC 40 index, on the period January 2, 1988, October 26, 2007. Under the historical measure, we estimate an EGARCH model with Generalized Hyperbolic innovations, using this dataset. We showed in Chorro, Guégan and Ielpo (2008) that when the pricing kernel is an exponential affine function of the state variables, the risk neutral distribution is unique and implies again a Generalized Hyperbolic dynamic, with changed parameters. Thus, using this theoretical result associated to Monte Carlo simulations, we compare our approach to natural competitors in order to test its efficiency. More generally, our empirical investigations analyze the ability of specific parametric innovations to reproduce market prices in the context of the exponential affine specification of the stochastic discount factor.

As soon as the assumption of market completeness is questionable in real-world equity markets, we need to work in an incomplete setting which is more problematic. Therefore, in an incomplete market with no arbitrage opportunities, there is more than one equivalent martingale measure and hence a range of no-arbitrage prices for a contingent claim. One crucial issue is to identify an equivalent martingale measure which gives an economically consistent and justifiable price for contingent claims compatible with market data.

To settle the latter problem, we proceed as follows:

- We use an EGARCH (Nelson (1991)) discrete time modeling for the underlying asset (under the historical distribution $\mathbb{P}$) with an appropriated distribution for the innovations to take into account the most important features which characterize financial time series (skewness, kurtosis).
- In order to price options we need to move from the historical to the risk neutral world. Under the assumption of an exponential affine Stochastic Discount Factor, we exhibit the exact representation of this model under the risk neutral probability measure $\mathbb{Q}$, proving the stability of the innovation distribution, with changed parameters.
- We show, throughout a deep empirical study based on Monte Carlo simulations on the CAC 40 French index that our model behaves fairly well to price options for a wide range of moneyness and maturities comparing to its natural competitors.

The first point is the choice of the model under the historical measure $\mathbb{P}$. Until recently, classical well known methods have been considered like the Black and Scholes (1973) model in continuous time, or the Duan (1995) and Heston and Nandi (2000) models in discrete time. These latter models are known to fail to capture the short term behavior of equity options smiles. In this paper we propose an EGARCH model with Generalized Hyperbolic innovations. This distribution introduced by Barndorff-Nielsen (1977) is known to fit financial datasets remarkably especially to handle the particular tail behavior found in equity indexes returns (see section 2). It has already been used with empirical successes to model the dynamic of several stock markets in discrete or continuous time (see e.g Eberlein and Prause (2002) or Guégan and Zhang (2007)).

For the second point we have to select a particular risk neutral probability with an interesting economic interpretation. One of the major attempt in this topic was proposed by Gerber and Siu (1994) that provide an elegant way to choose an equivalent martingale measure using the Esscher transformation. This tool has been introduced in actuarial science by Esscher (1932), in an incomplete market setting. This latter approach permits to choose a wide variety of parametric models within the class of infinite divisible distributions (see Chorro, Guégan and Ielpo, 2008 proposition 4 for a general result in the GARCH setting). This method has been used to price options in discrete financial models (Bühlmann, Delbaen, Embretchs and Shiryaev (1996), Siu, Tong and Yang (2004) or Christoffersen, Heston and Jacobs (2006)) and even in continuous time Lévy type ones (Eberlein and Prause (2002)). An equivalent formulation of the work of Gerber and Siu (1994) consists in using for the classical Stochastic Discount Factor an exponential affine parameterization. We have shown in Chorro, Guégan and Ielpo (2008) that in the framework of an EGARCH model with Generalized Hyperbolic innovations, this particular change of probability implies again a Generalized Hyperbolic dynamic with explicit parameters allowing for Monte Carlo simulations.

For the last point, we compare the pricing performances of our EGARCH model Generalized Hyperbolic innovations to the Black and Scholes (1973), Heston and Nandi (2000) and Bertolon, Monfort and Pegoraro (2003) ones. The choice of this competitors is natural because, for each of them, the exponential affine form of the pricing kernel is assumed (implicitly or explicitly) to move from the historical distribution to the risk neutral one. Especially, for the Bertolon, Monfort and Pegoraro (2003) model, the use of a mixture of two normal distributions to model the innovations may be interestingly compared to our choice of the Generalized Hyperbolic distribution because these two families have exactly the same number of parameters. With this study we want to highlight the importance of the choice of the Generalized Hyperbolic distribution to model stock price processes.

The paper is organized as follows. In section two, we describe the dataset on which we work. Section three is a summary of the methodology used in this paper. Mathematical details being provided in a previous paper (Chorro, Guégan and Ielpo, 2008), we especially focus on the explicit description of the carrying out of the theoretical method. The section four is devoted to the analysis of empirical results. Section five concludes.

2 The dataset

The dataset that we use contains the following time series. We consider daily log returns of the French CAC 40 whose value at time t is denoted S_t. The sample starts on January 2, 1988 and ends on October 26, 2007.

Number of observations	**Mean**	**Minimum**	**Maximum**
4159	0.0003	-0.0768	0.07
Median	**Annualized Volatility**	**Skewness**	**Kurtosis**
0.0005	0.0021	-0.15	2.87

Table 1: Descriptive statistics for the CAC 40 dataset.

Following this preliminary study we test various probability distributions in order to find the best one. The results are provided in Table 2. In this paper we are particularly interested in the Generalized Hyperbolic (GH) distributions introduced by Barndorff-Nielsen (1977). Let us remind briefly its definition.

For $(\lambda, \alpha, \beta, \delta, \mu) \in \mathbb{R}^5$ with $\delta > 0$ and $\alpha >\mid \beta \mid> 0$, the one dimensional $GH(\lambda, \alpha, \beta, \delta, \mu)$ distribution is defined by the following density function

$$d_{GH}(x, \lambda, \alpha, \beta, \delta, \mu) = \frac{(\sqrt{\alpha^2 - \beta^2}/\delta)^\lambda}{\sqrt{2\pi} K_\lambda(\delta\sqrt{\alpha^2 - \beta^2})} e^{\beta(x-\mu)} \frac{K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right)}{\left(\sqrt{\delta^2 + (x-\mu)^2}/\alpha\right)^{1/2-\lambda}}, \quad (1)$$

where K_λ is the modified Bessel function of the third kind. For $\lambda \in \frac{1}{2}\mathbb{Z}$, the basic properties of the Bessel function allow to find simpler forms for the density. In particular, for $\lambda = 1$, we get the Hyperbolic distributions (HYP) whose log-density is a hyperbola. For $\lambda = -\frac{1}{2}$, we obtain the Normal Inverse Gaussian distribution (NIG) which is closed under convolution. This family has already been suggested as a model for financial price processes because its exponentially decreasing tails seem to fit the statistical behavior of asset returns remarkably (Barndorff-Nielsen (1995), Eberlein and Prause (2002)).

From the results provided in Table 2 we observe that the Gaussian distribution is strongly rejected as a model for the conditional distribution of the CAC 40 log returns. On the contrary, the distributions belonging to the Generalized Hyperbolic family give

	CAC
KS p-value for NIG	0.3
KS p-value for HYP	0.73
KS p-value for GH	0.31
KS p-value for Gaussian	0
AD p-value for NIG	0.45
AD p-value for HYP	0.5
AD p-value for GH	0.23
AD p-value for Gaussian	0

Table 2: Kolmogorov-Smirnov (KS) and Andersen-Darling (AD) adequation tests

This table presents the Kolmogorov-Smirnov and Andersen-Darling adequation tests, testing the adequation of the NIG, Hyperbolic, Generalized Hyperbolic and Gaussian distributions to a dataset of the daily log returns of the French CAC 40. Time varying variance has been filtered out using an EGARCH process. The sample starts on January 2, 1988 and ends on October 26, 2007.

rise to satisfactory results. We provide several figures that confirm these results. On Figure 1, we present the empirical log-density (plain black line) vs. the estimated log density obtained with the NIG (red), Hyperbolic (green), Generalized Hyperbolic (dark blue) and Gaussian (light blue) distributions for the daily log returns of the French CAC 40. In Figure 2, we present the qq-plots comparing the empirical quantiles of the CAC 40 returns vs. the estimated quantiles obtained with the NIG (red), Hyperbolic (green), Generalized Hyperbolic (dark blue) and Gaussian (light blue) distributions. Based on the graphics and the tables previously mentioned, it is natural that a dynamic asset pricing model is based on a distribution that belongs to the Generalized Hyperbolic family.

The financial theory states that the price of any asset is equal to the present value of the expected pay-off, under a well chosen distribution. Thus, to compute option prices, we need to know the risk-free rate that can be used to compute the necessary discount factors. The risk free rate used in this paper is the zero rate obtained from the European interest rates swap (IRS). We use closing swap rates whose maturities range from 1 month to 3 years. Intermediate maturities required for option pricing are computed using the Svensson (1994) model[1].

Beyond this initial dataset, we use the available option contracts written on the French CAC 40. The most liquid contracts available are the quarterly ones, that is the con-

[1]We compared the results obtained with the Nelson and Siegel (1987) approach and the performances of the models presented here are globally unchanged.

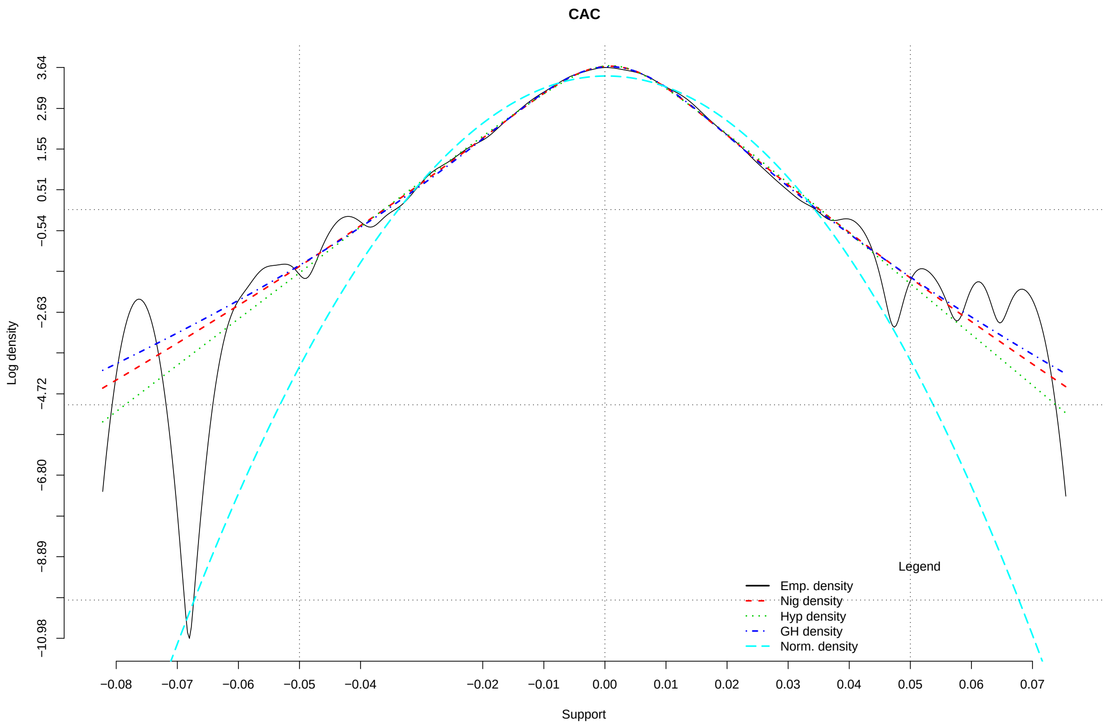

Figure 1: Log density

This figure presents the empirical log-density (plain black line) vs. the estimated log density obtained with the NIG (red), Hyperbolic (green), Generalized Hyperbolic (dark blue) and Gaussian (light blue) distributions using the CAC 40 returns dataset. Time varying variance has been filtered out using an EGARCH process. The sample starts on January 2, 1988 and ends on October 26, 2007.

tracts maturing on March, June, September and December for every available years. We focused on these maturities, neglecting the intermediate monthly maturities that are far less liquid on average. The option dataset starts on January 2, 2006 and ends on October 26, 2007. The strikes are chosen so that the moneynesses used here range from .8 to 1.2, which is the standard moneyness window used in the literature. The table 3 presents key statistics regarding the option dataset across moneynesses and maturities.

3 Methodology

3.1 Theoretical approach

In this section, we recall briefly the methodology we are going to use. Since we developed it in Chorro, Guégan and Ielpo (2008), we only focus on the main points here :

- Under the historical measure $\mathbb{P}$, the discrete time economy we consider is character-

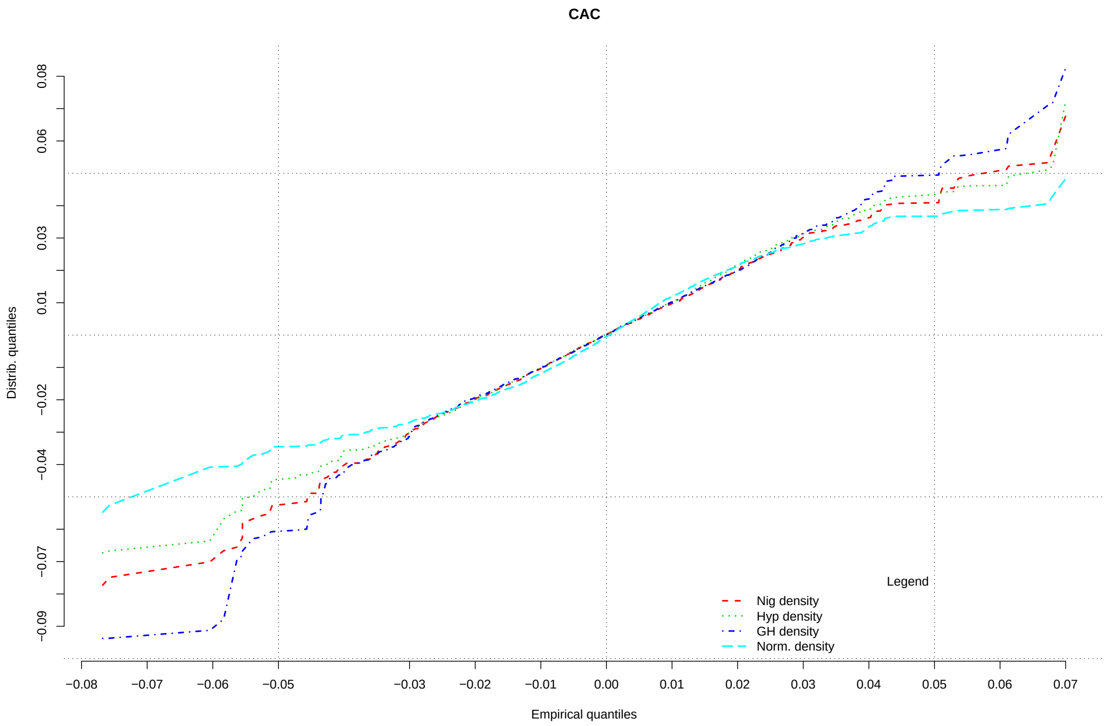

Figure 2: QQ-Plot

This figure presents the qq-plots comparing the empirical quantiles of the CAC 40 returns vs. the estimated quantiles obtained with the NIG (red), Hyperbolic (green), Generalized Hyperbolic (dark blue) and Gaussian (light blue) distributions. Time varying variance has been filtered out using an EGARCH process. The sample starts on January 2, 1988 and ends on October 26, 2007.

ized by the time series dynamic of the stock price process $(S_t)_{t\in\{0,1,\ldots,T\}}$ given by

$$Y_t = log\left(\frac{S_t}{S_{t-1}}\right) = r + m_t + \sqrt{h_t}z_t, \quad S_0 = s \in \mathbb{R}_+, \tag{2}$$

where r is the risk free rate expressed on a daily basis and supposed to be constant and where z_t is a sequence of independent and identically distributed centered random variables with variance 1. In this model we allow for non Gaussian innovations in order to model extreme returns behavior. Following our previous study, we assume that the random variables z_t follow a centered and reduced Generalized Hyperbolic (GH) distribution $GH(\lambda, \alpha, \beta, \delta, \mu)$ where $(\lambda, \alpha, \beta, \delta, \mu) \in \mathbb{R}^5$ with $\delta > 0$ and $\alpha >\mid \beta \mid> 0$. This distribution may be characterized by its moment generating function

$$\mathbb{G}_{GH}(u) = e^{\mu u}\left(\frac{\alpha^2-\beta^2}{\alpha^2-(\beta+u)^2}\right)^{\frac{\lambda}{2}} \frac{K_\lambda(\delta\sqrt{\alpha^2-(\beta+u)^2})}{K_\lambda(\delta\sqrt{\alpha^2-\beta^2})}, \quad \mid \beta + u \mid< \alpha, \tag{3}$$

where K_λ is the modified Bessel function of the third kind.

Number of available option contracts						
	<.8	**[.8-.9]**	**[.9-1]**	**[1-1.1]**	**[1.1-1.2]**	**>1.2**
.25<Maturity<.5	903	2579	2879	2835	1519	100
.5<Maturity<1	1294	4577	5530	5511	4065	1859
Maturity>1	1026	7262	15067	16010	14823	14688
Average option price						
Maturity/Moneyness	**<.8**	**[.8-.9]**	**[.9-1]**	**[1-1.1]**	**[1.1-1.2]**	**>1.2**
.25<Maturity<.5	1435.2837	949.245273	471.477398	139.330296	20.6086241	2.4202
.5<Maturity<1	1547.91826	1051.23124	621.277436	287.889459	94.3190627	19.5322431
Maturity>1	1651.09504	1158.02038	780.151686	483.018575	261.69954	124.69807

Table 3: Descriptive statistics for the CAC 40 option dataset used in the paper.

In equation (2), we consider a general time varying excess of return m_t that depends on the constant risk premium λ_0. In practice, it will be fixed for the empirical study and we retain the following classical form:

$$m_t = \lambda_0 \sqrt{h_t} - \frac{1}{2} h_t. \tag{4}$$

In order to capture asymmetry phenomena (as the leverage effect) observed on our dataset, we choose for the conditional variance of the returns h_t an EGARCH model (Nelson (1991)) that ensures positivity without restrictions on the coefficients

$$log(h_t) = a_0 + a_1 (\mid z_{t-1} \mid + \gamma \; z_{t-1}) + b_1 \; log(h_{t-1}). \tag{5}$$

According to Blaesild (1981), the conditional distribution of Y_t may be now deduced from the z_t one. In fact, given $\mathcal{F}_{t-1} = \sigma(z_u; 0 \leq u \leq t-1)$,

$$Y_t \hookrightarrow GH \left(\lambda, \frac{\alpha}{\sqrt{h_t}}, \frac{\beta}{\sqrt{h_t}}, \delta \sqrt{h_t}, M_t + \mu \sqrt{h_t} \right), \tag{6}$$

where $M_t = r + m_t$.

• Now, our model is entirely specified under $\mathbb{P}$. Since we want to use it to price contingent claims, we need to postulate an explicit risk premium to perform the change in distribution. Classically, in a discrete time dynamic equilibrium model (or in an arbitrage free continuous one), the price of any asset equals the expected present value of its future payoffs under an equivalent martingale measure $\mathbb{Q}$. For example, the price P_t at time t of an European asset paying Φ_T at T (Φ_T being $\mathcal{F}_T$ measurable) is given by

$$P_t = E_{\mathbb{Q}}[\Phi_T e^{-r(T-t)} \mid \mathcal{F}_t] \tag{7}$$

or equivalently

$$P_t = E_{\mathbb{P}}[\Phi_T M_{t,T} \mid \mathcal{F}_t]. \tag{8}$$

The $\mathcal{F}_{t+1}$ measurable random variable $M_{t,t+1}$ is the so called stochastic discount factor (SDF) (the quantity $M_{t,t+1} e^r$ is also known as the pricing kernel).

An important point of our approach concerns the assumption on the stochastic discount factor $M_{t,t+1}$ that is constrainted to be an exponential affine function of the log returns: $\forall t \in \{0, ..., T-1\}$

$$M_{t,t+1} = e^{\theta_{t+1} Y_{t+1} + \xi_{t+1}}, \tag{9}$$

where θ_{t+1} and ξ_{t+1} are $\mathcal{F}_t$ measurable random variables. Thus, we need to compute explicitly $(\theta_{t+1}, \xi_{t+1})$.

• Under our previous assumptions, we may show that there exists a unique risk neutral probability $\mathbb{Q}$. This result comes from the following facts. For all $t \in \{0, ..., T-1\}$, we denote by $\mathbb{G}_t$ the conditional moment generating function of Y_{t+1} given $\mathcal{F}_t$ defined on a non empty convex set. Applying the pricing equation (8) to the risky and the non risky assets we just have to solve the following system, to get θ_{t+1} and ξ_{t+1}:

$$\begin{cases} \mathbb{G}_t(\theta_{t+1}) = e^{-(r+\xi_{t+1})} \\ \mathbb{G}_t(\theta_{t+1}+1) = e^{-\xi_{t+1}}. \end{cases} \tag{10}$$

In the case where the innovations of the GARCH process (2) follow a GH distribution, the following proposition ensures under mild conditions the existence of a solution for (10) (see Chorro, Guégan and Ielpo, 2008 proposition 5 for the proof).

Proposition 1. *For a $GH(\lambda, \alpha, \beta, \delta, \mu)$ distribution with $\alpha > \frac{1}{2}$, then,*

1. *If $\lambda \geq 0$, the equation $log\left(\frac{\mathbb{G}_{GH}(1+\theta)}{\mathbb{G}_{GH}(\theta)}\right) = r$ has a unique solution,*
2. *If $\lambda < 0$, the equation $log\left(\frac{\mathbb{G}_{GH}(1+\theta)}{\mathbb{G}_{GH}(\theta)}\right) = r$ has a unique solution if and only if $\mu - C < r < \mu + C$ where*

$$C = log\left(\frac{\Gamma[-\lambda]}{2^{\lambda+1}}\right) - log\left(\frac{K_\lambda(\delta\sqrt{\alpha^2 - (\alpha-1)^2})}{\delta\sqrt{\alpha^2 - (\alpha-1)^2}}\right). \tag{11}$$

The constant C is strictly positive because $\frac{d}{dx}\frac{K_\lambda(x)}{x^\lambda} = -\frac{K_{\lambda+1}(x)}{x^\lambda} < 0$.

For the unicity we just have to apply the general result of Gerber and Siu (1994) and we denote by $(\theta^q_{t+1}, \xi^q_{t+1})$ the unique solution of (10). Then, the Stochastic discount factor

$$M_{t,t+1} = e^{\theta^q_{t+1} Y_{t+1} + \xi^q_{t+1}} \tag{12}$$

being explicitly known, we may deduce easily the form of the associated equivalent martingale measure $\mathbb{Q}$ and the dynamic of the log returns under $\mathbb{Q}$ (see Chorro, Guégan and Ielpo, 2008 propositions 3 and 4).

Proposition 2. *Under $\mathbb{Q}$, the distribution of Y_t given $\mathcal{F}_{t-1}$ is a*

$$GH\left(\lambda, \frac{\alpha}{\sqrt{h_t}}, \frac{\beta}{\sqrt{h_t}} + \theta^q_t, \delta\sqrt{h_t}, M_t + \mu\sqrt{h_t}\right) \tag{13}$$

where $M_t = r + m_t$.

We deduce from the preceding result that, under $\mathbb{Q}$,

$$Y_t = r + m_t + \underbrace{\sqrt{h_t} z_t}_{\varepsilon_t}, \quad S_0 = s, \tag{14}$$

where the z_t are $\mathcal{F}_t$ measurable random variables such that, conditionally to $\mathcal{F}_{t-1}$,

$$z_t \hookrightarrow GH(\lambda, \alpha, \beta + \sqrt{h_t}\theta_t^q, \delta, \mu). \tag{15}$$

In particular, we can observe that the GH distribution is stable under the change of measure, allowing us to simulate easily the sample paths of the risky asset and price contingent claims by Monte Carlo simulations.

We can also remark that under $\mathbb{Q}$, conditionally to $\mathcal{F}_{t-1}$, the random variable ε_t is no more centered and its variance is not h_t but

$$var(\varepsilon_t) = h_t \left(\frac{\delta K_{\lambda+1}(\delta\gamma_t)}{\gamma_t K_\lambda(\delta\gamma_t)} + \frac{(\beta + \sqrt{h_t}\theta_t^q)^2 \delta^2}{\gamma_t^2} \left(\frac{K_{\lambda+2}(\delta\gamma_t)}{K_\lambda(\delta\gamma_t)} - \frac{K_{\lambda+1}^2(\delta\gamma_t)}{K_\lambda^2(\delta\gamma_t)} \right) \right)$$

where $\gamma_t = \sqrt{\alpha^2 - (\beta + \sqrt{h_t}\theta_t^q)^2}$. Thus the GARCH structure of the volatility is modified in a nonlinear way when we move from $\mathbb{P}$ to $\mathbb{Q}$.

3.2 Carrying out

Now, we apply the preceding methodology to price european vanilla options on the CAC 40 index.

The strategy to price all available call options at a given date t between January 2, 2006 and October 26, 2007 unfolds as follows.

1. We first select a subsample containing 4000 working days[2] and ending on the date t.
2. Using this subsample, we estimate the EGARCH model with GH innovations (2) favoring a two-stages estimation procedure explained in details below. This estimation step also yields the conditional variance of returns for the date $t + 1$, h_{t+1}.
3. Now, starting from the date $t + 1$, we then simulate sampled paths, under the risk neutral distribution:
   (a) Start from the estimated conditional variance h_{t+1}.

[2]We compared the empirical results for samples of size 4000 and 4500, and the results work broadly the same.

(b) Compute θ^q_{t+1} and ξ^q_{t+1} by solving equation (10).

(c) Sample z_{t+1} from a distribution $GH(\alpha, \beta + \sqrt{h_{t+1}}\theta^q_{t+1}, \delta, \mu)$ using the methodology of Bibby and Sorensen, (2003).

(d) Compute the log return Y_{t+1} and the conditional variance h_{t+2}.

(e) Then go back to step (a), replacing t by $t+1$, until $t = T-1$, where T is the maturity of the option we need to price.

This simulation scheme gives the sample returns under the properly chosen risk neutral distribution, from time t to time T. The final price for the underlying asset at time T is given by:

$$S_T = S_t \prod_{k=t+1}^{T} e^{Y_k}. \tag{16}$$

4. Finally, to price a vanilla call option with time to maturity $T-t$ and strike price K, we simulate N paths for the underlying future price S_T. The i^{th} sampled final price for the underlying is denoted by $S_{T,i}$. Then using the Monte Carlo option pricing standard approach, we get the approximated option price $\hat{C}(.)$ as the sample average of the simulated final prices:

$$\hat{C}(t,T,K) = e^{-r(T-t)} \frac{1}{N} \sum_{i=1}^{N} (S_{T,i} - K)^+ . \tag{17}$$

In practice, the number of sampled paths N is equal to 10 000[3].

The previous option pricing strategy requires a few remarks. At step (2), a Quasi Maximum Likelihood Estimation (QMLE) is first used to determine the parameters $(\lambda_0, a_0, a_1, b_1, \gamma)$ of the EGARCH model. At the second stage, since we exactly know the form of the density function of a GH distribution (1) we adopt a classical maximum likelihood approach to estimate the unknown remaining parameters $(\lambda, \alpha, \beta, \delta, \mu)$ using the residuals obtained at the previous stage. The maximization of the log-likelihood of the EGARCH process is initialized using the unconditional variance of the process.

The option pricing methodology used here focuses on out-of-sample option pricing errors: for a given current date, we estimate the time series parameters using a dataset of constant size ending on the current date. We used each time 4000 observations. The key point in our approach is to maintain as much outliers in the dataset as possible: these extremal events are essential to fit the GH parameters and to control the tail behaviors.

[3] We compared the convergence of the estimator of the option price using 10 000, 15 000 and 20 000 simulations, and the results are globally the same. For the sake of numerical feasibility, we favored 10 000 simulations.

	a_0	b_1	a_1	γ	λ_0
Average	-5.51	0.47	0.01	0.91	0.08
Std. Dev.	0.33	0.04	0.02	0.02	0.03

Table 4: Descriptive statistics on the average estimated parameters for the EGARCH model.

	α	β	δ	μ	λ
Average	308.42	-2.77	335.64	3.04	-78.46
Std. Dev.	139.90	5.08	152.02	5.51	76.68

Table 5: Descriptive statistics on the average estimated parameters for the GH distribution.

It is essential to remark that the Monte Carlo simulation we used is indeed path-dependent: for any date t, we need to solve equation (10) to obtain θ^q_{t+1} and ξ^q_{t+1}, given the simulated volatility h_{t+1} and the GH parameters.

Finally, so as to reduce the option prices errors linked to the use of Monte Carlo methods, we follow the Duan and Simonato (1998) method that imposes martingality within the sampled processes. This approach makes it possible to reduce significantly the variance of the estimator of the option price.

For any available date t between January 2, 2006 and October 26, 2007, we reproduce the previous option pricing methodology. Thus we use in fact a rolling window estimator for both the EGARCH and GH parameters. So, we are able to check for the stability of the estimations across our dataset. Table 4 and Table 5 provide respectively descriptive statistics for the estimated parameters of the EGARCH model and the GH distribution. The estimated parameters for the EGARCH model seem to be quite stable over the different rolling windows. This is not exactly the case for the GH parameters : the standard deviation associated to each estimated parameters can be large. This is not a real problem for our option pricing framework, given the good pricing performances of our approach, when compared to market quotes.

Finally, to compare various option pricing models, we use the criterion introduced in Heston and Nandi (2000): it corresponds to the average absolute relative pricing errors criterion for the working days t between January 2, 2006 (τ_1) and October 26, 2007 (τ_2). Let $\hat{C}(t, T_j, K_i)$ be the estimated call option price with a time to maturity equal to $T_j - t$ and a strike price worth K_i. Let $C(t, T_j, K_i)$ be the corresponding quoted market option price. Then the criterion we use here is

$$AARPE = \sum_{t=\tau_1}^{\tau_2} \sum_{j=1}^{J_t} \sum_{i=1}^{G_{t,j}} \left| \frac{\hat{C}(t,T_j,K_i) - C(t,T_j,K_i)}{C(t,T_j,K_i)} \right|, \tag{18}$$

where J_t is the number of call option maturities T_j available at time t and $G_{t,j}$ the number of strikes K_i available at time t for this particular maturity T_j.

We chose this criterion for two reasons. First, it is the usual criterion selected in the empirical literature previously mentioned. We will thus be able to compare the range of errors found in the dataset used with our model with the remaining major models of the literature. Second, this criterion is robust to the well known fact that option pricing errors are proportional to the moneyness: out of the money call option prices are very low, and so are the usual errors found. The converse is true for deep in the money option prices. This criterion rescales the errors using the level of the market option price: it is therefore robust to this effect and the analysis is largely eased.

4 Results

Here, we present the pricing errors results obtained with our methodology and the dataset presented previously. A deep empirical study have been done. We classically compare our EGARCH-GH model with natural competitors for which the stochastic discount factor is also constrainted to be an exponential affine function of the log returns. Through this study we want to highlight the importance of the choice of the GH distribution to model stock price processes.

More precisely, we compare the pricing performances of our EGARCH-GH model to the Black and Scholes (1973) (BS), Heston and Nandi (2000) (HN) and Bertolon, Monfort and Pegoraro (2003) (MN) ones. Each model is estimated on an equal basis to make the inter-model comparison easier.

First, the choice of these competitors is natural because, in each of them, the exponential affine form of the pricing kernel is assumed to move from the historical distribution to the risk neutral one. Moreover, it is well known that the historical Black and Scholes (1973) model always appears as a benchmark in the financial literature and that the Heston and Nandi (2000) model is the discrete time counterpart of the main model used in the banking industry for option pricing: the Heston (1993) stochastic volatility model. Finally, for the Bertolon, Monfort and Pegoraro (2003) model, the use of a mixture of two normal distributions (MN) to model the innovations may be interestingly compared to our choice of the GH distribution because these two families have

.25<Maturity<.5						
Moneyness	**<.8**	**[.8-.9]**	**[.9-1]**	**[1-1.1]**	**[1.1-1.2]**	**>1.2**
BS	0.049	0.073	0.234	1.694	20.504	56.189
HN	0.043	0.069	0.219	1.560	25.422	73.120
EGARCH-MN	0.055	0.077	0.174	0.656	6.604	55.893
EGARCH-GH	0.046	0.058	0.118	0.294	0.989	1.492
.5<Maturity<1						
Moneyness	**<.8**	**[.8-.9]**	**[.9-1]**	**[1-1.1]**	**[1.1-1.2]**	**>1.2**
BS	0.067	0.122	0.252	0.749	2.818	9.852
HN	0.057	0.120	0.248	0.766	3.143	9.615
EGARCH-MN	0.069	0.101	0.152	0.384	1.245	4.542
EGARCH-GH	0.055	0.071	0.080	0.181	0.343	0.699
Maturity>1						
Moneyness	**<.8**	**[.8-.9]**	**[.9-1]**	**[1-1.1]**	**[1.1-1.2]**	**>1.2**
BS	0.132	0.215	0.361	0.664	1.379	3.064
HN	0.139	0.220	0.387	0.727	1.525	3.549
EGARCH-MN	0.125	0.150	0.177	0.272	0.534	1.287
EGARCH-GH	0.106	0.111	0.088	0.104	0.169	0.307

Table 6: Absolute average pricing errors for CAC 40 option prices.

exactly the same number of parameters.

Table 6 presents the values of the AARPE criterion given in (18) for each model. These results are graphically presented in figure 3.

Looking at these results, wo observe that

- The worst competitors are the Black and Scholes and Heston and Nandi models, yielding errors beyond 100 percent in this out-of-sample testing strategy.
- The EGARCH-GH and EGARCH-MN models behaved fairly well.

Thus, it appears that

- The models based on a fine understanding of the tail behaviors (EGARCH-GH and EGARCH-MN models) yield the best results.
- Our model outperforms the EGARCH-MN one for a wide range of moneynesses and maturities, or is equivalent to it.

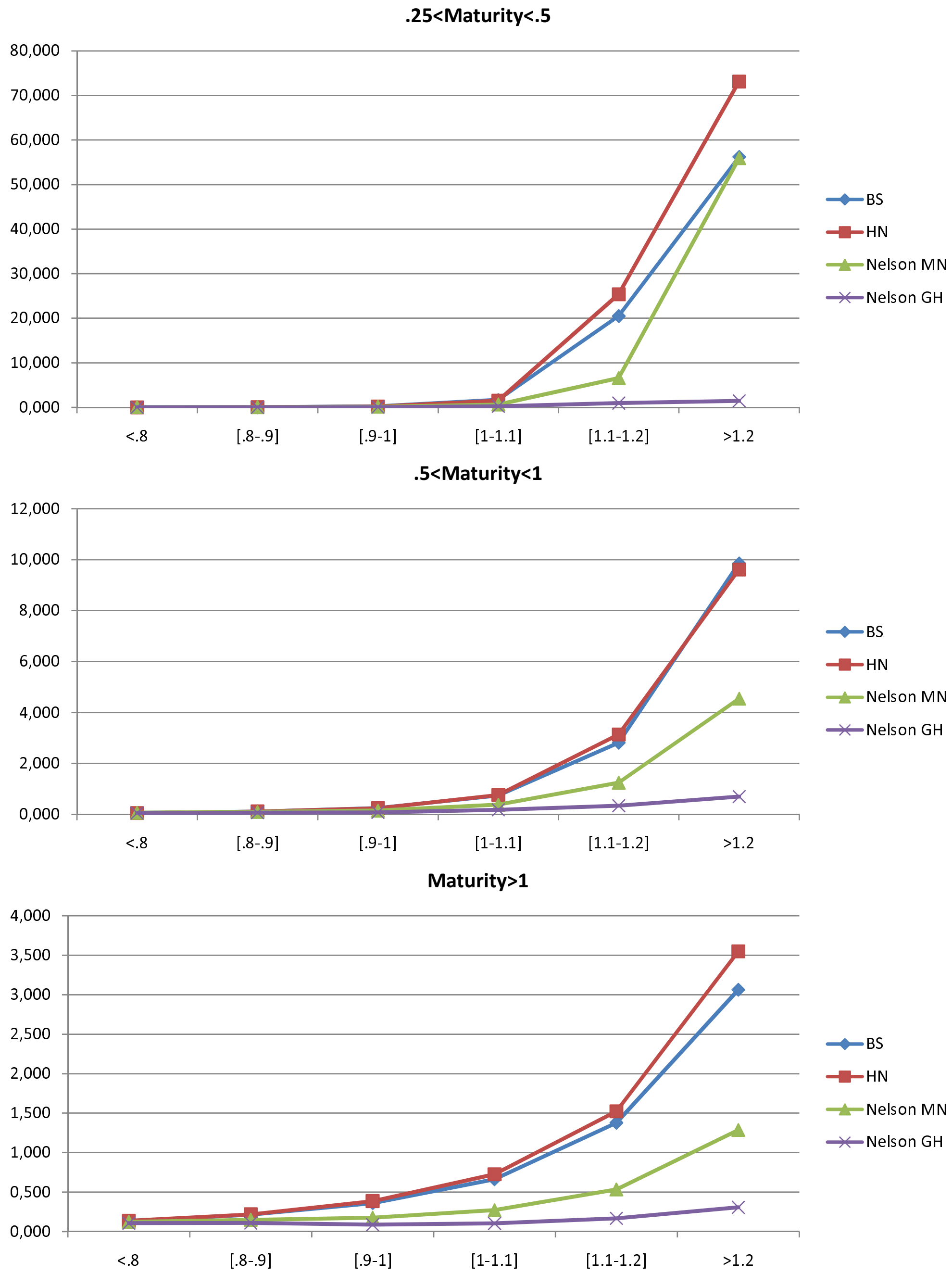

Figure 3: Absolute average pricing errors expressed in percentage for CAC 40 option prices using the exponential affine pricing kernel for different kinds of maturities.

- The EGARCH-GH model clearly outperforms the EGARCH-MN one for deep out-of-the-money option prices.

What is more, for the EGARCH-GH model, the size of the errors obtained is close to what is obtained during a calibration exercise, see for example Barone-Adesi, Engle and Mancini (2008). This result is very interesting because, in our approach, we do not perform any optimization exercise to call options: we especially focus on the direct modeling approach. In fact, starting from the historical distribution, we impose no arbitrage restrictions based on a specific choice on the shape of risk aversion and we compute after option prices, using an enhanced Monte Carlo method. We must underline that our approach is much less time consuming since it avoids the Monte Carlo based optimization. The key idea here is that a successful option pricing approach should focus on the tail behavior of the returns time series, under the historical distribution. The best competing models are the one based on the choice of a realistic distribution for the stock market returns especially the GH one.

5 Conclusion

In this paper, we presented a new model based on two assumptions. First, the pricing kernel is an exponential affine function of the log of the future value of the underlying asset. Second, returns in this economy are conditionally Generalized Hyperbolic distributed. Using these assumptions, we show how to price options and compare the empirical performances of our model with the one found in the existing literature. The performances of the model are found to be close to those found when performing a calibration exercise. Finally, we find that what seems to really matter for option pricing is the degree of fit in the tails of the historical distribution, which is a very interesting message for the econometrics of asset pricing framework.

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